### Transition times through the black hole bounce

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### Introduction

In the last decade, there have been many concrete examples of singularity resolution using loop quantum gravitational effects. Central singularities in black holes can be eliminated (Ashtekar, Bojowald

(06); Modesto (06); Boehmer, Vandersloot (07); Campiglia, Gambini, Pullin, Olmedo, Rastgoo (07-15)); Corichi, PS (16)).

• Focus so far has been on the details of singularity resolution and mathematical consistency. Details of the physics of the bounce regime and transition to white hole geometries not explored.

Independently, various directions explored to understand the implications of quantum gravity in gravitational collapse and black hole to white hole transitions (Hajiceck, Kiefer (01); Bojowald, Goswami, Maartens, PS (05); Husain, Winkler (06); Goswami, Joshi, PS (06); Ziprick, Kunstatter (10); Barcelo, Carballo-Rubio, Garay, Jannes (11-15); Barrau, Bolliet, Christodoulou, Haggard, Perez, Rovelli, Speziale, Vidotto,... (14-16) )

• Very interesting phenomenology but often with assumptions about nature of singularity resolution.

# Understanding black hole to white hole transitions

• It is generally assumed, using isotropic LQC, that bounce is symmetric. This simplifying assumption does not hold for anisotropic and black hole spacetimes. For the only known model of loop quantization of Schwarzschild interior having consistent UV and IR limits and fiducial cell independence, white hole horizon highly asymmetric compared to parent black hole. How does the asymmetric bounce regime affect the time scales of black hole to white hole transitions?

• It is assumed that there is only one quantum gravity regime in the collapse. Is it really so? Or does the anisotropic evolution implies multiple distinct regimes?

• How do the details of the quantum gravitational regime affect conclusions about the transition times?

• Is there any mass dependence in time scales? Do tiny black holes behave the same way as larger black holes? Should size matter?

#### Goal

Using a concrete quantized model of black hole interior understand details of quantum gravity regime and black hole to white hole transition. Insights on answers to above questions can help in making robust models and precise quantum gravity predictions.

Caveats: Exterior picture not available. Effective dynamics assumed (recent rigorous verification for Bianchi-I spacetime which has similar Hamiltonian structure (Diener, Joe, Megevand, PS (17)))

# Outline:

- Loop quantization of Schwarzschild interior (Corichi, PS (16))
- Details of the quantum gravitational regime
- Transition times: Bounce time(s) for crossing quantum gravity regime/transition from interior BH to WH geometry; Delivery time from parent black hole to white hole horizon
- Mass (in)dependence and some implications
- Summary

### Schwarzschild interior

The interior can be described by a Kantowski-Sachs cosmological spacetime without matter. The spatial manifold has topology  $\mathbb{R} \times \mathbb{S}^2$ . Radius of  $\mathbb{S}^2$  associated with the Schwarzschild radius. A fiducial length scale  $L_o$  needed for coordinate x in  $\mathbb{R}$ .

Using the symmetries, and imposing Gauss constraint, the connection and triads become:

$$A_a^i \tau_i \, \mathrm{d}x^a = \frac{c}{L_o} \tau_3 \, \mathrm{d}x + b \, \tau_2 \mathrm{d}\theta - b\tau_1 \sin\theta \, \mathrm{d}\phi + \tau_3 \cos\theta \, \mathrm{d}\phi$$

$$E_i^a \tau^i \frac{\partial}{\partial x^a} = p_c \tau_3 \sin \theta \frac{\partial}{\partial x} + \frac{p_b}{L_o} \tau_2 \sin \theta \frac{\partial}{\partial \theta} - \frac{p_b}{L_o} \tau_1 \frac{\partial}{\partial \phi}$$

The connection and triad components are invariant under freedom to rescale coordinates and satisfy:  $\{c, p_c\} = 2G\gamma$ ,  $\{b, p_b\} = G\gamma$ 

However freedom to rescale fiducial length  $L_o$  present. Under rescaling  $L_o \rightarrow \alpha L_o$ ,  $p_c$  and b invariant, but  $c \rightarrow \alpha c$ ,  $p_b \rightarrow \alpha p_b$ . (Only Boehmer-Vandersloot and Corichi-Singh quantizations independent of this freedom). Spacetime metric:

$$\begin{split} \mathrm{d}s^2 &= -N^2 \mathrm{d}t^2 + \frac{p_b^2}{|p_c|L_o^2} \,\mathrm{d}x^2 + |p_c| \,(\mathrm{d}\theta^2 + \sin^2\theta \,\mathrm{d}\phi^2) \\ \text{where} \qquad \frac{p_b^2}{|p_c|L_o^2} &= (2m/t-1), \qquad |p_c| = t^2 \end{split}$$

