Numerical indications for a phase transition in 4D spin foam models
based on w.i.p. with Benjamin Bahr

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Motivation

- **Spin foams** are a **promising** path integral approach to quantum gravity
  - Many flavours / formulations [Barrett, Crane, Engle, Pereira, Rovelli, Livine, Freidel, Krasnov, Speziale, Wieland, Dupuis, Han, Haggard, Kamiński, Hnybida, Banburski, Chen, ...]
  - Lots of potential for new development!

- Many **ambiguities**: Are spin foams a **consistent theory**?
  - Summing à la **Group field theory**? Is the theory renormalisable? [Oriti, Carrozza, Rivasseau, Gurau, Ben Geloun, Bonzom, Lahoche,...]
  - **Refining**? How are theories related? **Consistent**? [Dittrich, Bahr, S.St., ...]

- Which configurations **dominate** in the path integral for **larger spin foams**?

- Consider ‘**hypercuboid’ spin foams** [Bahr, S.St. ’15]
  - 4D EPRL on hypercubic lattice + restrictions.
  - See also Benjamin’s ILQGS from September 1st 2015
  - Hypercubes / -cuboids also used in other approaches [Alesci, Cianfrani ’15]
An observable of ‘fine’ geometries

- 16 ‘quantum cuboids’ + embedding maps $\sim$ coarse grained amplitude
- Compute expectation values by Monte Carlo techniques

Indications for phase transition:
Either ‘irregular’ or ‘regular’ configurations dominate.
Outline

1. Review: Quantum Cuboids
2. Review: Refinement and embedding maps
3. The setup: from fine to coarse cuboids
4. First results and renormalization prescriptions
5. Summary and Outlook
Quantum cuboid – coherent intertwiners

- **Quantum cuboids: 4D EPRL model** [Engle, Pereira, Rovelli, Livine ’08] On hypercubic 2-complex [Kamiński, Kisielowski, Lewandowski ’10]

- **Coherent SU(2) intertwiner** [Livine, Speziale ’07] for cuboids:

\[
|\nu_{j_1,j_2,j_3}\rangle = \int_{SU(2)} dg \ g \triangleright \bigotimes_{i=1}^{3} |j_i, e_i\rangle \otimes |j_i, -e_i\rangle
\]

- \(e_i\): normal unit vectors in \(\mathbb{R}^3\).
- \(e_1 = \exp(-i\pi \sigma_2/4) \triangleright e_3\) and \(e_2 = \exp(i\pi \sigma_1/4) \triangleright e_3\).

- **Restriction:** same spin \(j\) for ‘opposite’ faces.

- Boost map + SU(2) \times SU(2) group averaging: \(|\nu_{j_1,j_2,j_3}\rangle \rightarrow |\Phi \nu_{j_1,j_2,j_3}\rangle\).

\(j \rightarrow (j^+, j^-) = \left(\frac{1}{2} (1 + \gamma) j, \frac{1}{2} (1 - \gamma) j\right)\)
Quantum cuboids – amplitudes

- **Vertex amplitude** $A_v$: contract 8 coherent intertwiners
  \[ A_v = A_v^+ A_v^- \text{ for } \gamma < 1. \]
  \[ A_v^\pm = \int_{\text{SU}(2)^8} dg_a e^{S^\pm [g_a]} , \text{ with} \]
  \[ S^\pm [g_a] = \frac{1}{2} \pm \gamma \sum_l 2j_l \ln \langle -\vec{n}_{ab} | g_a^{-1} g_b | \vec{n}_{ba} \rangle \]

- **Edge amplitude** $A_e$:
  \[ A_e = \| \Phi_{j_1,j_2,j_3} \|^2 \]

- **Face amplitude** $A_f$:
  \[ A_f = ((2j_f^+ + 1)(2j_f^- + 1))^\alpha \]
Quantum cuboids – Large $j$ asymptotics

- **Large $j$ asymptotics** of partition function \([\text{Freidel, Conrady '08, Barrett, Dowdall, Fairbairn, Gomes, Hellmann '09}]\) (to leading order):

\[
Z = \sum \prod_f A_f \prod_e A_e \prod_v A_v \\
\sim \left( \frac{1 - \gamma^2}{4} \right)^{\alpha F - \frac{3}{2} E + \frac{21}{2} V} \sum \prod_f j_f^{2\alpha} \prod_e (j_1 + j_2)(j_1 + j_3)(j_2 + j_3) \prod_v \tilde{A}_v^2 \\
= \left( \frac{1 - \gamma^2}{4} \right)^{(6\alpha - \frac{9}{2})V} \sum \prod_v \tilde{A}_v(\alpha, \{j_f\})
\]

