Effect of ambiguities in loop cosmology on primordial power spectrum

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(Feb 5, 2019)

Based on work with Bao-Fei Li and Anzhong Wang (arXiv:1912.08225)

Outline

- Introduction
- Ambiguities in background dynamics resulting from different regularizations of Hamiltonian constraint – modified loop quantum cosmologies: mLQC-I, mLQC-II
- Ambiguity in choice of momentum of the scale factor in the Hamiltonian for scalar perturbations
- Comparison of primordial power spectrum in different models with initial conditions imposed in the contracting branch
- Summary

Goal: What is the effect of different ambiguities on the primordial scalar/tensor power spectrum in different regimes?

Caveat: Assuming validity of dressed metric approach as understood in LQC so far for modified LQC models.

Introduction

Inflationary paradigm resolves several puzzles in the standard cosmological model. Provides a framework to explain the formation of large scale cosmic structure in the universe.

But, inflation is past incomplete **(Borde, Guth, Vilenkin (03))**. Quantum gravity expected to resolve big bang singularity and provide a Planck scale extension of the inflationary paradigm.

Such a non-singular extension exists in LQC **(Agullo, Ashtekar, Nelson (12))**. Existence of attractors for isotropic as well as anisotropic LQC **(Barrau, Gupt, Linsefors, Martineau, PS, Ranken, Schander, Vandersloot, Vereshchagin (07-16))**. Inflation natural in LQC **(Ashtekar, Corichi, Karami, Sloan (10-13))**.

QG effects not washed out by inflation. Non Bunch-Davies vacuum states at onset of inflation result in stimulated particle creation leading to departures from GR **(Agullo, Navarro-Salas, Parker (11))**

Non-trivial pre-inflationary dynamics potentially provides a window to observe quantum gravity effects in CMB.

LQC and perturbations

In the last decade, cosmological perturbations in LQC explored using different approaches (Agullo, Ashtekar, Barrau, Bojowald, Bolliet, Bonga, Cleaver, De Blas, F-Mendez, Gomar, Grain, Gupt, Hossain, Kagan, Kirsten, Li, Marugan, M-Benito, F-Mendez, Mielczarek, Morris, Nelson, Olmedo, Shankaranarayanan, Sheng, Sreenath, Vidotto, Wang, Wilson-Ewing, Zhu, ... (08-..))

Dressed metric (Agullo, Ashtekar, Nelson (13)) and the hybrid approach (Gomar, M-Benito, F-Mendez, Marugan, Olemdo (13)) have gained most attention recently. Both utilize Fock quantized perturbations over loop quantized background. Qualitative results in agreement.

Quite non-trivial that results agree with standard cosmology while providing a window to test LQC effects. Many interesting novel and robust results, which include:

- Suppression of power for $\ell \leq 30$ for a choice of initial states (De Blas, Olmedo (16); Ashtekar, Gupt (17))
- Signatures in non-gaussianities due to bounce (Agullo, Bolliet, Sreenath (18))
- Overcoming tension in CMB data concerning power anomaly and lensing amplitude (Ashtekar, Gupt, Jeong, Sreenath (20))

Dressed metric approach (Agullo, Ashtekar, Nelson (13))

Based on QFT on quantum spacetimes **(Ashtekar, Kaminski, Lewandowski (09))**. Quantum state of form: $\psi_0 \otimes \psi_1$ with $-i\hbar\partial_\phi \psi_0(v,\phi) = \hat{H}_0 \psi_0(v,\phi)$ and $\psi_1(Q,T,\phi)$ is such that backreaction of perturbations on background is negligible (test-field approximation).

Dynamics of $\psi_1(Q,T,\phi)$ is completely equivalent to that of a state evolving on an metric "dressed" with QG corrections

$$\tilde{g}_{ab} dx^a dx^b = \tilde{a}\left(-d\tilde{\eta}^2 + dx_i dx^i\right)$$

where

$$\tilde{a}^4 = \frac{\langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle}{\langle \hat{H}_0^{-1} \rangle}, \quad d\tilde{\eta} = \left(\langle \hat{H}_0^{-1} \rangle \langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle \right)^{1/2} d\phi$$

Expectation values computed using sharply peaked ψ_0 states.

