Modified Loop Quantum Cosmologies

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Based on: PRD **97** (2018) 084029 and PRD **98** (2018) 066016 (with B-F. Li and A. Wang),

1812.08937 and 1901.01279 (with S. Saini)

Outline

- Modified Loop Quantum Cosmology (mLQC): origins
- Modified Friedmann dynamics in mLQC-I and mLQC-II
- Infra red behavior of quantum Hamiltonian constraints in mLQC-I and mLQC-II. Ruling out μ_o scheme in mLQC-I and mLQC-II using von Neumann stability analysis
- Inflationary attractors and qualitative dynamics
- Generic resolution of strong singularities and a phenomenological demonstration for exotic singularities

Goals:

Do some key results of LQC survive in mLQC-I and mLQC-II?

How different qualitatively is Planck scale physics of these loop cosmologies?

Introduction

LQC based on following quantization procedure in LQG to quantize homogeneous spacetimes after classical symmetry reduction.

Various quantization ambiguities can arise in the process of obtaining the quantum Hamiltonian constraint.

Standard LQC only one such quantization. Many robustness and consistency tests passed. With in LQC, restrictions on quantization ambiguities understood (eg. μ_o vs. $\bar{\mu}$ scheme).

Some key results:

- Second order non-singular quantum difference equation.
- Exactly solvable for spatially flat universe with a massless scalar.
- Big bang replaced by a bounce at a universal values of spacetime curvature. Bounce symmetric in isotropic universe.
- Modified effective Friedmann-Raichaudhuri equations with ρ^2 correction.
- Naturalness of inflation. Phenomenological predictions for CMB.
- Geodesic completeness and generic resolution of strong singularities.
- Bounce in presence of anisotropies and Fock quantized inhomogeneities.

(Agullo, Alesci, Ashtekar, Barrau, Brahma, Bojowald, Cartin, Corichi, Craig, Date, Dapor, Diener, Engle, Grain, Gupt, Henderson, Hossain, Joe, Kaminski, Karami, Li, Ma, Martin-Benito, Mena-Marugan, Mielczarek, Montoya, Nelson, Olmedo, Saini, Sloan, Szulc, Taveras, Thiemann, Pawlowski, Perez, PS, Vandersloot, Wang, Wilson-Ewing, ... (01–..))

Modifications of standard LQC

Hamiltonian constraint composed of Euclidean and Lorentzian terms:

$$\mathcal{C}_{\rm grav} = \mathcal{C}_{\rm grav}^{(E)} - (1+\gamma^2)\mathcal{C}_{\rm grav}^{(L)}$$

where

$$\mathcal{C}_{\rm grav}^{(E)} = \frac{1}{2}\int \mathrm{d}^3x\, \epsilon_{ijk} F^i_{ab} \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}}$$

and

$$\mathcal{C}_{\rm grav}^{(L)} = \int \mathrm{d}^3x\, K^j_{[a} K^k_{b]} \frac{E^{aj}E^{bk}}{\sqrt{\det(q)}}$$

In LQC, quantization of spatially flat models obtained after combining $\mathcal{C}_{\rm grav}^{(E)}$ and $\mathcal{C}_{\rm grav}^{(L)}$. If terms are treated distinct, then form of quantum Hamiltonian constraint significantly different.

Two ambiguities at this level:

- Quantize $\mathcal{C}_{\rm grav}^{(L)}$ as above after using identities on classical phase space and expressing in terms of holonomies. Leads to mLQC-I.
- Use $K^i_a = \gamma^{-1} A^i_a$ in $\mathcal{C}_{\rm grav}^{(L)}$, and then quantize. Results in mLQC-II.

mLQC-I and mLQC-II (timeline)