- $\tilde{A}_v := \left( \frac{1+\gamma}{2} \right)^{\frac{21}{2}} A_v^+ = \left( \frac{1-\gamma}{2} \right)^{\frac{21}{2}} A_v^-.$

- In detail $\tilde{A}_v$ is of the form:

\[
\tilde{A}_v \sim \sum_c \sqrt{\frac{1}{-\det(H(\vec{g}_c))}} e^{S[g_c]} \text{ with } S[g_c] = 0
\]
On the amplitude $\hat{A}_v$

- $\hat{A}_v(\alpha, \{j_f\})$ contains **all important ingredients** of the model.
- $\hat{A}_v(\alpha)$ via face amplitude $A_f(\alpha) \sim j_f^{2\alpha}$
  - $\alpha \sim$ coupling constant
- $\hat{A}_v(j_1, j_2, j_3, j_4, j_5, j_6)$ depends on 6 spins $\{j\}$
  - Normally hypercuboid has 24 areas, 4 per pair of directions
  - ‘Cuboid symmetry’: assign same spin to these areas
- $\hat{A}_v(\alpha, \{\lambda j\}) = \lambda^{12\alpha-9} \hat{A}_v(\alpha, \{j\})$

- **Quantum cuboids** are a **restriction** of the EPRL model
  - But they are **contained** in the EPRL model.
- Quantum cuboids **cannot** capture **curvature** degrees of freedom.
  - Quantum cuboids can be **non-geometric**: cuboid not described by 4 edge lengths $\rightarrow$ twisted geometries [Freidel, Speziale ’10, Freidel, Ziprick ’14]
- **We only discuss the large $j$ limit** today – already interesting!
Consider two hypercuboids glued along a 3D cube in \((xyz)\) direction.

Spins in \((xyz)\) direction set to \(j\), in \(t\)-direction to \(j(1 + x) / j(1 - x)\), \(x \in [-1, 1]\).

\[
I(\alpha, x) = \hat{A}_v(\alpha, \{j\}, \{j(1 + x)\}) \times \hat{A}_v(\alpha, \{j\}, \{j(1 - x)\})
\]

- \(\alpha < 0.6\): **irregular** configuration preferred
- \(\alpha > 0.6\): **regular** configuration preferred

**Face amplitudes** suppress irregular configurations. Cause?
No free variations, we keep the total area fixed.
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Spin foams as a theory

- What does ‘summing over all spin foams’ mean?
- Two conceptually different approaches tackling these questions:

**Group field theory approach**: Summing over all triangulations and topologies [Oriti, Carrozza, Rivasseau, Ben Geloun, Gurau, Bonzom, Lahoche,...]
  - GFT generates spin foam amplitudes as Feynman diagrams
  - Renormalisable (as a QFT)?
  - Spin foam vertices $\sim$ ‘atoms of spacetime’

**Refining approach**: renormalize the spin foam amplitudes by coarse graining [Dittrich, Bahr, Martin-Benito, S.St., Mizera,...]
  - How is a coarse spin foam related to a fine spin foam? Boundary data?
  - Results consistent? Discretization dependent?
  - 2-complex is a regulator.

Complementary approaches, not opposing!
Suppose $\psi_b \in \mathcal{H}_b$ and $\phi_{b'} \in \mathcal{H}_{b'}$.

- $b$ and $b'$ can be boundaries of different complexity.
- $\mathcal{H}_b$ and $\mathcal{H}_{b'}$ are different boundary Hilbert spaces [Oeckl '03].

Need embedding maps $\iota_{b,b''} : \mathcal{H}_b \hookrightarrow \mathcal{H}_{b''}$ and $\iota_{b',b''} : \mathcal{H}_{b'} \hookrightarrow \mathcal{H}_{b''}$.

- $b \prec b''$ and $b' \prec b''$, $b''$ common refinement of $b$ and $b'$.

Cylindrical consistency conditions: $\forall b \prec b' \prec b''$

$$\iota_{b',b''} \circ \iota_{b,b'} = \iota_{b,b''}$$

Compare states in different Hilbert spaces and define equivalence classes:

$$\psi_b \sim \phi_{b'} \iff \iota_{b,b''}(\psi_b) = \iota_{b',b''}(\phi_{b''})$$

Inductive limit construction: $\mathcal{H} := \bigcup_b \mathcal{H}_b / \sim$
Embedding maps and cylindrical consistency

- Different embedding maps in **loop quantum gravity**
- **Ashtekar–Lewandowski vacuum** [Ashtekar, Isham ’92, Ashtekar, Lewandowski ’95]:
  - Arbitrary refinement of graphs
  - New edges carry spin $j = 0$.
- ‘**New**’ **BF-vacuum** [Dittrich, Geiller ’15, Bahr, Dittrich, Geiller ’15]:
  - Refinement of triangulations by Pachner / Alexander moves
  - New closed holonomies are flat.
- Cylindrical consistency: how to **refine boundaries** and how to **add degrees of freedom in vacuum state**.
  - Vacuum refers to representing the same state (w.r.t. an inner product) on a finer boundary.
- Consider as well variational discrete systems [Dittrich, Höhn ’12, ’13, Höhn ’14].