Assumptions in our analysis: We consider sharply peaked states on effective geometry. We assume implications from subtle infrared issue in LQC **(Kaminski, Kolanowski, Lewandowski (19))** can be successfully addressed. Assume no backreaction effects **(Gomar, M-Benito, Marugan (15); Schander and Thiemann (19))**

Hamiltonian for cosmological perturbations

Dressed metric approach (as well as hybrid approach) utilizes canonical formulation introduced by Langlois (94).

Massive scalar field ϕ with potential $V(\phi) = \frac{1}{2}m^2\phi^2$ minimally coupled to gravity in spatially flat FLRW spacetime.

Hamiltonians (using Mukhanov-Sasaki variable Q and its momentum P):

$$H_S = \frac{N(t)}{2} \int d^3k \left(\frac{P_s^2}{a^3} + a\left(\Omega_Q^2 + k^2\right) Q_s^2 \right)$$

with

$$\Omega_Q^2 = 3\kappa \frac{p_\phi^2}{a^4} - 18 \frac{p_\phi^4}{a^6 \pi_a^2} - 12 a V_\phi \frac{p_\phi}{\pi_a} + a^2 V_{\phi\phi}$$

and

$$H_T = N(t) \int d^3k \left(2\kappa \frac{P_t^2}{a^3} + \frac{a}{8\kappa} k^2 Q_t^2 \right)$$

Perturbations

Equation of motion (scalar perturbations):

$$\ddot{Q}_k + 3H\dot{Q}_k + \frac{k^2 + \tilde{\Omega}^2}{\tilde{a}^2} Q_k = 0$$

with $\tilde{\Omega}^2$ determined from dressed metric approach.

$$\tilde{\Omega}^2 = \frac{\langle \hat{H}_0^{-1/2} \hat{a}^2 \hat{\Omega}^2 \hat{a}^2 \hat{H}_0^{-1/2} \rangle}{\langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle}$$

In the test-field approximation using sharply peaked states, background quantities in Mukhanov-Sasaki equation can be replaced by their analogs in the effective spacetime description.

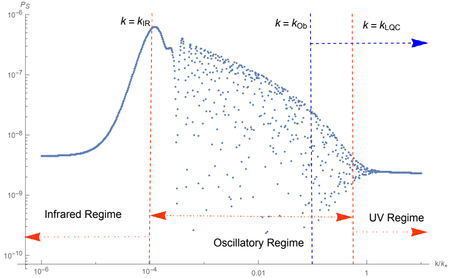
Primordial power spectrum computed at the end of inflation:

$$P_{\mathcal{R}} = \frac{k^3}{2\pi^2} \frac{|Q_k|^2}{z^2}, \quad \text{with} \quad z = \dot{\phi}/H$$

Similarly,

$$P_{\mathcal{T}} = \frac{16k^3}{\pi} |Q_k|^2$$

Primordial scalar power spectrum: three regimes



QG effects define a characteristic scale $k_{\mathrm{LQC}} = \sqrt{\frac{a''}{a}} \approx 3.20$

Smallest interesting k value determined by ϕ_B: $k_{Ob} \approx 0.89$ for LQC (when $\phi_B = 1.15$)

Together they provide a window to capture QG effects in power spectrum. This window changes on modification to background evolution.

Regularization ambiguities in Hamiltonian constraint

In LQC, Euclidean and Lorentzian parts of the Hamiltonian constraint combined before quantization. Modified versions of LQC arise if these terms are treated distinct.

$$\mathcal{C}_{\rm grav} = \mathcal{C}^{(E)}_{\rm grav} - (1+\gamma^2)\mathcal{C}^{(L)}_{\rm grav}$$

where

$$\mathcal{C}^{(E)}_{\rm grav} = \frac{1}{2}\int \mathrm{d}^3x\, \epsilon_{ijk} F^i_{ab} \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}}$$

and

$$\mathcal{C}^{(L)}_{\rm grav} = \int \mathrm{d}^3x\, K^j_{[a} K^k_{b]} \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}}$$