- First considered by Bojowald (02). Obtained non-singular fourth order difference equation for μ_o type quantization for mLQC-I.
- Forgotten till 2009. Thought to be bringing little change to LQC.
- Analyzed by Yang, Ding, Ma (09) for $\bar{\mu}$ scheme, for mLQC-I and mLQC-II. Symmetric bounce claimed for both mLQC-I and mLQC-II. Modified Friedmann dynamics unknown. Forgotten again till 2017.
- mLQC-I re-discovered while understanding cosmological sector of LQG by Dapor, Liegener (17). Asymmetric bounce in contrast to results of Yang, Ding, Ma (09)! Emergent cosmological constant in pre-bounce branch. Modified Friedmann dynamics unknown.
- Quantization of mLQC-I following LQC with a massless scalar field for $\bar{\mu}$ scheme (Assanioussi, Dapor, Liegener, Pawlowski (18)).
- Modified Friedmann dynamics with higher order corrections than ρ^2 in LQC, and subtle behavior of solutions explaining discrepancy in claims found (Li, PS, Wang (18)). Asymmetric (symmetric) bounce in mLQC-I (mLQC-II). Emergent Planckian cosmological constant and rescaled Newton's constant in pre-bounce regime for mLQC-I.
- Inflationary models (Li, PS, Wang (18-..)), CMB (Agullo (18)), generic singularity resolution (Saini, PS (18)), Von Neumann stability (Saini, PS (19))

Quantum Hamiltonian constraint in mLQC-I

(Yang, Ding, Ma (09); Saini, PS (19))
For massless scalar as matter in $\bar{\mu}$ scheme (recall: $\bar{\mu}^2 = (4\sqrt{3}\pi\gamma\ell_{\rm Pl}^2)/|p|$, arises from taking physical metric into account for minimum areas of loops in field strength **(Ashtekar, Pawlowski, PS (06))**), quantum Hamiltonian constraint turns out to be:

$$\begin{aligned}\frac{C_2}{2}|v|\Big||v+1|^{1/3}-|v-1|^{1/3}\Big|^3\partial_\phi^2|v\rangle \quad &= \quad F'_+(v)|v+8\rangle + f'_+(v)|v+4\rangle \\ &\quad +(F'_o(v)+f'_o(v))|v\rangle \\ &\quad +f'_-(v)|v-4\rangle + F'_-(v)|v-8\rangle,\end{aligned}$$

with

$$\begin{aligned}
f'_+(v) \quad &= \quad -C_3(v+2)(|v+3|-|v+1|), f'_-(v) = f'_+(v-4), f'_o(v) = -f'_+(v) - f'_-(v), \\
F'_+(v) \quad &= \quad \frac{4\kappa^2 C_1}{\gamma^4}[M_v(1,5)f'_+(v+1) - M_v(-1,3)f'_+(v-1)](v+4)M_v(3,5)[M_v(5,9)f'_+(v+5) - M_v(3,7)f'_+(v+3)], \\
F'_+(v) \quad &= \quad \frac{4\kappa^2 C_1}{\gamma^4}[M_v(1,-3)f'_-(v+1) - M_v(-1,-5)f'_-(v-1)](v-4)M_v(-5,-3)[M_v(-3,-7)f'_-(v-3) - M_v(-5,-9)f'_-(v-5)], \\
F'_o(v) \quad &= \quad \frac{4\kappa^2 C_1}{\gamma^4}[M_v(1,5)f'_+(v+1) - M_v(-1,3)f'_+(v-1)](v+4)M_v(3,5)[M_v(5,1)f'_-(v+5) - M_v(3,-1)f'_-(v+3)] \\
&\quad + \frac{4\kappa^2 C_1}{\gamma^4}[M_v(1,-3)f'_-(v+1) - M_v(-1,-5)f'_-(v-1)](v-4)M_v(-5,-3)[M_v(-3,1)f'_+(v-3) - M_v(-5,-1)f'_+(v-5)]
\end{aligned}$$

$$M_v(a,b) := |v+a| - |v+b|,$$

$$C_1 = 2(1+\gamma^2)\frac{\sqrt{6}\gamma^{\frac{3}{2}}}{2^8 3^3 \kappa^{\frac{3}{2}}\hbar^{\frac{1}{2}}\alpha}, C_2 = \left(\frac{3}{2}\right)^3\left(\frac{6}{\kappa\hbar\gamma}\right)^{3/2}\alpha, C_3 = \frac{\gamma^2}{2\kappa}\frac{27}{16}\left(\frac{8\pi}{6}\right)^{1/2}\frac{\alpha l_{\rm Pl}}{\gamma^{\frac{3}{2}}}, \quad \alpha := 2/3(3\times 3^{1/2})^{1/2}$$

Quantum Hamiltonian constraint in mLQC-II

(Yang, Ding, Ma (09); Saini , PS (19))

