Big Bang and Loop Quantum Cosmology

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Loop Quantum Gravity World Seminar

- Standard model of cosmology provides an excellent agreement with the observational data. Age of precision cosmology.
- Though we have a ‘working model’ (= hot big bang + inflation + dark energy + dark matter + ...) in cosmology, important questions remain unanswered:
  - The backward evolution of our Universe leads to a Big Bang singularity (result of powerful singularity theorems). General Relativity inadequate to provide correct physics at high curvature scales. Occurrence of singularity $\longrightarrow$ limit of validity of the theory
  - Justification of various assumptions in models of very early Universe ? Primordial perturbations ? Dark energy ?
- Need for inputs from quantum gravity.

What does Quantum Gravity tell us about the physics near the Big Bang ?

(Based on work with Abhay Ashtekar and Tomasz Pawlowski)

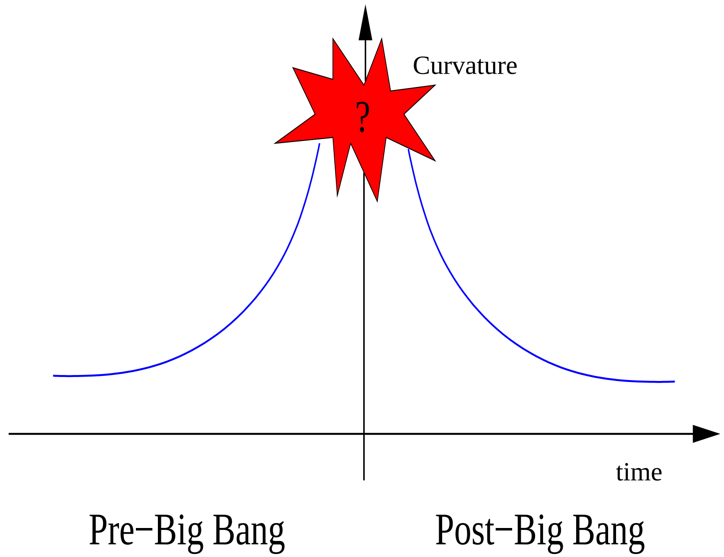
Some age-old questions

- Does the theory provides a non-singular evolution through the big bang ?
- What is on the other side of the big bang ? Quantum foam or a classical spacetime ?
- What is the scale at which the spacetime ceases to be classical ? Does the spacetime continuum exists at all scales, especially when primordial fluctuations were generated ?
- What are the modifications to the Friedmann dynamics at scales of high curvature and at what scales these become important ?
- Does the theory make testable predictions ?

  (complex set of questions $\rightarrow$ difficult to answer without detailed analytical, phenomenological and numerical investigations)

Various Innovative Ideas

- Pre big bang scenarios: Based on the scale factor duality of the string dilaton action. Big bang not the beginning. The post big bang phase with $H = \dot{a}/a > 0, \dot{H} < 0$ preceded by a pre big bang phase with $H > 0, \dot{H} > 0$.
  Major Problem: No non-singular evolution between the pre big bang and the post big bang phase.



Similar problems with Ekpyrotic/Cyclic models

- Various models propose modifications to Friedmann dynamics at high energies. Classical general relativity is envisioned as a low energy effective theory. It is modified at high energies.

  Popular Ideas: Modifications may arise due to higher order terms in the gravity action, large extra dimensions, ...

  Example: Randall-Sundrum model. Our Universe exists on a 3-brane embedded in 5 dimensional anti-de Sitter bulk.

$$H^2 = \frac{\kappa}{3}\,\rho\left(1 + \frac{\rho}{2\sigma}\right)$$

  $H = \dot{a}/a$, $\rho \propto a^{-n}$. As $a \to 0$, $H, \rho \to \infty$.

  Singular evolution. Problems related to big bang as in standard cosmology.

  Extensions yield limited success (unnatural conditions to get a bounce).

- Hope has been that higher order perturbative corrections on the continuum spacetime might be enough to capture the details of singularity resolution – limited success
- Ideas assume the existence of a spacetime continuum at all scales
- Non-perturbative quantum gravitational modifications not included

Quantum nature of dynamical spacetime not captured in these approaches

Do non-perturbative quantum gravitational effects resolve the big bang singularity ?