- mLQC-I: (Yang, Ding, Ma (09); Dapor, Liegener (17))
  Quantize $\mathcal{C}^{(L)}_{\rm grav}$ using identities on classical phase space and expressing in terms of holonomies.
- mLQC-II: (Yang, Ding, Ma (09))
  Use $K^i_a = \gamma^{-1}A^i_a$ in $\mathcal{C}^{(L)}_{\rm grav}$, and then quantize.

mLQC-I:

$$\mathcal{H}_{\mathrm{mLQC-I}} = \frac{3v}{8\pi G\lambda^2}\left\{\sin^2(\lambda b) - \frac{(\gamma^2+1)\sin^2(2\lambda b)}{4\gamma^2}\right\} + \mathcal{H}_M, \quad \lambda^2 = 4\sqrt{3}\pi\gamma\ell_{\mathrm{Pl}}^2$$

Two branches for b. Switch from one to another at bounce.

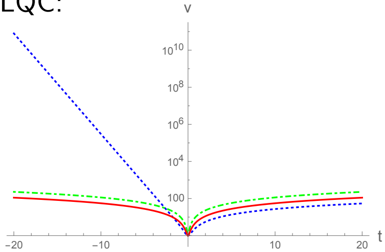
Bounce density $\rho_c^{\mathrm{I}} = \frac{\rho_c}{4(\gamma^2+1)}$ with $\rho_c = \frac{3}{8\pi G\lambda^2\gamma^2}$

mLQC-II:

$$\mathcal{H}_{\mathrm{mLQC-II}} = -\frac{3v}{2\pi G\lambda^2\gamma^2}\sin^2\left(\frac{\lambda b}{2}\right)\left\{1+\gamma^2\sin^2\left(\frac{\lambda b}{2}\right)\right\} + \mathcal{H}_M$$

Bounce at $\rho_c^{\mathrm{II}} = 4(\gamma^2+1)\rho_c$

Comparison with LQC:



Modified loop quantum cosmology: main results

- Unlike LQC where the difference equation is of second order, quantum Hamiltonian constraint of mLQC-I and mLQC-II is a difference equation of fourth-order. Consistency of infra-red behavior checked using von-Neumann stability analysis **(Saini, PS (19))**
- mLQC-I results in an asymmetric bounce. Universe has a Planck curvature in pre-bounce branch with an emergent cosmological constant for $\bar{\mu}$-scheme **(Assanioussi, Dapor, Liegener, Pawlowski (18))**. Rescaled Newton's constant **(Li, PS, Wang (18))**. Emergent matter depends on the way area of the loops are assigned. String gas type emergent matter for μ_0-scheme **(Liegener, PS (19))**.
- mLQC-II results in a symmetric bounce with a classical pre-bounce regime as in LQC but with a different bounce density.
- Modified Friedmann dynamics far more non-trivial than LQC. Higher order corrections than ρ^2 in LQC **(Li, PS, Wang (18))**.
- Inflationary attractors found for various potentials. Probability for inflation to occur extremely large **(Li, PS, Wang (19))**.
- Effects in primordial scalar power spectrum studied for mLQC-I **(Agullo (18))**.
- Generic resolution of strong curvature singularities **(Saini, PS (18))**

Initial conditions for different models

If initial conditions of the background are given at the bounce then the only free parameter is ϕ_B. For existence of a non-trivial window to capture QG effects, choose ϕ_B such that $k_{\mathrm{LQC}} > k_{Ob}$ (similarly for mLQC-I and mLQC-II).

For all models, we require 72 e-folds during inflation. This determines ϕ_B in different models.

In LQC: $\phi_B = 1.15 m_{\mathrm{Pl}}$ $(k_{\mathrm{LQC}} = 3.20; k_{Ob} \approx 0.89)$
In mLQC-I $\phi_B = 1.27 m_{\mathrm{Pl}}$ $(k_{\mathrm{mLQC-I}} = 1.60; k_{Ob} \approx 0.65)$
In mLQC-II $\phi_B = 1.04 m_{\mathrm{Pl}}$ $(k_{\mathrm{mLQC-II}} = 6.40; k_{Ob} \approx 1.11)$

Evolve universe backward from bounce at ϕ_B and specify initial conditions for perturbations as 4th-order adiabatic states at $t \approx -1.1 \times 10^5\ t_{\mathrm{Pl}}$ for LQC and mLQC-II.