For massless scalar as matter in $\bar{\mu}$ scheme, quantum Hamiltonian constraint is:

$$\begin{aligned} \frac{C_2}{2}|v|\Big||v+1|^{1/3}-|v-1|^{1/3}\Big|^3\partial_\phi^2|v\rangle &= f'_+(v)|v+4\rangle + S'_+(v)|v+2\rangle \\ &+(f'_o(v)+S'_o(v))|v\rangle \\ &+S'_-(v)|v-2\rangle + f'_-(v)|v-4\rangle \end{aligned}$$

with

$$S'_+(v) = 4\frac{(1+\gamma^2)}{\gamma^2}C_3(v{+}1)(|v{+}2|{-}|v|), S'_-(v) = S'_+(v{-}2), S'_o(v) = -S'_+(v){-}S'_-(v)$$

- Unlike LQC, difference equation in mLQC-I and mLQC-II is a fourth order difference equation.
- Similar structure of quantum Hamiltonian constraint when one uses μ_o scheme, but with very different properties.

Effective Hamiltonian for mLQC-I

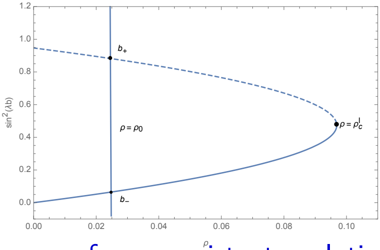
Assume one can obtain effective Hamiltonian as in LQC using coherent states for $\bar{\mu}$ scheme **(eg. Taveras (08), Liegener (to appear))**.

Assume validity of effective dynamics **(Diener, Gupt, Joe, Megevand, PS (14-..))**

Effective Hamiltonian:

$$\mathcal{H} = \frac{3v}{8\pi G\lambda^2}\left\{\sin^2(\lambda b) - \frac{(\gamma^2+1)\sin^2(2\lambda b)}{4\gamma^2}\right\} + \mathcal{H}_M, \quad \lambda^2 = 4\sqrt{3}\pi\gamma\ell_{\rm Pl}^2$$

$$\mathcal{C_H}^{\rm eff} \approx 0 \Rightarrow \sin^2(\lambda b_\pm) = \frac{1 \pm \sqrt{1-\rho/\rho_c^{\rm I}}}{2(\gamma^2+1)}, \quad \rho_c^{\rm I} \equiv \frac{3}{32\pi\lambda^2\gamma^2(\gamma^2+1)G} = \frac{\rho_c}{4(\gamma^2+1)}$$



Both branches necessary for a consistent evolution across the bounce. Switch from b_- to b_+ at the bounce.

Modified Friedmann equations for mLQC-I

For b_- branch (post-bounce):

$$H^2 = \frac{8\pi G\rho}{3}\left(1 - \frac{\rho}{\rho_c^{\mathrm{I}}}\right)\left[1 + \frac{\gamma^2}{\gamma^2+1}\left(\frac{\sqrt{\rho/\rho_c^{\mathrm{I}}}}{1+\sqrt{1-\rho/\rho_c^{\mathrm{I}}}}\right)^2\right],$$

For b_+ branch (pre-bounce):

$$H^2 = \frac{8\pi G\beta\rho_\Lambda}{3}\left(1 - \frac{\rho}{\rho_c^{\mathrm{I}}}\right)\left[1 + \left(\frac{1-2\gamma^2+\sqrt{1-\rho/\rho_c^{\mathrm{I}}}}{4\gamma^2\left(1+\sqrt{1-\rho/\rho_c^{\mathrm{I}}}\right)}\right)\frac{\rho}{\rho_c^{\mathrm{I}}}\right],$$

$$\beta := \frac{1-5\gamma^2}{1+\gamma^2}, \rho_\Lambda := \frac{3}{8\pi G\beta\lambda^2(1+\gamma^2)^2}$$

At very early times in pre-bounce regime:

$$H^2 \approx \frac{8\pi G_\beta}{3}\left(\rho + \rho_\Lambda\right), G_\beta = \beta G$$

The pre-bounce phase asymptotes to a branch with a Planckian cosmological constant and rescaled Newton's constant. Important to use different sets of Friedmann-Raichaudhuri equations before and after bounce. Else one finds an unphysical symmetric bounce.

Effective Hamiltonian for mLQC-II

$$\mathcal{H} = -\frac{3v}{2\pi G\lambda^2\gamma^2}\sin^2\left(\frac{\lambda b}{2}\right)\left\{1+\gamma^2\sin^2\left(\frac{\lambda b}{2}\right)\right\}+\mathcal{H}_M$$

Vanishing of Hamiltonian constraint yields

$$\sin^2(\lambda b_\pm/2) = \frac{-1\pm\sqrt{1+\gamma^2\rho/\rho_c}}{2\gamma^2}$$

Unlike mLQC-I, only one physical branch possible. The b_+ branch covers the entire evolution across the bounce.

