A priori likelihood of slow roll inflation in LQC

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Previous Work by: Gibbons & Turok. Germani, Nelson & Sakellariadou

Introduction

- Loop Quantum Cosmology:
  Big Bang singularity is replaced by Quantum Bounce.
- Quantum theory of Cosmology
  Supposed to be trusted at higher energy scales than GR.
- Would like to ask an *a priori* questions:
  Does the theory allow inflation?
  If so how likely is it?
- Follow Laplace's Principle of Indifference

Outline

Inflation

- Accelerated expansion of the early universe
- Typically fuelled by 'slow roll' of scalar field down a potential
- Inflation mechanism 'solves' physical problems...
  - Flatness
  - Horizon problem
- ... but introduces others
  - Appears to need fine tuning
  - Gibbons & Turok: Probability of N e-foldings suppressed by $Exp(-3N)$
  - Typically need 60-70 e-foldings

Probability

Want to measure *a priori* probabilities:
Idea (Gibbons & Turok, ’06): Solution Counting

Solution Counting

Liouville Measure on phase space, want to count solutions: Fix a surface I in phase space that all solutions cross exactly once
Pull back symplectic structure ω to this surface to form a measure

$$P(X) = \frac{\int_A \omega}{\int_I \omega}$$

Problem: The surface I is typically non-compact, so we need cut-offs, regularizations etc

Probability

Why use *a priori* probabilities?
Can introduce a probability distribution:

$$P(A) = \frac{\int_A F(u)\ \omega}{\int_I F(u)\ \omega}$$

Concept of *Information*

$$I(F) = -\int F(u) ln(F(u)) du$$

Extremized by uniform distribution.

- *A priori* probabilities useful when very high or very low.

Need a lot of information to justify choice of F for an event.

Warm up problem: Orbital Dynamics

Planet orbiting sun (equatorial): 4D phase space: $\{r, P_r; \phi, P_\phi\}$ Fix total energy E gives constraint:

$$C = 2E - P_r^2 - \frac{P_\phi^2}{r^2} + \frac{1}{r}$$

Where $E < 0$ for a bounded orbit. The symplectic structure is

$$\omega = dP_\phi \wedge d\phi + dP_r \wedge dr$$

Each orbit crosses $\phi = 0$ once, hence solve constraint for P_r and pull back to $\phi = 0$ slice:

$$\overleftarrow{\omega} = \frac{P_\phi \, dP_\phi \wedge dr}{\sqrt{2Er^4 - P_\phi^2 r^2 + r^3}}$$

$$r \in [0, \frac{-1}{2E}] \quad P_\phi \in [0, \sqrt{r + 2Er^2}]$$

Warm up problem: Orbital Dynamics

Total measure:

$$N = \int_I \overleftarrow{\omega} = \frac{\pi}{2\sqrt{-2E}}$$

Apply to physical problem: Eccentricity

$$e = \sqrt{1 + 8EP_\phi^2}$$

Region of phase space for a eccentricity $e > e_0$:

$$A = \{r \in [\frac{-1+e_0}{4E}, \frac{-1-e_0}{4E}] \quad , \quad P_\phi > \sqrt{\frac{e_0^2 - 1}{4E}}\}$$

$$\begin{aligned} P(e > e_0) &= \frac{1}{N}\int_A \overleftarrow{\omega} \\ &= \sqrt{1 - e_0^2} \end{aligned}$$

Warm up problem: Orbital Dynamics

Interpretational Issues

- Almost circular orbits unlikely
- Solar system populated by almost circular orbits
- Solar system unlikely?

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Resolution

- No - Consider the question asked.
- Orbits picked out by other physics
- Low probability of event -> Heavy burden on model to explain.

Loop Quantum Cosmology

Work from the effective Hamiltonian for LQC (Ashtekar, Pawlokwski & Singh; Bojowald; Willis; Taveras etc)
4D Phase Space: $\{\phi, P_\phi; \nu, b\}$

$$\mathcal{H} = \frac{P_\phi^2}{2\nu} - 3\pi\nu \frac{sin^2(\lambda b)}{\lambda^2} + 4\pi^2\gamma^2\nu V(\phi)$$

Equations of motion:

$$\dot{\phi} = \frac{P_\phi}{2\pi\gamma\nu} \qquad \dot{P_\phi} = -2\pi\gamma\nu V_{,\phi}$$

$$\dot{\nu} = \frac{3\nu}{2\gamma}\frac{sin(2\lambda b)}{\lambda} \qquad \dot{b} = -\frac{P_\phi^2}{\pi\gamma\nu^2} = -4\pi\gamma\dot{\phi}^2$$

Notice: b monotonic non-increasing, system symmetric under

$$\{\nu, b, \phi, P_\phi\} \rightarrow \{\alpha\nu, b, \phi, \alpha P_\phi\}$$

Useful Symmetry

The system has a useful symmetry:

$$\{\nu, b, \phi, P_\phi\} \to \{\alpha\nu, b, \phi, \alpha P_\phi\}$$

Under transformation:

$$\begin{aligned} \phi(t) &\to \phi(t) \\ \nu(t) &\to \alpha\nu(t) \end{aligned}$$

Space-time physics is unchanged. Symmetry can be viewed as gauge.