Pre-bounce regime in mLQC-I mimics deSitter evolution. Choose Bunch-Davies vacuum as initial state, as in **(Agullo (18))**.

Ambiguity in treatment of π_a

Subtlety: Ω_Q^2 contains inverse powers of π_a.

Classically $\pi_a = -6a\dot{a}/\kappa N$. If used in LQC it vanishes at bounce making Ω_Q^2 singular.

Strategy: **(Agullo, Ashtekar, Nelson (13))**
Vanishing of the background classical Hamiltonian constraint

$$\mathcal{H}^{(0)} = -\frac{\kappa\pi_a^2}{12a} + \frac{p_\phi^2}{2a^3} + a^3 V \approx 0$$

implies $1/\pi_a^2 = \kappa/(12a^4\rho)$. Leads to

$$\Omega_+^2 = a^2\left(V_{\phi\phi} + 2fV_\phi + f^2V\right), \quad \text{where} \quad f = \sqrt{24\pi G/\rho}\,\dot{\phi}$$

with a and ρ determined from effective dynamics.

Above expression true only for expanding branch where π_a is negative. **(Zhu et al (18); Navascues, de Blas, Marugan (18))**

Another strategy: Use solution of effective dynamics. Differences appeared significant only in infra-red regime **(Agullo, Bolliet, Sreenath(18))**

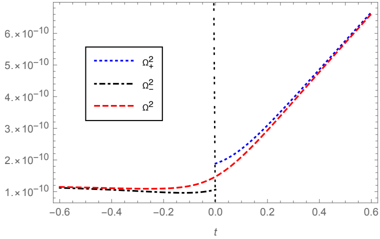
In the contracting branch,

$$\Omega_{-}^{2}=a^{2}\left(V_{\phi\phi}-2fV_{\phi}+f^{2}V\right)$$

Discontinuity in Ω_Q^2 at the bounce.

Inspired by a strategy used in the hybrid approach, we can assume a smooth extension

$$\Omega^{2}=a^{2}\left(V_{\phi\phi}+2\cos\left(\lambda b\right)fV_{\phi}+f^{2}V\right)$$



Ambiguity in treatment of π_a

Strategy used in Hybrid approach: **(Gomar, Mendez, Marugan, Olmedo (15))**

$$\frac{1}{\pi_a^2} \rightarrow \frac{16\pi^2 G^2 \gamma^2 \lambda^2}{9a^4 \sin^2(\lambda b)}$$

This replacement in classical background Hamiltonian constraint, leads to the effective background Hamiltonian constraint of LQC.

For $1/\pi_a$ term in Ω_Q^2:

$$\frac{1}{\pi_a} \rightarrow \frac{-4\pi G \gamma \lambda \cos(\lambda b)}{3a^2 \sin(\lambda b)}$$

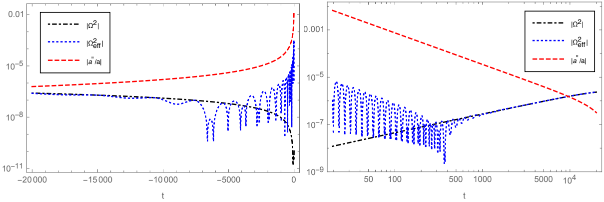
This choice takes into account different signs of π_a across the bounce. Using these replacements one obtains a candidate expression of Ω_Q^2 in effective dynamics (Ω_{eff}^2).

Similar constructions can be made for mLQC-I and mLQC-II using their effective Hamiltonians.

Effect of Ω^2 vs Ω^2_{eff}

Equation of motion can be rewritten as (for $\Omega_o^2 = (\Omega^2; \Omega^2_{\text{eff}})$)

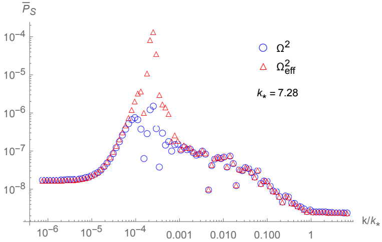
$$\nu_k'' + (k^2 + s)\nu_k = 0, \quad \text{with} \quad \nu = aQ, \quad s = \Omega_o^2 - \frac{a''}{a}$$

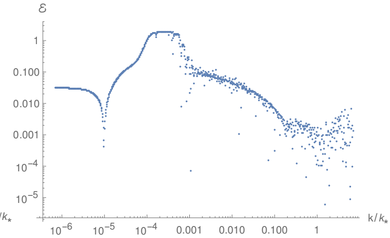


Ω^2_{eff} is 10^6 times greater than Ω^2 at bounce. Curvature term dominates both except near $t \sim 10^4 t_{\text{Pl}}$. At late times it is 100 times larger than Ω_o^2 term.