Classical dynamics:

$$\mathcal{H}_{\text{class}} = -\frac{N \text{sgn}(p_c)}{2G\gamma^2} \left( (b^2 + \gamma^2) \frac{p_b}{\sqrt{|p_c|}} + 2bc|p_c|^{1/2} \right)$$

For lapse  $N = \gamma \operatorname{sgn}(p_c) |p_c|^{1/2}/b$ , (for a black hole of mass m):

$$p_b(T) = 2mL_o e^T \sqrt{e^{-(T-T_o)} - 1}, \quad p_c(T) = 4m^2 e^{2T}$$

$$b(T) = \pm \gamma \sqrt{e^{-(T-T_o)} - 1}$$
 and  $c(T) = \mp \frac{\gamma L_o}{4m} e^{-2T}$ 

Horizon (T = 0):  $p_b = 0$ ,  $p_c = 4m^2$ ; singularity at  $p_b = p_c = 0$ .

# Quantization

Holonomies of connections b and c generate an algebra of almost periodic functions with elements:  $\exp(i(\mu b + \tau c)/2)$ . Kinematical Hilbert space:  $L^2(\mathbb{R}_{Bohr}, d\mu_b)$ .

Action of triad operators:

$$\hat{p}_{b} |\mu, \tau\rangle = \frac{\gamma l_{\rm Pl}^{2}}{2} \mu |\mu, \tau\rangle, \quad \hat{p}_{c} |\mu, \tau\rangle = \gamma l_{\rm Pl}^{2} \tau |\mu, \tau\rangle$$

Classical Hamiltonian constraint:

$$C_{\text{Ham}} = -\int d^3x \, e^{-1} \varepsilon_{ijk} E^{ai} E^{bj} (\gamma^{-2} F^k_{ab} - \Omega^k_{ab})$$

 $F^i_{ab}$  expressed in terms of holonomies over loops in  $x-\theta$ ,  $x-\phi$  and  $\theta-\phi$  planes.

Departure from 'improved dynamics' by using fixed areas of loops. The edge of the loop along x direction has length  $\delta_c L_o$ , and the edges along  $\mathbb{S}^2$  have length  $2m\delta_b$ . Quantum geometry fixes minimum area of loops ( $\Delta = 4\sqrt{3}\pi\gamma l_{\rm Pl}^2$ ).

### Quantum Hamiltonian constraint

Quantization results in an anisotropic difference equation with unequal spacings in volume  $V = 4\pi |p_b| |p_c|^{1/2}$ :

$$\begin{split} \hat{C}\Psi(\mu,\tau) &= \left[ \left( V_{\mu+\delta_{b},\tau} - V_{\mu-\delta_{b},\tau} + V_{\mu+3\delta_{b},\tau+2\delta_{c}} - V_{\mu+\delta_{b},\tau+2\delta_{c}} \right) \Psi(\mu+2\delta_{b},\tau+2\delta_{c}) \\ &+ \left( V_{\mu-\delta_{b},\tau} - V_{\mu+\delta_{b},\tau} + V_{\mu+\delta_{b},\tau-2\delta_{c}} - V_{\mu+3\delta_{b},\tau-2\delta_{c}} \right) \Psi(\mu+2\delta_{b},\tau-2\delta_{c}) \\ &+ \left( V_{\mu-\delta_{b},\tau} - V_{\mu+\delta_{b},\tau} + V_{\mu-3\delta_{b},\tau-2\delta_{c}} - V_{\mu-\delta_{b},\tau+2\delta_{c}} \right) \Psi(\mu-2\delta_{b},\tau+2\delta_{c}) \\ &+ \left( V_{\mu+\delta_{b},\tau} - V_{\mu-\delta_{b},\tau} + V_{\mu-\delta_{b},\tau-2\delta_{c}} - V_{\mu-3\delta_{b},\tau-2\delta_{c}} \right) \Psi(\mu-2\delta_{b},\tau-2\delta_{c}) \\ &+ \frac{1}{2} \Big[ \left( V_{\mu,\tau+\delta_{c}} - V_{\mu,\tau-\delta_{c}} + V_{\mu+4\delta_{b},\tau+\delta_{c}} - V_{\mu+4\delta_{b},\tau-\delta_{c}} \right) \Psi(\mu-4\delta_{b},\tau) \\ &+ \left( V_{\mu,\tau+\delta_{c}} - V_{\mu,\tau-\delta_{c}} + V_{\mu-4\delta_{b},\tau+\delta_{c}} - V_{\mu-4\delta_{b},\tau-\delta_{c}} \right) \Psi(\mu-4\delta_{b},\tau) \Big] \\ &+ 2(1+2\gamma^{2}\delta_{b}^{2})(V_{\mu,\tau-\delta_{c}} - V_{\mu,\tau+\delta_{c}})\Psi(\mu,\tau) \Big] / (2\gamma^{3}\delta_{b}^{2}\delta_{c}l_{\mathrm{Pl}}^{2}) \\ &(\delta_{b} = \sqrt{\Delta}/2m \text{ and } \delta_{c} = \sqrt{\Delta}/L_{o}) \end{split}$$