Background independent non-perturbative quantum gravity – LQG is leading approach.
Our Strategy: Construct a quantum cosmological model based on LQG $\longrightarrow$ LQC

Caveats:

- Homogeneous and Isotropic setting
- Relationship with full theory unclear

Hopes:

- Glimpses of singularity resolution and physics of very early Universe
- Learning/Testing ground for model building, phenomenological applications, making testable predictions
- Valuable insights/lessons to complete the program in LQG and other non-perturbative approaches

LQC: Homogeneous and Isotropic setting

- Spatial homogeneity and isotropy: fix a fiducial triad ${}^o e^a_i$ and co-triad ${}^o\omega^i_a$. Symmetries $\rightarrow$

$$A^i_a = c\,{}^o\omega^i_a, \quad E^a_i = p(\det {}^o\omega)\,{}^o e^a_i$$

- Basic variables: c and p satisfying $\{c, p\} = 8\pi G\gamma/3$.
  Relation to scale factor:
  $|p| = a^2$ (two possible orientations for the triad)
  $c = \gamma\dot{a}$ (on the space of physical solutions of GR).

- Elementary variables – Holonomies:
  $h_k(\mu) = \cos(\mu c/2)\mathbb{I} + 2\sin(\mu c/2)\tau_k$, $\mu \in (-\infty, \infty)$
  Elements of form $\exp(i\mu c/2)$ – generate algebra of almost periodic functions

- Hilbert space: $\mathcal{H}_{\rm kin} = L^2(\mathbb{R}_{\rm B}, d\mu_{\rm B})$
  Orthonormal basis: $N(\mu) = \exp(i\mu c/2)$; $\langle N(\mu)|N(\mu')\rangle = \delta_{\mu,\mu'}$
  Hilbert Space different from the Wheeler-DeWitt theory.

Hamiltonian Constraint

$$C_{\text{grav}} = -\gamma^{-2} \int_{\mathcal{V}} d^3x \, N \, \varepsilon_{ijk} \, F^i_{ab} \, (E^{aj} E^{bk} / \sqrt{|\det E|})$$

Strategy: Express C_{grav} in terms of elementary variables and their Poisson brackets

$\varepsilon_{ijk}(E^{aj} E^{bk} / \sqrt{|\det E|}) \longrightarrow \mathrm{Tr}(h_k^{(\bar\mu)} \{ h_k^{(\bar\mu)-1}, V \} \tau_i)$ (Thiemann's trick)

($V = |p|^{3/2}$, $\bar\mu$ can be a function of p)

$F^i_{ab} \longrightarrow$ Limit of the holonomy around a loop divided by the area of the loop, in the limit area going to zero.

Limit well defined in full theory on Diff-Inv states.

In LQC: do not have Diff-Inv. We need a new strategy.

Exploit the area gap in LQG (ABL 2003): $\Delta = 2\sqrt{3}\pi\gamma\ell_{\mathrm{P}}^2$

- Old approach: Area operator associated with the face of elementary cell: $\widehat{Ar} = \widehat{|p|}$: $\quad \widehat{Ar}\, h_k^{(\bar\mu)} \;=\; (8\pi\gamma\ell_{\rm P}^2/6)\,\bar\mu\, h_k^{(\bar\mu)}$
  Put eigenvalue = $\Delta \implies \bar\mu = \mu_o = 3\sqrt{3}/2$
- New approach: Shrink the physical area enclosed by the loop. Area associated with a square loop of length $\bar\mu$ = $\bar\mu^2|p|$

$$\widehat{Ar}|\mu\rangle \;=\; (8\pi\gamma\ell_{\rm P}^2/6)|\mu|\,\bar\mu^2\,|\mu\rangle$$

Put eigenvalue = $\Delta \implies \bar\mu = \sqrt{3\sqrt{3}/2|\mu|}$

Problem: What is the action of $\widehat{e^{i\bar\mu c/2}}$?