These concepts work well in the **kinematical** setting.
Can they be extended to the dynamical / physical setting?
Spin foam amplitudes and embedding maps

- The question of cylindrical consistency also arises for spin foam models.
- Consider two spin foam amplitudes, where $b < b'$:
  - $A_b : \mathcal{H}_b \to \mathbb{C}$
  - $A'_{b'} : \mathcal{H}_{b'} \to \mathbb{C}$
- Relate / renormalize spin foam amplitudes:
  \[
  A'_{b'}(\iota_{b, b'}(\psi_b)) := A'_{b'}(\psi_{b}) = A_b(\psi_b)
  \]

- $\iota_{b, b'}$ relates amplitudes $A_b$ and $A'_{b'}$, but need not be dynamical.
- **Dynamical** embedding maps [Dittrich '12, '14] from spin foam amplitudes
  [Dittrich, S.St. '14]
  - Realized for **analogue spin foam models** [Dittrich, Eckert, Martin-Benito '12,
    Dittrich, Martin-Benito, Schnetter '13, Dittrich, Martin-Benito, S.St. '14, S.St. '15] and LGT
    [Dittrich, Mizera, S.St. '14] via tensor network renormalization [Levin, Nave '07,
    Gu, Wen '09, Vidal, Evenbly '15].
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Quantum cuboid renormalization: Strategy

- Renormalize quantum cuboid amplitude $\hat{A}_v$
- Combine several quantum cuboids (in most cases: 16): $\hat{A}_v^{(16)}$
- Employ embedding maps $\Upsilon$ to $\hat{A}_v^{(16)}$ to get $\hat{A}_v'$
Before employing embedding maps

Consider the product of 16 hypercuboids:

\[ \hat{A}_v^{(16)}(\{j_{i,\beta}\}) := \prod_{k=1}^{16} \hat{A}_v(\{j_{i,\beta}\}) \]

- ‘Coarse’ face subdivided into four ‘fine’ faces
- 4 ‘fine’ spins \( j_{i,\beta}, \beta = 1, \ldots, 4 \), for each \( i = 1, \ldots, 6 \)

‘Cuboid symmetry’: all spins \( \{j_{i,\beta}\} \) are fixed as boundary data

Compared to original cuboid: More boundary data
- 24 spins \( \{j_{i,\beta}\} \) compared to 6 spins \( \{j_i\} \)
- More complicated ‘boundary graph’: 8 ‘subdivided’ 3D cuboids / intertwiners instead of 8 cuboids / intertwiners.

Construct an embedding map relating 8 ‘fine’ (coherent) intertwiners to one ‘coarse’ (coherent) intertwiner by relating ‘fine’ spins to ‘coarse’ spins.
A geometric embedding map I

- **Recall ‘coherent cuboid’ construction:**
  - SU(2) coherent states labelled $\vec{n}_i = j_i e_i$
  - $\vec{n}_i \cdot \vec{n}_j = 0$ if $i, j$ neighbours

- **Glued hypercuboids in large $j$ limit:**
  - Glued along a shared 3D cuboid
  - Hypercuboid is flat

- **Geometry highly peaked on cuboids**

- **Geometric intuition in large $j$ limit:** ‘Coarse’ cuboids from ‘fine’ cuboids
  - Normal vectors $\vec{n}_i$ in the same subdivided face point in the same direction.

- **Embed** $|\Phi_{\nu J_1, J_2, J_3} \rangle \hookrightarrow \bigotimes_{k=1}^{8} |\Phi_{\nu \{j(i,\beta)\}_{(i,\beta) \in k}} \rangle$
  - $J_i, i = 1, \ldots, 3$ label ‘coarse’ spins.
  - $j(i,\beta), \beta = 1, \ldots, 4$, label fine spins in ‘coarse’ face $i$.