Choice of Ω^2 vs Ω^2_{eff} naively does not seem to matter for power spectrum.

Similar conclusions about Ω^2 vs Ω^2_{eff} for mLQC-I and mLQC-II.





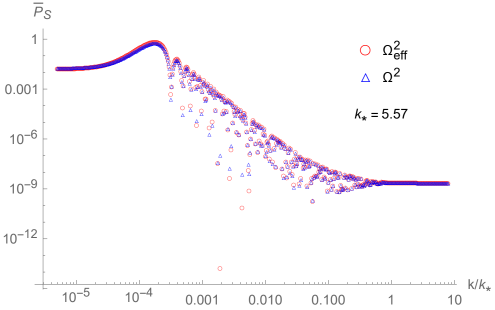
$\bar{P}_s$ computed as average taken in range $k \in (5 \times 10^{-6}, 50)$ over small bins with 5 wavenumbers each.

Relative difference $\mathcal{E} = 2|P_s - P_s'|/|P_s + P_s'|$

Relative difference quite significant in interval $k/k_* \in (10^{-4}, 10^{-3})$.

Relative difference less than 10% in IR regime and about 0.1% in UV regime.

Primordial scalar power spectrum in mLQC-I



For $\Omega^2_{\rm eff}$ we use

$$\frac{1}{\pi_a^2} \to \frac{64\pi^2 G^2 \lambda^2 \gamma^2}{9a^4 \left[(1+\gamma^2)\sin^2(2\lambda b) - 4\gamma^2 \sin^2(\lambda b) \right]}$$

and

$$\frac{1}{\pi_a} \to -\frac{8\pi G \lambda \gamma \left(1 - 2(\gamma^2+1)\sin^2(\lambda b)\right)}{3a^2 \sqrt{(1+\gamma^2)\sin^2(2\lambda b) - 4\gamma^2 \sin^2(\lambda b)}}$$

Also,

$$\Omega^2 = a^2 \left(V_{\phi\phi} + 2\left(1 - 2(\gamma^2+1)\sin^2(\lambda b)\right) f V_\phi + f^2 V \right)$$

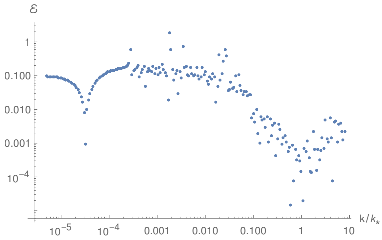
Primordial scalar power spectrum in mLQC-I

Large amplification in power in IR regime compared to LQC and mLQC-II. This is due to Planck scale emergent cosmological constant in pre-bounce epoch.

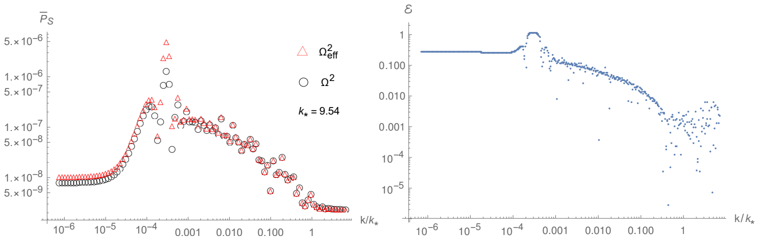
Faster growth for amplitude in intermediate regime.

Relative difference in amplitude of power spectrum for Ω^2 vs $\Omega^2_{\rm eff}$ varies significantly in IR and intermediate regime. Maximum relative difference is 10% in IR, and greater than 100% in intermediate regimes (as in LQC).

Smaller than 0.1% difference in UV regime.