Idea: Since

$$\widehat{e^{i\mu_o c/2}}\,\tilde\Psi(\mu) = \tilde\Psi(\mu+\mu_o)$$

action is to drag the state a unit affine parameter distance along the vector field $\mu_o\frac{d}{d\mu}$

Set $\widehat{e^{i\bar{\mu}c/2}}\,\tilde{\Psi}(\mu)$ as Lie drag of the state by a unit affine parameter distance along the vector field $\bar{\mu}\frac{d}{d\mu}$.
Affine parameter of this vector field:

$$v = K\mathrm{sgn}(\mu)|\mu|^{3/2}, \text{ where } K = 2\sqrt{2}/3\sqrt{3\sqrt{3}}$$

v are proportional to the eigenvalues of the Volume operator. $v(\mu)$ is invertible, C^1 function of μ.

$$\hat{V}|v\rangle = \left(\frac{8\pi\gamma}{6}\right)^{3/2}\frac{|v|}{K}\ell_{\mathrm{P}}^3|v\rangle$$

Orthonormal basis in $\mathcal{H}_{\mathrm{kin}}$: $\langle v_1|v_2\rangle = \delta_{v_1,v_2}$
Consider $\Psi(v) = \tilde{\Psi}(\mu)$:

$$\widehat{e^{i\bar{\mu}c/2}}\,\Psi(v) = \Psi(v+1)$$

Quantum Constraint

Quantization of the constraint leads to difference equation:

$$\hat{C}_{\rm grav}\Psi(v) = f_+(v)\Psi(v+4) + f_o(v)\Psi(v) + f_-(v)\Psi(v-4) = \hat{C}_{\rm matt}\Psi(v)$$

where

$$f_+(v) \;=\; \frac{27}{16}\sqrt{\frac{8\pi}{6}}\,\frac{K\ell_{\rm P}}{\gamma^{3/2}}\,|v+2|\;\big||v+1|-|v+3|\big|$$

$$f_-(v) = f_+(v-4), \quad f_o(v) = -f_+(v) - f_-(v)$$

$\hat{C}_{\rm grav}$ is self-adjoint and negative definite.

- Evolution in steps which are constant in eigenvalues of the volume operator. Earlier constraint based on μ_o had steps which are constant $(4\mu_o)$ in eigenvalues of the triad operator.
- Evolution non-singular across $v = 0$ for all states.
- $\hat{C}_{\rm grav} \longrightarrow \hat{C}_{\rm grav}^{\rm WDW}$ with natural factor ordering for $|v| \gg 1$.

What is the physics of singularity resolution ?
What are the physical predictions in LQC ?

Algorithm to extract Physics in LQC

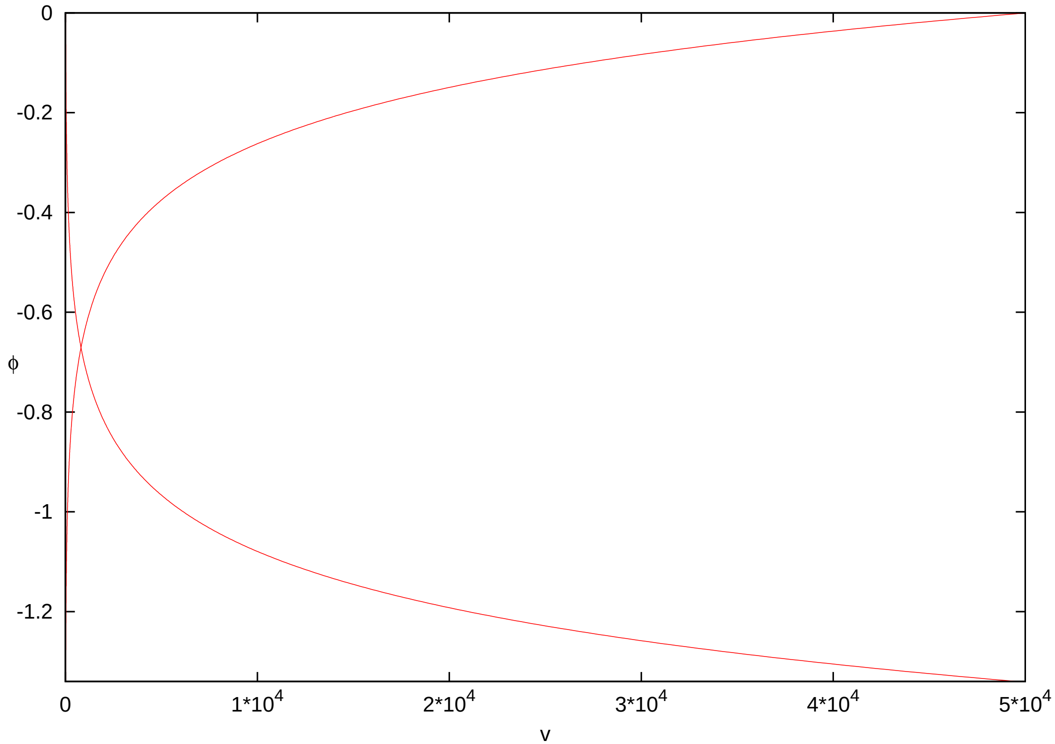
- Seek a dynamical variable that can play the role of internal time (physical interpretations more transparent).
- Introduce Inner Product, find Physical Hilbert Space.
- Construct Dirac Observables, raise them to operators.
- Construct physical semi-classical states representing a large classical Universe.
- Evolve these states backward towards Big Bang using quantum constraint. Analyze the behavior of expectation values and fluctuations of Dirac observables.
- Compare with classical dynamics, extract predictions.