- In k=0 cosmologies have to introduce fiducial cell
- Symmetry is rescaling fiducial cell
  or
- Between solutions with fixed cell.

New Features

There are two new key features of LQC which we will exploit:

- Unique Bounce Point at $b = \frac{\pi}{2\lambda}$ (recall b monotonic)
- Superinflation ($\dot{H} > 0$)

From EoM:

$$H = \frac{1}{3}\frac{\dot{\nu}}{\nu} = \frac{1}{2\gamma}\frac{sin(2\lambda b)}{\lambda}$$

- Before bounce $H < 0$ after bounce $H > 0$
  We must have superinflation.
- All solutions bounce at a unique value of b
  Define our gauge fixed 2D surface I to be $b = \frac{\pi}{2\lambda}, H = 0$

Bounce point

Recall we have fixed a 2D surface for counting solutions.

- 'Non-prejudiced' point
  Can be done for any $b = const$ surface.
- Contains all solutions
- Non-compact constraint surface

Following the methods above we pull back our symplectic structure to obtain:

$$\overleftarrow{\omega} = \sqrt{\frac{3\pi}{\lambda^2} - 4\pi^2\gamma^2 V(\phi)}\, d\nu \wedge d\phi$$

Recall: Looking for fractional volume of I that leads to sufficient inflation.

Calculating Probabilities

We want to find the number of solutions that have property A.

$$P(A) = \frac{\int_A \omega}{\int_I \omega}$$

Recall physics invariant under $\{\phi, \nu\} \rightarrow \{\phi, \alpha\nu\}$:

- Integrals over ν are gauge
- Physical questions only depend on ϕ.

$$P(A) = \frac{\int_{\phi \in A} \sqrt{\frac{3\pi}{\lambda^2} - 4\pi^2\gamma^2 V(\phi)}\, d\phi}{\int_{\phi \in I} \sqrt{\frac{3\pi}{\lambda^2} - 4\pi^2\gamma^2 V(\phi)}\, d\phi}$$

Robust Superinflation in LQC

In LQC we see a robust phase of superinflation:

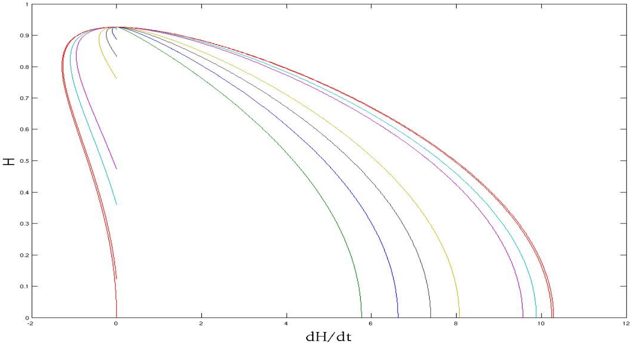
- Begins at $b = \frac{\pi}{2\lambda}$ $H = 0$
- Ends at $b = \frac{\pi}{4\lambda}$ $H = H_{max}$

Was originally thought this could entirely replace inflation even without potential.

- Amount of superinflation determined by value of ϕ and potential during this phase.
- Minimum amount $\nu \to \sqrt{2}\nu_{bounce}$
- Ends with H large, $\dot{H} = 0$
  Good conditions for slow roll.

Superinflation in LQC

- Superinflation can be very short lived.
- Driven to set H on exit



Inflation

We want a situation in which we see *slow roll* inflation.
Approximately described by $H = const$ ie $\dot{H}/H^2 << 1$
Introduce a specific potential: $V(\phi) = \frac{m^2\phi^2}{2}$ resembles damped harmonic oscillator:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

Physically motivated inflaton mass: $m = 6 * 10^{-7} m_{pl}$ (COBE normalization)
H is dynamical, but for slow roll inflation should be approximately constant.
with damping parameter $\zeta = \frac{H}{2m}$

Dynamics

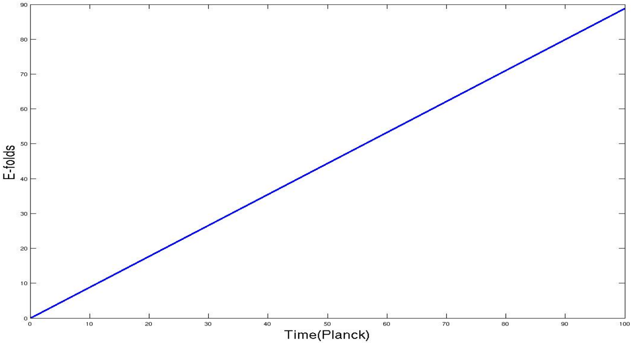
To understand dynamics of the system, split into three cases at the bounce point:

- Potential Domination ($V(\phi) > \dot{\phi}^2$)
- Small Kinetic Domination ($\dot{\phi}^2 > V(\phi) >> 0$)
- Extreme Kinetic Domination ($V(\phi) \approx 0$)

Note that all cases can be expressed in terms of ϕ_b.
To simplify algebra: Define $F = \frac{V(\phi)}{\rho_c}$

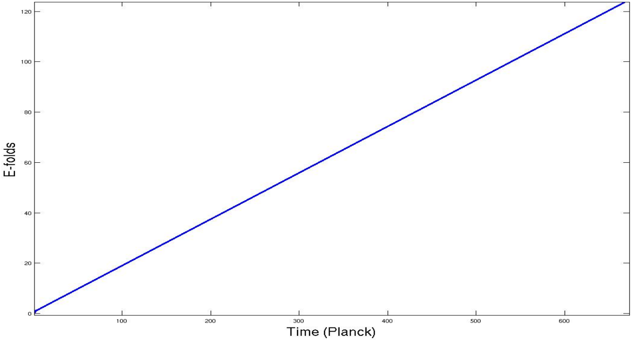
Potential Domination

- $F > 1/2$
- Long superinflation
- Slow roll begins during superinflation
- Universe undergoes 'super-exponential' phase
- Superinflation followed by long inflation phase.



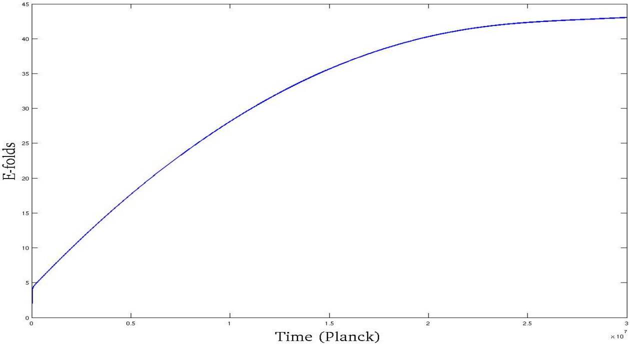
Kinetic Domination

- $10^{-}10 < F < 1/2$
- Short superinflation
- Inflaton undergoes slow roll
- Slow decay of Hubble leads to 'almost-exponetial' expansion
- Outside extreme kinetic domination achieve >68 e-foldings



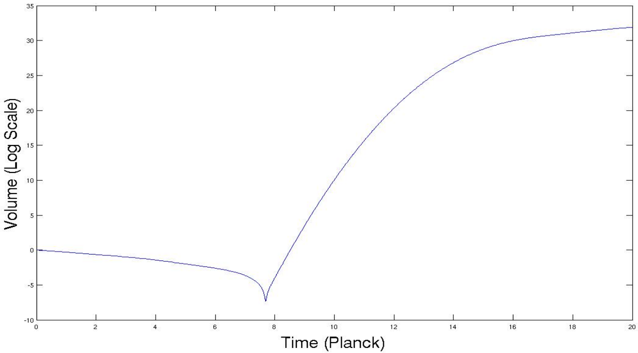
Extreme Kinetic Domination

- $F < 10^{-10}$ for <68 e-foldings
- Inflaton starts near minimum of potential
- Rolls up to small value, begins slow roll down
- Very short superinflation



Summary of Dynamics

- Universe starts pre-bounce
- Bounce
- Superinflation
- Inflation



Probability Distribution

As we have a measure we can form our probability distribution on F where $F = V(\phi)/\rho_c$:

$$P(F > F_0) = \frac{1}{N} \int_{F_0}^{1} \sqrt{F^{-1} - 1}\, dF$$

Where

$$N = \int_{0}^{1} \sqrt{F^{-1} - 1}\, dF$$

And since we get 68 e-foldings for $F > 10^{-10}$

$$P(68e - foldings) \approx 0.9999$$

Robustness

- Decrease mass, results approximately same
  $m \approx 10^{-8} m_{pl} \rightarrow P(68efolds) > 0.99$
- Increase mass, results approximately same
  $m \approx 10^{-5} m_{pl} \rightarrow P(68efolds) > 0.99$
- Can change form of potential (work in progress)
  $V(\phi) \approx \phi^4, e^{\phi}$

Conclusions

- We can define a measure of probability a la Gibbons & Turok in LQC
- Inflation now appears very likely
- Key features of LQC required for calculation
- Inflation can begin at high densities
  Needs physics close to planck scale.
- Bounce point gives unique place to calculate probabilities
- Superinflation sets conditions for long slow roll inflation

Future Work

- Apply to more complicated systems
- More complicated potentials
  - Higgs $V(\phi) = m^2(\phi^2 - \phi_0^2)$
  - Quartic $V(\phi) = \Lambda\phi^4$
- Bianchi Cosmologies
- Understand superinflation phase in inhomogeneous cosmology