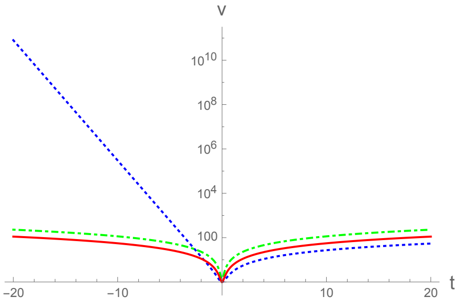
Modified Friedmann equation:

$$H^2 = \frac{8\pi G\rho}{3}\left(1+\gamma^2\frac{\rho}{\rho_c}\right)\left(1-\frac{(\gamma^2+1)\rho/\rho_c}{(1+\sqrt{\gamma^2\rho/\rho_c+1})^2}\right)$$

Bounce at $\rho_c^{\mathrm{II}} = 4(\gamma^2+1)\rho_c$.

Comparison of mLQC-I and mLQC-II with LQC

Non-trivial modifications to Friedmann dynamics for mLQC-I and mLQC-II in comparison to LQC in Planck regime.



In mLQC-I, spacetime curvature remains Planckian before the bounce yet satisfies Einstein field equations but with a quantum gravitational origin matter. Similar earlier result in a loop quantization of a Kantowski-Sachs model **(Dadhich, Joe, PS (15))**

In mLQC-II, spacetime curvature decreases quickly on both sides of the bounce as in LQC. No emergent matter or a rescaled G.

Properties of difference equations at large volumes

Using stability properties of finite difference equations we can find whether quantum Hamiltonian constraints in mLQC-I and mLQC-II for μ_o and $\bar{\mu}$ schemes are compatible with GR at large volumes.

Many insights on viability of quantizations in LQC and black holes **(Bojowald, Date (04); Date (05); Cartin, Khanna (05); Bojowald, Cartin, Khanna (07); Nelson, Sakellariadou (09); Tanaka, Amemiya, Shimano, Harada, Tamaki (11); PS (12); Yonika, Khanna, PS (18))**

Results in agreement/complement effective dynamics **(Corichi, PS (08))**

Von Neumann stability of a finite difference equation ensures that any mismatch between finite difference equation and PDE (Einstein field eqs.) remains small in time evolution to large volumes. With a typical ansatz: $|v\rangle = g^{\frac{v}{2}}\, e^{i\omega\phi}$ the problem reduces to finding growth of amplification factor g in the Fourier space.

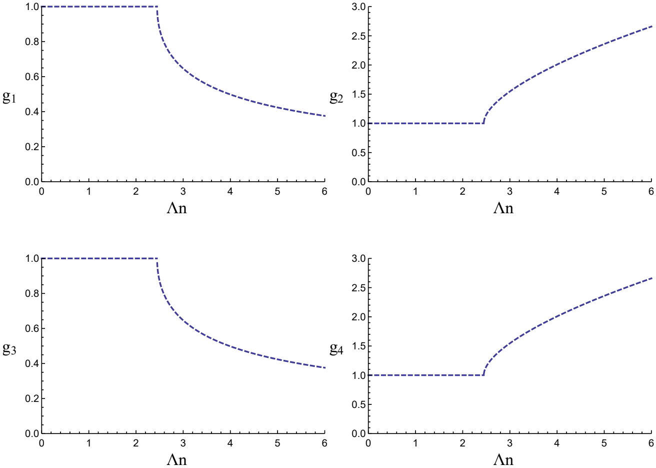
Amplification factor must be bounded by unity. If larger than unity, quantum Hamiltonian constraint has large departures from GR at small curvatures. Generally signals unphysical effects.