Massless Scalar Field Model

Phase space: (c, p, ϕ, p_ϕ), $\{\phi, p_\phi\} = 1$

$$\hat{C}_{\rm grav} + \hat{C}_{\rm matt} = -6\frac{c^2}{\gamma^2}\sqrt{|p|} + 8\pi G\,\frac{p_\phi^2}{|p|^{3/2}} = 0$$

$$p_\phi = \text{constant}, \quad \phi \sim \log \mu$$



All solutions are singular

- ϕ is a monotonic function – can play the role of internal time. Evolution refers to relational dynamics – the way geometry changes with 'time' (or as ϕ evolves).
- Dirac Observables: p_ϕ, $|v|_{\phi_o}$
- For $\hat{C}_{\rm matt}$, use Thiemann's trick and get:

$$\frac{\widehat{\mathrm{sgn}(p)}}{\sqrt{|p|}}|v\rangle = \left(\frac{6}{8\pi\gamma\ell_{\rm P}^2}\right)^{1/2} B^{1/3}(v)\,|v\rangle$$

$$B^{1/3}(v) = \frac{3}{2}K^{1/3}|v|^{1/3}\left(|v+1|^{1/3} - |v-1|^{1/3}\right)$$

For $|v| \gg 1$, eigenvalue $\sim \mathrm{sgn}(p)/\sqrt{|p|}$.

  - $\widehat{|p|^{-3/2}}\Psi(v) = \left(\frac{6}{8\pi\gamma\ell_{\rm P}^2}\right)^{3/2} B(v)\Psi(v)$

- Gravitational constraint:

$$\partial_\phi^2 \Psi(v,\phi) = B(v)^{-1}\Big[C^+(v)\Psi(v+4,\phi) + C^o(v)\Psi(v,\phi) + C^-(v)\Psi(v-4,\phi)\Big] =: \Theta\Psi(v,\phi)$$

with

$$C^+(v) = \frac{3\pi KG}{8}\,|v+2|\ \big||v+1| - |v+3|\big|$$

$$C^-(v) = C^+(v-4), \quad C^o(v) = -C^+(v) - C^-(v)$$

- Constraint similar to the massless Klein-Gordon equation in static spacetime. ϕ plays the role of time, Θ of Laplacian-type operator.
- Θ is self-adjoint and positive definite.

- For $|v| \gg 1$, $B(v) \sim \underline{\mathrm{B}}(v) := K|v|^{-1}$.
  Wheeler-DeWitt limit for the constraint:

  $$\partial_\phi^2 \Psi(v,\phi) = 12\pi G\, v\, \partial_v (v \partial_v \Psi(v,\phi))$$

  In geometrodynamics classical constraint: $G^{AB} p_A p_B = 0$
  In quantum theory natural choice of factor ordering $\rightarrow$ Laplacian for G^{AB}.
  LQC Hamiltonian constraint automatically yields the natural factor ordering for Wheeler-DeWitt.