Considering  $\tau$  as a clock, evolution occurs in steps  $2\delta_c$ . One can evolve across the central singularity at  $\tau = 0$  starting with initial conditions at  $\tau = 2n\delta_c$  and  $\tau = 2(n-1)\delta_c$ . Non-singular evolution.

Agreement with classical theory near the horizon.

### Effective dynamics

#### Effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = -\frac{N \text{sgn}(p_c)}{2G\gamma^2} \left[ 2 \frac{\sin(\delta_c c)}{\delta_c} \frac{\sin(\delta_b b)}{\delta_b} |p_c|^{1/2} + \left( \frac{\sin^2(\delta_b b)}{\delta_b^2} + \gamma^2 \right) p_b |p_c|^{-1/2} \right]$$

Using lapse  $N = \gamma \text{sgn}(p_c) |p_c|^{1/2} \delta_b / \sin(\delta_b b)$ , Hamiltonian dynamics yields:

$$\begin{split} b(T) &= \pm \frac{1}{\delta_b} \, \cos^{-1} \left[ b_o \tanh \left( \frac{1}{2} \left( b_o T + 2 \tanh^{-1}(1/b_o) \right) \right) \right] \\ &\text{with} \quad b_o = (1 + \gamma^2 \delta_b^2)^{1/2} \end{split}$$

$$\begin{split} c(T) &= \frac{2}{\delta_c} \, \tan^{-1} \left( \mp \frac{\gamma L_o \delta_c}{8m} e^{-2T} \right), \qquad p_c(T) = 4m^2 \left( e^{2T} + \frac{\gamma^2 L_o^2 \delta_c^2}{64m^2} e^{-2T} \right) \\ p_b(T) &= -2 \frac{\sin(\delta_c c)}{\delta_c} \frac{\sin(\delta_b b)}{\delta_b} \frac{|p_c|}{\frac{\sin^2(\delta_b b)}{\delta_b^2} + \gamma^2} \end{split}$$

Modified Hamilton's equations yield a minimum allowed value of  $p_c$ :  $p_{c\,(\min)} = \gamma \, \Delta^{1/2} \, m$ . Central singularity replaced by bounces.

Important features:

- Recovery of GR at infra-red scales
- Bounce of  $p_b$  and  $p_c$  at well defined scales
- Independence from fiducial length scale  $L_o$

No other loop quantization of Schwarzschild interior shares all these features.



The final white hole mass is approximately a quartic power of the initial black hole mass.



Bounce turns out to be highly asymmetric due to anisotropic shear.

Such an asymmetry common in all anisotropic models in LQC. Examples: Kasner transitions in Bianchi-I spacetime (Gupt, PS (11)), No white holes in Boehmer-Vanderloot quantization (Dadhich, Joe, PS (15)) etc.

# A closer look at quantum gravitational regime(s)

Differences between classical and effective Hamiltonian due to

$$c \to \frac{\sin(\delta_c c)}{\delta_c}, \quad b \to \frac{\sin(\delta_b b)}{\delta_b}$$

Quantum regime can be characterized by relative departure of  $\sin(\delta_b b)/\delta_b$  from classical *b*, and of  $\sin(\delta_c c)/\delta_c$  from classical *c*.

Most significant non-perturbative effects contained in regime when relative difference greater than 1%.