Von Neumann stability/instability for μ_o scheme

(Saini, PS (19))

- In LQC, μ_o scheme is von Neumann stable for massless scalar field as matter content. Two real roots for g which are bounded by unity. In presence of any $+\Lambda$ there exists a finite volume at which one of the roots becomes greater than unity.
- In mLQC-I and mLQC-II, von Neumann stability results in four roots in two separate pairs. Both pairs needed for a physical solution in mLQC-I, but only one pair needed in mLQC-II.
- Without $+\Lambda$: In mLQC-I one pair has real roots equalling unity, and the other pair has complex conjugate roots with unit magnitude. In mLQC-II, physical pair has real roots equalling unity. Unphysical pair has complex conjugate roots with magnitude greater than unity.
- With $+\Lambda$: For any value of Λ there exists a volume for which both pairs of roots in mLQC-I and mLQC-II are greater in magnitude than unity. No viable physical solution is possible.

Von Neumann instability for μ_o scheme with $+\Lambda$



mLQC-I and mLQC-II are von Neumann unstable in presence of $+\Lambda$. Do not yield GR at large volumes. Result in an unphysical recollapse of the universe at large volumes (as in μ_o LQC).

Von Neumann stability for $\bar{\mu}$ scheme

(Saini, PS (19))

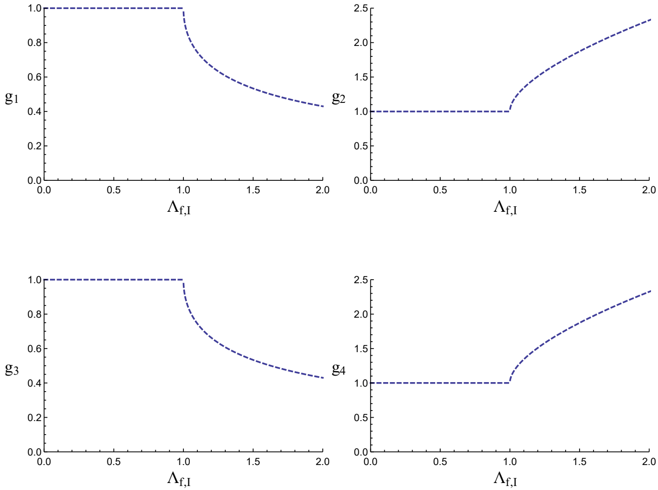
- The $\bar{\mu}$ scheme results in von Neumann stable finite difference equation in LQC for all matter with equation of state ranging from massless scalar field to $+\Lambda$. One of the roots of amplification factor becomes greater than unity if matter content chosen with energy density greater than ρ_c. (No such states in the physical Hilbert space. Corresponds to unphysical solution in effective dynamics since Hubble rate becomes imaginary).

- For mLQC-I, both pair of roots have unit magnitude with or without Λ when energy density is less than or equal to the bounce density.

- Same situation for mLQC-II, for the pair of roots corresponding to the physical solution.

- mLQC-I and mLQC-II are von Neumann stable in $\bar{\mu}$ scheme for physically allowed values of Λ.

Von Neumann stability for $\bar{\mu}$ scheme with $+\Lambda$



mLQC-I is von Neumann stable for $\bar{\mu}$ scheme in presence of $+\Lambda$ till $\Lambda_{f,\mathrm{I}} = \Lambda/\Lambda_{c,\mathrm{I}} = 1$ when $\rho = \rho_c^{\mathrm{I}}$. Same situation for mLQC-II.

Qualitative dynamical analysis of mLQC-I and mLQC-II

(Li, PS, Wang (18))

When analytical solutions are difficult to obtain, qualitative dynamical systems analysis, phase space portraits and stability of fixed points provides valuable hints on general features of dynamical evolution.

Makes easy to find inflationary attractors in effective dynamics. Found for LQC in PS, Vandersloot, Vereshchagin (06); PS, Ranken (12); Gupt, PS (13)

Phase space variables: For $m^2\phi^2$ potential, introduce X and Y:

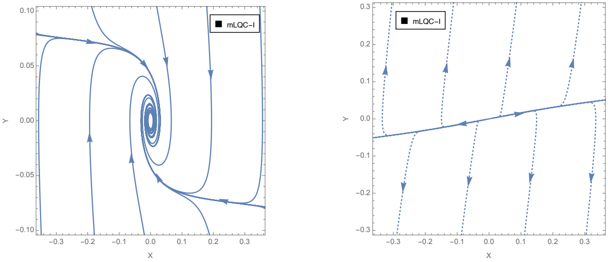
$$X = \frac{m\phi}{\sqrt{2\rho_c^i}}, \qquad Y = \frac{\dot\phi}{\sqrt{2\rho_c^i}}; \qquad X^2 + Y^2 = \rho/\rho_c^i \leq 1$$

which satisfy equations of motion:

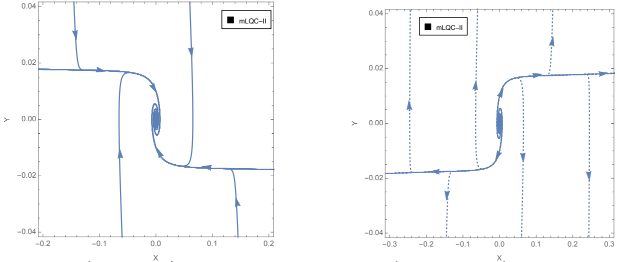
$$\dot X = mY, \qquad \dot Y = -mX - 3H^i Y$$

Find the fixed point(s) and introduce perturbations around fixed points in X and Y to determine the nature of fixed points. Extract the behavior of phase space trajectories near the fixed points.

Inflationary attractors for ϕ^2 potential



mLQC-I: Stable spiral in post-bounce. Unstable repelling node in pre-bounce branch.



mLQC-II: Stable (unstable) spiral in post-bounce (pre-bounce) branch, as in LQC.

Inflationary attractors for Starobinsky potential

In GR, Starobinsky inflation based on adding R^2 term to action, equivalent to adding $V = \frac{3m^2}{32\pi G}\left(1 - e^{-\sqrt{16\pi G/3}\phi}\right)^2$ in Einstein frame.

Assume the same potential for mLQC-I and mLQC-II, as considered earlier in LQC (Bonga, Gupt (16); Ashtekar, Gupt (16))

Asymmetric potential. Slow-roll inflation only for $\phi \geq 0$.

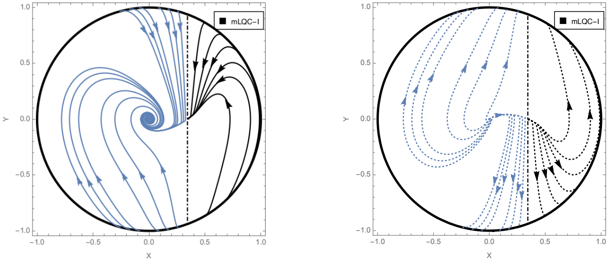
Phase space variables: $X = \sqrt{V/\rho_c^i}$, $Y = \frac{\dot{\phi}}{\sqrt{2\rho_c^i}}$

Phase space for Starobinsky potential only for $X < \chi_0$. For $X > \chi_0$, $V(\tilde{\phi}) = \frac{3m^2}{32\pi G}\left(1 + e^{-\sqrt{16\pi G/3}\tilde{\phi}}\right)^2$ with $\tilde{\phi} \equiv -\sqrt{\frac{3}{16\pi G}} \ln\left(\left|1 - \frac{X}{\chi_0}\right|\right)$.

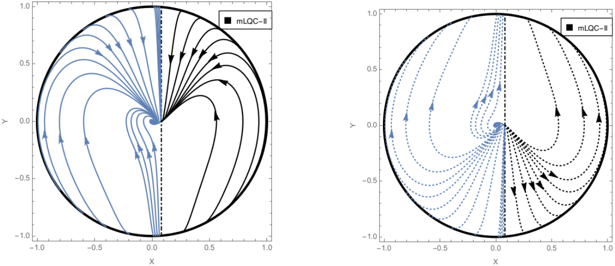
Two fixed points:

- $(X, Y) = (0, 0)$. Similar properties as in ϕ^2 inflation.
- $(X, Y) = (\chi_0, 0)$. Perturbative analysis leads to a characteristic equation with one eigenvalue vanishing. Degenerate, non-simple fixed point. Higher order analysis becomes necessary. Turns out to be unstable.

Inflationary attractors for Starobinsky potential



mLQC-I: $(X, Y) = (0, 0)$ is stable spiral in post-bounce. Unstable repelling node in pre-bounce branch for $m \leq m_{\mathrm{Pl}}$, else unstable spiral.



mLQC-II and LQC: $(X, Y) = (0, 0)$ is stable (unstable) spiral in post (pre)-bounce.