- Action of parity operator: $\hat{\Pi}\, \Psi(v,\phi) = \Psi(-v,\phi)$
  $\hat{\Pi} \rightarrow$ large gauge transformation on the space of solutions. No observable which can detect the change in orientation of the triad. Physical considerations require symmetric states.

- Inner Product:
  - Demand that action of operators corresponding to Dirac observables is self-adjoint
  - Group averaging
- Result: $\Psi(v,\phi)$ are positive frequency: $-i\,\partial_\phi \Psi = \sqrt{\Theta}\Psi$

$$\langle \Psi_1 | \Psi_2 \rangle = \sum_v B(v) \bar{\Psi}_1(v,\phi_o) \Psi_2(v,\phi_o)$$

Dirac observables act as:

$$\hat{p}_\phi \Psi = -i\hbar\,\partial_\phi \Psi, \;\; \hat{v}|_{\phi_o} \Psi(v,\phi) = e^{i\sqrt{\Theta}(\phi-\phi_o)}\, |v|\, \Psi(v,\phi_o)$$

Physical Hilbert space divided into sectors: $\mathcal{H}_{\rm phy} = \oplus_\varepsilon \mathcal{H}_\varepsilon$, $\varepsilon \in (0,2)$.
States have support on $v = |\varepsilon| + 4n$. Predictions insensitive to the choice of sectors.

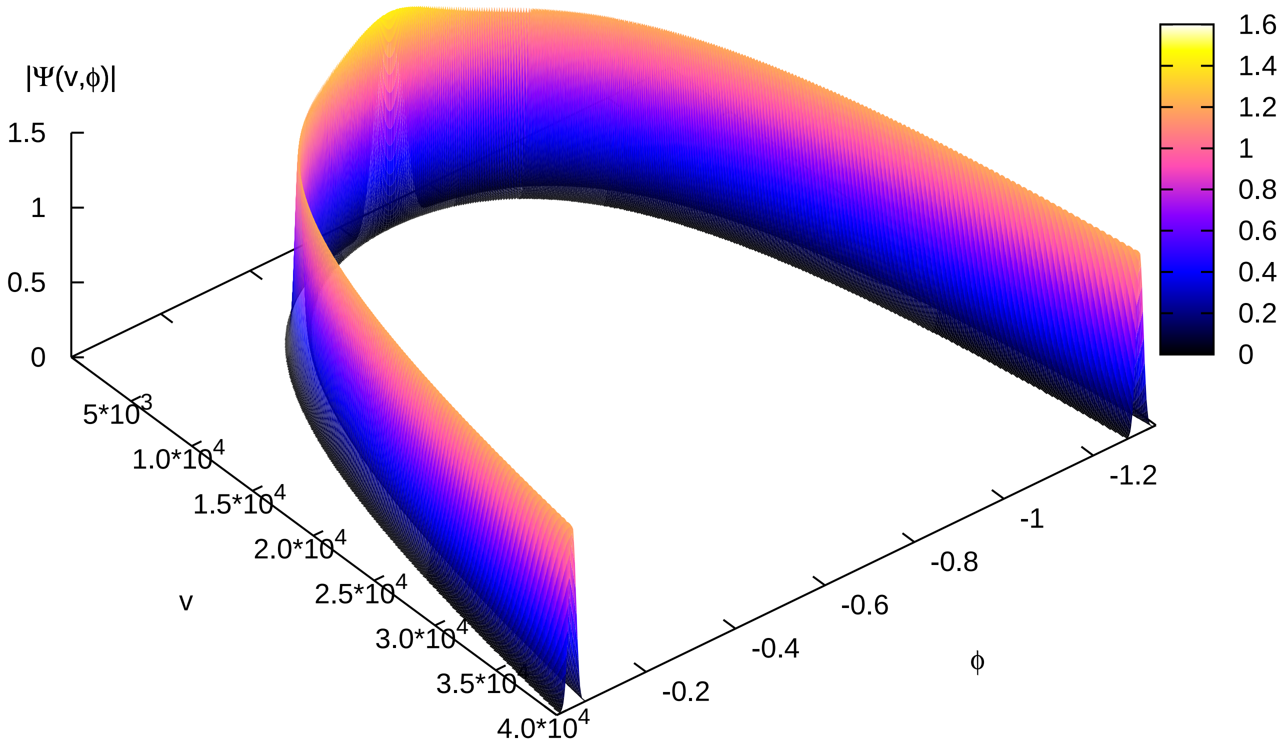
Numerics

Evolution in ϕ:

- Initial value problem in ϕ. Solve $\partial_\phi^2 \Psi = -\Theta\Psi \rightarrow$ countable number of coupled ODE's.
- Initial data at $\phi = \phi_o$ consists of $\Psi(v, \phi_o)$ and $\partial_\phi \Psi|_{\phi_o}$. Initial data specified in three different ways using classical equations.
- For semi-classical states choose a large value of $p_\phi = p_\phi^\star$. Fix a point $(v^\star, \phi_o)$ on the classical trajectory for $v^\star \gg 1$ (large classical Universe). Using quantum constraint follow its evolution backward.
  Example of initial state:

$$\Psi(v, \phi_o) = \int_{-\infty}^{\infty} dk\, \Psi'(k)\, e_k(v)\, e^{iw(\phi_o - \phi^\star)}$$

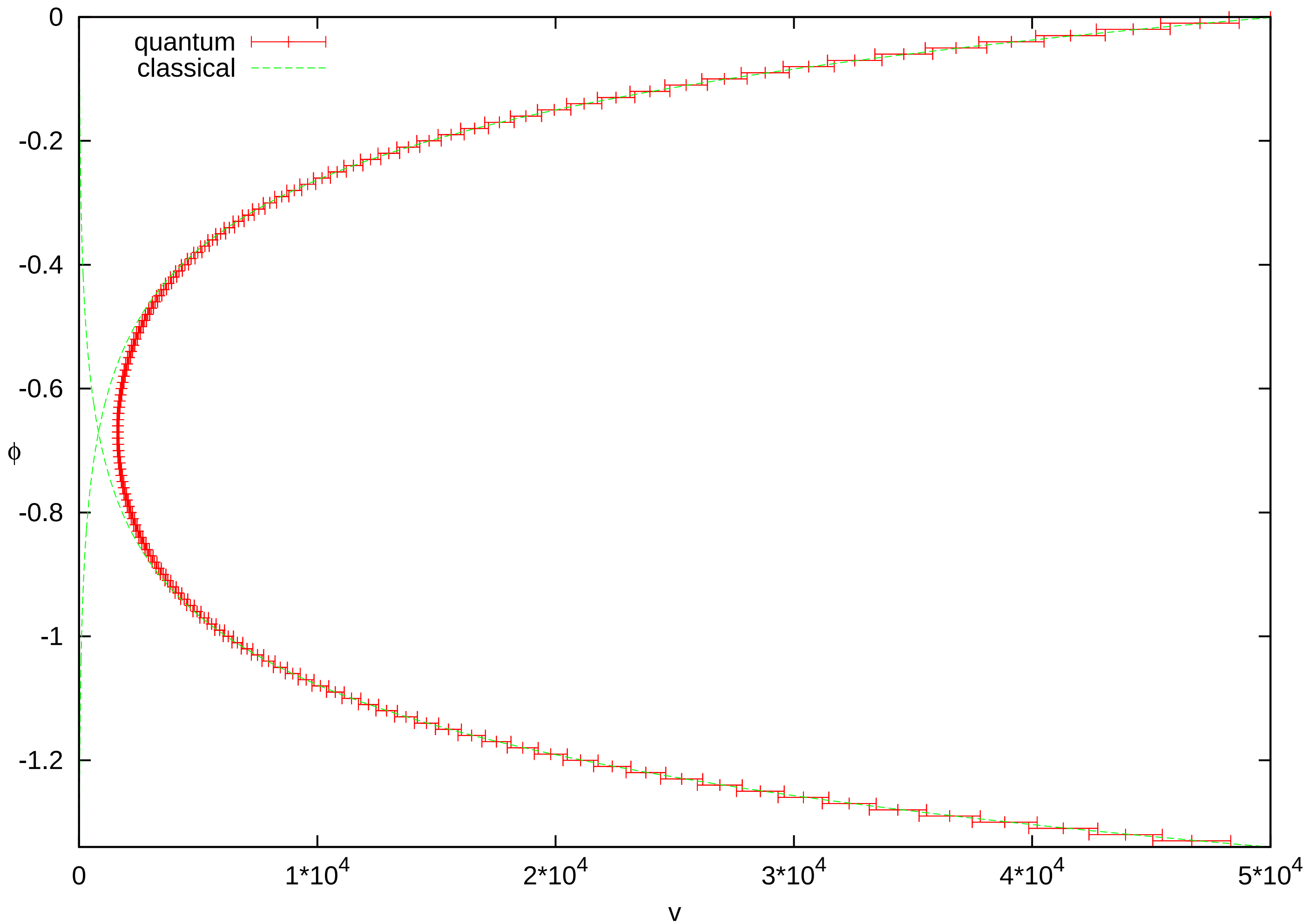
Quantum Bounce



Results of Loop Quantum Evolution

- States remain sharply peaked through out evolution.
- Expectation values of $|v|_\phi$ and p_ϕ are in good agreement with classical trajectories until energy density becomes of the order of a critical density (of the order Planck).
- The state bounces at critical density from expanding branch to the contracting branch with same value of $\langle \hat{p}_\phi \rangle$. This phenomena is generic. Big bang replaced by a big bounce at Planck scale.
- Norm and expectation value of $\hat{p}_\phi$ remain constant.
- Fluctuations of observables remain small. Some differences arise near the bounce point depending on the method specification.

Comparison of Evolution



Effective Theory

- An effective Hamiltonian description can be obtained by using the geometric methods.
- Results in the effective Friedmann equation:

$$H^2 = \frac{\kappa}{3}\,\rho\left(1 - \frac{\rho}{\rho_{\rm crit}}\right), \quad \rho_{\rm crit} = \frac{\sqrt{3}}{16\pi^2\gamma^3 G^2\hbar}$$

Features:

Similar to the Friedmann equation in Randall-Sundrum braneworlds, except for the (-) sign.

Small difference in Friedmann equation $\to$ Profound implications for Physics.

- For $\rho \ll \rho_{\rm crit}$, classical Friedmann dynamics recovered.

Answers to some long pending questions

- Does the theory provides non-singular evolution through the big bang ? Yes, for all solutions.
