Black holes and reversibility

Matteo Smerlak

Perimeter Institute for Theoretical Physics

ILQGS
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Setup: gravitational collapse

- A black hole forms from ingoing matter.
- Trapping horizon forms and peels off outgoing geodesics.
- Thermal Hawking radiation is emitted.
- Breakdown of predictability?
Information loss as a physical problem

“Information loss violates a basic tenet of quantum mechanics.”

- Information loss happens all the time:
  - with open systems (decoherence)
  - with non-Cauchy “out” surfaces

- Information loss does not mess up with conservation laws.

- The only real question is: what difference would it make?

What physical effects relate to the information loss problem?
Outline

Black holes as squeezers

Past/future entanglement

(A)cyclic processes
The Hawking effect

- Time dilation
- Classical vs. quantum
- Vacuum vs. squeezed vacuum
- Hawking pairs

Open questions
Two-mode squeezed vacuum

\[ |\psi_{AB}\rangle \propto \sum_{n=0}^{\infty} (\tanh r)^n |n, n\rangle \quad \Rightarrow \quad \rho_A \propto \sum_{n=0}^{\infty} (\tanh r)^n |n\rangle \langle n| \]

Stronger squeezing, higher temperature \((e^{-\hbar \omega / kT} = \tanh r)\).
Observing TMSV

LETTER

Observation of the dynamical Casimir effect in a superconducting circuit

ARTICLES

Observation of self-amplifying Hawking radiation in an analogue black-hole laser

BEC

SQUID

Nonlinear optics

Hydrodynamics
Open questions

The evaporation problem is a runaway problem

\[
\text{radiation} \implies \text{mass loss} \implies \text{smaller hole} \implies \text{higher squeezing} \implies \text{more radiation}...
\]

The questions for us are

- does this lead to an explosive behavior?
- does thermality break down at late times?
- what astrophysical signatures should we look for?
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Entanglement in finite systems

In finite dimensions, entanglement entropy

\[ S[\rho_A] \equiv -\text{tr}_A[\rho_A \ln \rho_A] \quad \text{with} \quad \rho_A \equiv \text{tr}_B[\rho_{AB}] \]

is unitarily invariant and satisfies the triangle inequality

\[ |S[\rho_A] - S[\rho_B]| \leq S[\rho_{AB}] \leq S[\rho_A] + S[\rho_B]. \]
Page’s conjecture

Hawking phase

Page time?

purification?

\[ \frac{1}{4} \frac{S}{\pi M_0^2} \]

\[ \frac{1}{8895} \frac{t}{M_0^3} \]

[Page (93,13)]
Reversibility: three open questions

Is the evaporation process

1. **unitary**, viz. is purity preserved?

   \[ S_{vN}[\rho_{\text{out}}] = S_{vN}[\rho_{\text{in}}] ? \]

   [Hawking (76)]

2. **cyclic**, viz. does entanglement return to its initial value?

   \[ \lim_{u \to +\infty} S_P(u) = \lim_{u \to -\infty} S_P(u) ? \]

   [Page (93)]

3. **conservative**, viz. do energy input and output match?

   \[ \lim_{u \to +\infty} M(u) = 0 ? \]
Working assumptions

Neglect

- angular momentum (of spacetime and fields)
- backscattering
- non-conformal interactions

but not

- semiclassical backreaction (even strong).

Reduces field dynamics to 2d CFT:

\[
\phi(t, r) = r^2 \int_{S^2} d\Omega^2 \Phi(t, r, \Omega)
\]
Renormalized entanglement entropy

In QFT, entanglement entropy is UV-divergent. Subtract vacuum contribution

\begin{align*}
\rho_{\psi}(u) - \rho_0(u)
\end{align*}

Defines renormalized entanglement entropy

\begin{align*}
S_P(u) = [\rho_{\psi}(u)] - S[\rho_0(u)]
\end{align*}

[Holzhey, Larsen, Wilczek (94)]
The Page curve

Starting from the (non-covariant) CFT formula for a segment

\[ S[\rho(R)] = \frac{1}{3} \log \frac{L(R)}{\epsilon} \]

[Holzhey, Larsen, Wilczek (94)]

we obtain the geometric formula

\[ S(u) = \frac{1}{12} \ln \chi(u) \]

[Bianchi, MS 14]

with \( \chi = \omega_+ / \omega_- \) the in-out redshift factor.
Vaidya spacetime: the Hawking phase

\[ S(u) = \frac{1}{12} \log \left( \frac{1 + W(e^{-u/4M})}{W(e^{-u/4M})} \right) \sim \frac{u}{48M} \]

[Bianchi, de Lorenzo, MS 14]
“Hawking spacetime”: thunderbolt

\[ S(u) \sim \frac{1}{12} \log \left( \frac{4M}{u - u_H} \right) \]

[Bianchi, de Lorenzo, MS 14]
From spacetime to the Page curve

More examples illustrate the connection between geometry and entanglement...