Inflationary dynamics for other potentials

(Li, PS, Wang (18))

Inflationary attractors found in addition for

- Fractional monodromy potential inspired by supergravity: $V = V_1 \left|\frac{\phi}{\phi_0}\right|^p \frac{1}{[1+(\frac{\phi_0}{\phi})^n]^{\frac{2-p}{n}}}$
- Non-minimal Higgs potential: $V = V_0 \left(1 - e^{-\sqrt{\frac{16\pi G}{3}}|\phi|}\right)^2$
- Regardless of potential, origin $(X, Y) = (0, 0)$ found to be late time attractor in LQC, mLQC-I and mLQC-II, showing naturalness of inflation after bounce.
- Exponential potential $V = V_0 e^{-\sqrt{8\pi G}q\phi}$:
  - Scaling solutions between potential and kinetic energy found.
  - Existence of a new fixed point for mLQC-I in pre-bounce phase $(X, Y) = (0, 0)$, not found in LQC and mLQC-II. $(X = \frac{\dot{\phi}}{H}\sqrt{\frac{4\pi G}{3}}, Y = \frac{1}{H}\sqrt{\frac{8\pi G V}{3}})$.
  - Absence of kinetic dominated fixed points, $(X = \pm 1, Y = 0)$ for mLQC-I in pre-bounce regime.

General features of singularity resolution

(Saini, PS (18))

- As in LQC, in mLQC-I and mLQC-II scale factor remains finite and non-zero for all finite time evolution.
- Hubble rates are universally bounded as in LQC, but $\dot{H}$ can potentially diverge. For eg. in mLQC-II:

$$\dot{H} = \frac{4\pi G(\rho + P)}{3}\left[3 + 2\gamma^2(1+\rho/\rho_c) - 3(\gamma^2+1)\sqrt{1+\gamma^2\rho/\rho_c}\right]$$

  If $P \to \pm\infty$ at a finite ρ, $\dot{H} \to \pm\infty$.
- Spacetime curvature is not generically bounded in mLQC-I and mLQC-II.
- However, it can be proved that for all such events geodesics can be extended in the effective spacetime. mLQC-I and mLQC-II spacetimes turn out to be geodesically complete.
- For any singularity at $\tau = \tau_o$, $\int_0^\tau \mathrm{d}\tau |R^i{}_{0j0}|$ computed in a parallely propagated frame along any non-spacelike geodesic is finite as $\tau \to \tau_o$. There exist no strong singularities in the effective spacetime. All potential divergences in spacetime curvature correspond to weak singularities (Saini, PS (18))

Types of exotic singularities

- Type I singularity (Big Rip): In a finite time, scale factor, energy density and pressure become infinite. All curvature invariants diverge. Strong singularity.
- Type II singularity (Sudden): At a finite value of scale factor and energy density, pressure and curvature invariants diverge. Weak singularity.
- Type III singularity (Big Freeze): At a finite value of scale factor, energy density, pressure and curvature invariants diverge. Strong singularity.
- Type IV singularity: Occurs at a finite value of scale factor. Energy density, pressure and curvature invariants finite. Time derivatives of spacetime curvature diverge. Weak singularity.

Type-I and III singularities resolved in LQC, but type-II and IV singularities not resolved **(Sami, PS, Tsujikawa (06); Samart, Gumjudpai(07); Naskar, Ward (07); Cailleteau, Cardoso, Vandersloot, Wands (08); Wu & Zhang (08); PS (09); PS, Vidotto (11))**.

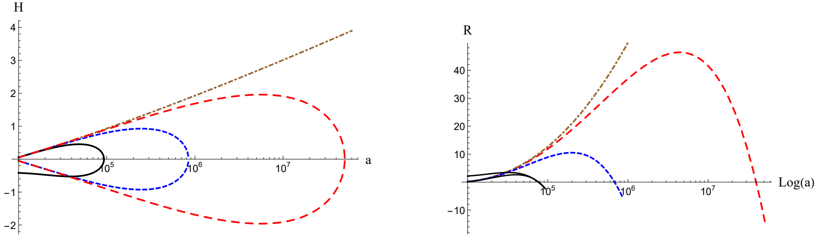
Resolution of type-I singularity

(Saini, PS (18))

Use ansatz $P = -\rho - f(\rho), \quad f(\rho) = \frac{AB\rho^{2\alpha-1}}{A\rho^{\alpha-1}+B}$ **(Nojiri, Odintsov, Tsujikawa (05))**

Type-I singularity occurs when parameters: $3/4 < \alpha < 1$, $A > 0$.