Effective dynamics matches the black hole and white solutions symmetrically in time T. Quantum regime bridges two classically disjoint classical geometries.

# Some observations from the quantum regimes

- Black holes of mass less than m = 0.7 are already quantum from the horizon.
- Generally there are two distinct quantum gravitational regimes. The departures of  $\sin(\delta_c c)/\delta_c$  from classical connection c begin and end quickly in comparison to departures of  $\sin(\delta_b b)/\delta_b$  from classical connection b.
- For some time, the effective geometry is a mixture of classical black hole and white hole geometries. In this period, quantum regime in c has passed and that in b is yet to begin.
- Quantum regime in *b* very asymmetric in proper time  $\tau$ . Very short regime in the quantum black hole geometry, but a very long regime in quantum white hole geometry.

For a black hole of mass m = 50: Time to cross quantum regime in c:  $\approx 3.6$  Planck seconds Time to cross quantum regime in b:  $\approx 37000$  Planck seconds (Only 126 Planck seconds to reach bounce from black hole horizon) Time to white hole horizon:  $\approx 1.36 \times 10^{12}$  Planck seconds

# Transition times

The detailed picture of the bounce(s) in the Schwarzschild interior lead us to identifying three different transition times:

- Bounce time to cross quantum regime in c ( $\tau_B^c$ )
- Bounce time to cross quantum regime in b, into white hole geometry  $(\tau^b_B)$
- Time to form the white hole horizon. (Delivery time,  $\tau_D$ , from the observable parent black hole to the observable offspring white hole)

(In literature so far, no such clear demarcation because bounce assumed to be symmetric and quantum gravity regime not probed).

Quantum regime (or the bounce regime) significantly dominated by  $\tau_B^b$ . This time also directly linked to white hole geometry formation. Its properties determine the features of the 'bounce time' through the quantum gravitational regime.

# How do bounce times change with mass of the black hole?

#### Size matters!

Depending on whether the black hole has mass close to Planck scale or is much larger, bounce time can be quite different.

Bounce times to cross quantum gravity regime in connection c and in connection b have different qualitative behavior.



When Schwarzschild radius approaches underlying quantum gravitational discreteness determined by area gap, mass dependence becomes important in quantum gravity regimes.

Different quantum gravity regimes have strikingly different dependence on mass of the black hole.



Proper time for bounce scales exactly as m for large black holes  $(\forall m \ge 10)$  (universal relationship). It scales roughly as  $m^2$  for Planck size black holes  $(m \sim 0.7 - 5)$ 

Proper time for quantum gravity regime in c scales roughly as  $m^{0.22}$  for large black holes. Weak growth in scaling as mass increases.

Unlike the bounce times which depend on mass of the black hole, a universal behavior is found for delivery time.



Delivery time scales exactly as  $m^5$  for black holes of all masses, from very Planckian to large masses.

# Some implications

- Bounce time for large masses surprisingly same as the one found by Barcelo, Carballo-Rubio, Garay and Jannes (14) using heuristic ideas of propagation of non-perturbative effects.
- Bounce time for small masses has same relationship as proposed in Planck stars by Haggard and Rovelli (15).
- Bounce time from classical black hole geometry to white hole geometry in the interior always less than Hawking evaporation time  $\tau_H$ .
- For large black holes, delivery time much larger than  $\tau_H$ . For an external observer a black hole horizon completely disappears long before a white hole horizon forms. White holes would appear in the universe without any traces of their parent black holes. A baby universe from no where!
- If a black hole has mass  $m \lesssim 1$  then delivery time can be much smaller than Hawking evaporation time. Planck scale black holes explode before they can evaporate!

# Conclusions

- Due to a concrete quantum gravitational model available, we are able to perform a quantitative study of the transition times from black hole to white holes. No such study either for black hole bounces or for quantum gravitational regimes in LQC.
- Transition from the classical black hole geometry to white hole geometry in the interior given by  $\tau \sim m$  for all black holes with large masses.
- For Planck scale black holes, this transition time  $au \sim m^2$ .
- The delivery time is much longer because of the huge asymmetry in the sizes of the parent black hole and the offspring white hole. Scales as  $\tau \sim m^5$ .
- Size matters for bounce times, does not for delivery time.
- Many open questions: How do these results improve existing models of black hole bounce? Phenomenological consequences? Are all or some of these results model dependent? Consequences for black hole evaporation paradigm?