Primordial power spectrum in mLQC-II



With

$$\Omega^2 = a^2\left(V_{\phi\phi} + 2\cos(\lambda b/2) f V_\phi + f^2 V\right)$$

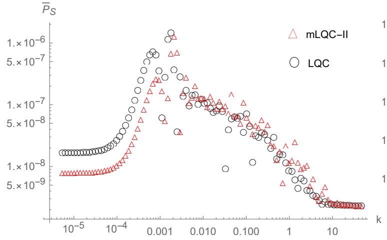
Ω^2_{eff} obtained using:

$$\frac{1}{\pi_a^2} \to \frac{4\pi^2\gamma^2\lambda^2}{9a^4\sin^2(\lambda b/2)\left(1+\gamma^2\sin^2(\lambda b/2)\right)}$$

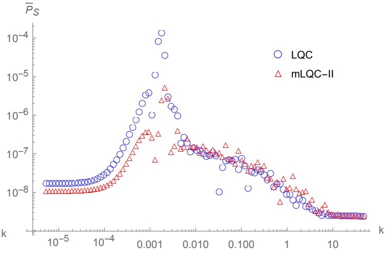
$$\frac{1}{\pi_a} \to \frac{-2\pi\gamma\lambda\cos(\lambda b/2)}{3a^2\sin(\lambda b/2)\sqrt{\left(1+\gamma^2\sin^2(\lambda b/2)\right)}}$$

Relative difference in magnitude of power spectrum can be quite large in intermediate regime. Is 30% in IR regime. And less than 0.1% in UV regime.

Comparison of scalar power spectrum for LQC and mLQC-II



(with Ω^2)



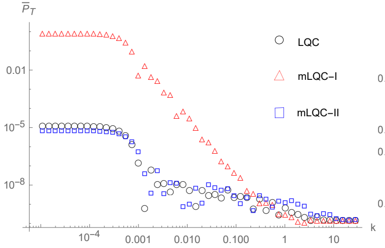
(with Ω^2_{eff})

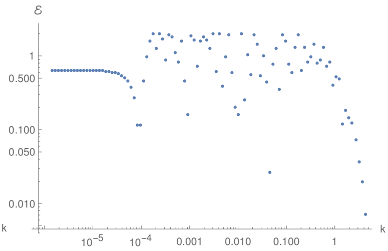
Relative difference between LQC and mLQC-II larger in IR regime for Ω^2 (greater than 60%) than Ω^2_{eff} (less than 40%).

Significant difference at the boundary of IR and intermediate regimes, especially for Ω^2_{eff}.

Relative difference varies between 1-100% in the intermediate regime. In the UV regime the difference is less than 1%.

Comparison of tensor power spectrum for LQC, mLQC-I and mLQC-II





Amplitude in mLQC-I and its growth in intermediate regime significantly different from LQC and mLQC-II.

Relative difference between amplitude of tensor power spectrum for LQC and mLQC-II larger than 50% in IR regime.

Relative difference varies widely in intermediate regime, and is less than 1% in IR regime.

Summary and conclusions

- Different treatments of the Lorentzian term in the Hamiltonian constraint, result in non-trivial changes in physics of bounce and pre-bounce. Still, mLQC-II bears qualitative similarity with LQC. However, mLQC-I results in an asymmetric bounce with a pre-bounce universe with Planckian curvature.
- Treatment of π_a in the scalar perturbation Hamiltonian has been performed in different ways. We used approaches used widely in dressed metric as well as one inspired from hybrid approach.
- Results between LQC, mLQC-I and mLQC-II agree in UV regime (contributing most to observable modes) where ambiguity in π_a is also of little significance.

Qualitative predictions for observable modes for linear perturbations in dressed metric approach robust to considered Hamiltonian regularization and π_a ambiguities.

Summary and conclusions

- However, these ambiguities do leave a significant trace in IR and intermediate regimes.
- Relative difference between LQC and mLQC-II can be as large as 50-100% in IR and in intermediate regimes depending on choice of π_a.
- mLQC-I in IR and intermediate regime leaves qualitatively different signatures with a very large amplitude and its rapid growth of amplitude for scalar as well as tensor power spectrum.
- Different choices of π_a generally result in at least 10% relative difference in IR and intermediate regimes for the same model.

Do these differences due to ambiguities translate to qualitatively different observable signatures, such as in non-gaussianities?