- What is on the other side of the big bang ? Quantum foam or a classical spacetime ? Universe escapes big bang and bounces at Planck scale to a pre-big bang classical contracting branch.
- What is the scale at which the spacetime ceases to be classical ? Does the spacetime continuum exists at all scales ? When the energy density becomes of the order of a critical density ($\sim 0.82\ \rho_{\rm Planck}$), deviations from classical dynamics are significant and spacetime ceases to be classical. Picture of a continuum spacetime breaks down in the deep Planck regime.
- What about the modifications to Friedmann equation ? ρ^2 modifications at high energy scales with a negative sign.

Summary and Open Issues

- Loop quantum cosmology provides a new picture of the Universe near and at the big bang and beyond. Big bang not the beginning, big crunch not the end. Two classical regions of spacetime joined by a quantum geometric bridge.
- Quantum gravitational effects play important role at Planck scales to yield non-singular evolution for generic initial data. No need to introduce exotic matter or ad-hoc assumptions.
- Long standing problem of singularity resolution in homogeneous and isotropic spacetimes resolved. New avenues to test non-perturbative techniques opened.
- Important lessons learned in quantization of simple models about ambiguities. Example: In μ_o evolution, critical density not constant ($\propto 1/p_\phi$), bounce could occur at small curvatures ! Physical ramifications can narrow down the ambiguities $\rightarrow$ lead to the choice of v evolution.

- Results obtained in flat homogeneous and isotropic model, *devoid* of perturbations.

  Does the picture of bounce survive with addition of perturbations ?

  Relax symmetries: Work in progress. Early indications from more general situations suggest singularity resolution but a detailed picture yet to be worked out.

- What happens in case of closed models, anisotropies and potentials ?
  For closed model a similar picture appears. Effective theory gives useful insights on picture of bounce in presence of potentials and anisotropies.

- Connection with full theory ?

- Testable predictions ? Confirmation with observations ??