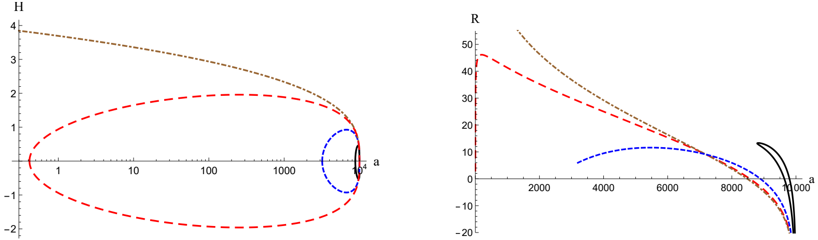
In GR, there is no big bang but only a future big rip singularity.



ρ universally bounded in LQC, mLQC-I and mLQC-II. Big rip is generically avoided and replaced by quantum recollapse. Quantum recollapse asymmetric in mLQC-I. Post recollapse universe with Planckian spacetime curvature and an emergent Λ.

Non-resolution of type-II singularity

Past big bang (or future big crunch) and a future (or past) sudden singularity in GR occurs for $A/B < 0$.

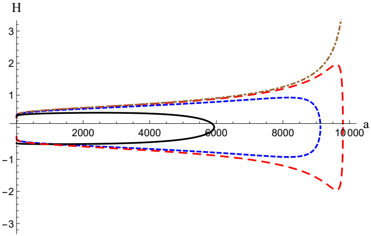


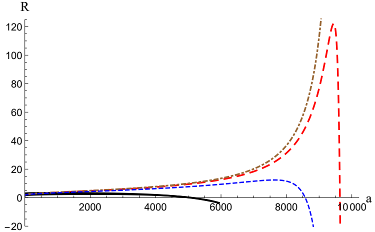
In LQC, mLQC-I and mLQC-II past big bang singularity replaced by big bounce because ρ is bounded. But, future sudden singularity not resolved.

Past unresolved sudden singularity in LQC and mLQC-II. No past-sudden singularity in mLQC-I. Universe asymptotes to a Planckian spacetime curvature phase in pre-bounce.

Resolution of type-III singularity

Occurs for $\alpha > 1$. In GR, ρ, P, and curvature invariants diverge at a finite scale factor. No big bang in past evolution.

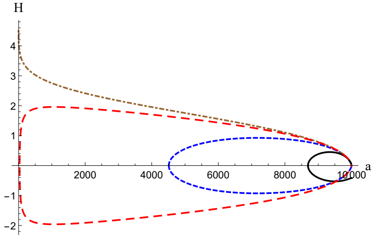


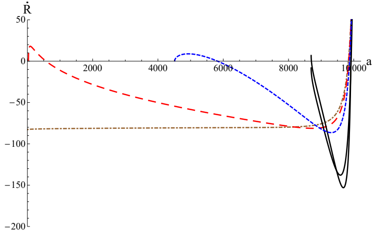


Boundedness of ρ resolves big freeze singularity. Big freeze replaced by quantum recollapse.

LQC and mLQC-II universes are symmetric in pre and post-recollpase branches. Post-recollapse universe in mLQC-I asymmetric with an emergent Λ.

Occurs for $0 < \alpha < 1/2$. Only time derivatives of curvarture invariants diverge at a finite scale factor. Since ρ is finite (and very small) at these singularities, quantum geometry effects play no role at these events.





Past big bang singularity replaced by bounce in LQC, mlQC-I and mLQC-II. But future type-IV singularity not resolved.

LQC and mLQC-II have a past type-IV singularity. Absent in mLQC-I.

Summary

- Treating Euclidean and Lorentzian terms independently during loop quantization results in non-trivial changes to the Planck scale physics in loop cosmologies.
- In mLQC-I universe has Planckian curvature before bounce. Asymmetric bounce with an emergent Λ and a rescaled G. In contrast, mLQC-II yields a symmetric bounce like in LQC.
- Both mLQC-I and mLQC-II have fourth order quantum difference equations, and higher order terms in energy density in modified Friedmann and Raichaudhuri equations.
- Inflation natural in both mLQC-I and mLQC-II. Phase space dynamics qualitatively similar for mLQC-I, mLQC-II and LQC in post-bounce. Pre-bounce similarities only for LQC and mLQC-II.
- Generic resolution of strong singularities and geodesic completeness for mLQC-I and mLQC-II.
- Key qualitative features of LQC hold in mLQC-I and mLQC-II, but quantitative differences remain to be fully explored.