... but in this approach, where

\[ \text{spacetime} \implies \text{entropy}, \]

backreaction is an input. Next best thing after blind guess!

Importance of other, less narrow approach.

[Bianchi, de Lorenzo, MS 14]
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Unitarity violations?

Several authors propose that evaporation is non-unitary (in the QFT sector):

- decoherence without dissipation: spin bath model
  [Unruh, Wald 95; Unruh 12]

- quantum gravity decoherence: defects in spacetime weave
  [Perez 14]

Here I’ll explore another possibility: unitary but acyclic evaporation.
The moving mirror

Mirror starts at rest...

... then accelerates...

... then is inertial again.

\[ \Delta S \propto (\text{relative rapidity}) \]

Unitary but acyclic.

What does cyclicity imply?
Outgoing energy flux

Other natural observable at $\mathcal{I}^+$: energy flux

$$F(u) \equiv 4\pi r^2 \langle \text{in} | T_{uu} | \text{in} \rangle$$

and Bondi mass

$$M(u) \equiv M_0 - \int_{-\infty}^{u} du' F(u').$$

In the 2d approximation,

$$F(u) = -\frac{1}{24\pi} \left( \frac{\ddot{p}(u)}{\dot{p}(u)} - \frac{3}{2} \frac{\ddot{p}(u)^2}{\dot{p}(u)^2} \right)$$

[Fulling, Davies, Unruh (76)]
The it from bit equation

\[
2\pi F(u) = 6\dot{S}(u)^2 + \ddot{S}(u)
\]

- “Page curve” \( S(u) \) determines energy flux \( F(u) \)
- Energy flux \( F(u) \) determines Page curve \( S(u) \), via
  \[ -\ddot{\psi}(u) + 12\pi F(u) \psi(u) = 0 \quad \text{where} \quad \psi \equiv e^{6S} \]
- Flux \( F(u) \) is “exceptional”: \( F(u) + \delta F(u) \) not a flux
- Implies quantum inequality: \( |F|\tau^2 \lesssim 1 \)
It-from-bit and the GSL

\[ 2\pi F(u) = 6\dot{S}(u)^2 + \ddot{S}(u) \]

Generalizes GSL in two ways:

- Includes non-adiabatic term (identity rather than ineq.)
- Does not require special causal structure (event horizon)
- Gives back GSL when \(|\dot{S}| \ll \dot{S}^2\). For a Schwarzschild black hole, with

\[ \dot{S} = \frac{1}{48M_B} \quad \text{and} \quad F = -\dot{M}_B = -\frac{\dot{S}_{BH}}{32\pi M_B} \]

you get

\[ dS_{BH} + dS = \frac{u}{96M} > 0. \]
A black hole’s last gasp

\[ 2\pi F(u) = 6\dot{S}(u)^2 + \ddot{S}(u) \]

At the “Page time” \( u_* \), the flux is negative: \( F(u_*) < 0 \).

Black hole’s “last gasp”.
Time scales

\[ F(u_*) < 0 \]

- Hawking phase
- \( u_1 \) and \( u_2 \)
- \( \tau_H \) and \( \tau_P \)
- An open problem
Lifetime of a black hole

From the it-from-bit equation we get that if

- the evaporation process is cyclic
- energy is conserved: $M_B(u) > 0$,

then the purification time must be large:

$$\tau_P \geq \xi \frac{(M_0^2 - M_1^2)^2}{M_1 m_P^2} = \begin{cases} \mathcal{O}(M_0^4/m_P^3) & \text{if } M_1 = \mathcal{O}(m_P) \\ \mathcal{O}(M_0^3/m_P^2) & \text{if } M_1 = \mathcal{O}(M_0/2) \end{cases}$$

[Carlitz-Willey 87; Bianchi, MS 19]

Recent nonsingular black hole models fail to respect this bound.

[Frolov, Vilkoviski 91; Hayward 06; Bardeen 14; Rovelli, Vidotto, Haggard 14]
An open problem

1. Is there a nonsingular black hole spacetime such that evaporation is cyclic and (sub)-conservative?
2. What kind of spacetime does Page’s curve describe?
Conclusions

- Focus on asymptotic observers (us).
- In field theory, *unitarity* is not equivalent to *cyclicity*.
- From a (guessed) geometry, can *compute* the Page curve.
- Inverse problem seems insightful, thanks to *it-from-bit*.

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