- **3 conditions** $J_i = \sum_{\beta=1}^{4} j(i,\beta)$.
A geometric embedding map II

- $J_i = \sum_{\beta=1}^{4} j_{(i,\beta)}$: ‘fine areas sum up to coarse area’

- Choice! Embedding map non-dynamical!

\[
\Upsilon : \text{Inv}(V_{J_1} \otimes V_{J_2} \otimes V_{J_3}) \rightarrow \bigotimes_{k=1}^{8} \text{Inv} \left( \bigotimes_{(i,\beta) \in k} V_{j_{(i,\beta)}} \right)
\]

\[
\sum_{\beta} j_{(i,\beta)} \overset{1}{=} J_i
\]

\[
|\Phi \iota_{J_1,J_2,J_3} \rangle \mapsto \sum_{\{\hat{j}_{(i,\beta)}\}} \sum_{k=1}^{8} c_{\{\hat{j}_{(i,\beta)}\}}^{J_1,J_2,J_3} \bigotimes_{k=1}^{8} |\Phi \iota_{\{\hat{j}_{(i,\beta)}\}} \rangle_{(i,\beta) \in k} \rangle
\]

- $c_{\{\hat{j}_{(i,\beta)}\}}^{J_1,J_2,J_3}$ fixed by $\|\Phi \iota_{J_1,J_2,J_3}\| = 1$

- $|\Phi \iota_{\{\hat{j}_{(i,\beta)}\}} \rangle_{(i,\beta) \in k} \rangle$ already normalized (edge amplitude!)

\[
\left( c_{\{\hat{j}_{(i,\beta)}\}}^{J_1,J_2,J_3} \right)^{-1} = \sqrt{\prod_{i=1}^{3} N_{J_i}} = \sqrt{\prod_{i=1}^{3} \left( \frac{2J_i-1}{4-1} \right)} \sim \left( \frac{4}{3} J_1 J_2 J_3 \right)^{\frac{3}{2}}
\]
The renormalized amplitude

- Conversely, we use the embedding map to **project on coarse boundary data**.

- Sum over ‘fine’ configurations \( \{j(i,\beta)\} \) contributing to the ‘coarse’ configuration \( \{J_i\} \).

- Relation ‘fine’ \( \rightarrow \) ‘coarse’ essentially captured in factors \( c^{J_1, J_2, J_3}_{\{j(i,\beta)\}} \).

- Large \( j \) limit: \( \sum \{j(i,\beta)\} \sim \int \prod_{(i,\beta)} dj(i,\beta) \)

\[
\hat{A}'_v(\alpha, J_1, J_2, J_3, J_4, J_5, J_6) := \\
\int_{0}^{J_i} \prod_{(i,\beta)} dj(i,\beta) \prod_{i=1}^{6} \delta(\sum_{\beta=1}^{4} j(i,\beta) - J_i) \prod_{k=1}^{8} c^{(k)}_{\{j(i,\beta)\}} \hat{A}_v^{(16)}(\alpha, \{j(i,\beta)\})
\]
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What can we learn about $\hat{A}_v'$?

- $\hat{A}_v'$: 18-dim. integral – use **numerical methods**

- **Monte Carlo integration:**
  - **Sample** $N$ points in 18-dim. configuration space (w.r.t. a probability distribution) to **approximate** integral.
  - **Error drops off** with $1/\sqrt{N}$.

- **Possible to compute** $\hat{A}_v'$ in such a way, but **not naively**.
  - Sampling randomly over configuration space **converges very slowly**. Advanced techniques necessary, e.g. dividing into smaller volumes.
  - **Mathematica quote:** “Catastrophic loss of precision.”

Instead, we evaluate **expectation values of observables** $O$:

$$
\langle O \rangle = \frac{\int (\sum_{\beta=1}^{4} j_{(i,\beta)}=J_i) \prod_{(i,\beta)} dj_{(i,\beta)} O(J_i, j_i) \mathcal{I}(j_i, J_i)}{\int (\sum_{\beta=1}^{4} j_{(i,\beta)}=J_i) \prod_{(i,\beta)} dj_{(i,\beta)} \mathcal{I}(j_i, J_i)}
$$

with $\mathcal{I}(j_i, J_i) := \prod_{k=1}^{8} c^{(k)}_{\{j_{(i,\beta)}\}} \hat{A}_v^{(16)}(\alpha, \{j_{(i,\beta)}\})$. 
Sampling w.r.t. $\mathcal{I}$ and observables

- **Compute** $\langle \mathcal{O} \rangle$ with **Monte Carlo methods**.

- **Random walk** sampling w.r.t. $\mathcal{I}(j_i, J_i)$:
  - $\{j(i,\beta)\}_n \rightarrow \{j(i,\beta)\}_{n+1}$ randomly generated
  - Define $p_{n\rightarrow n+1} := |\mathcal{I}(\{j(i,\beta)\}_{n+1})| / |\mathcal{I}(\{j(i,\beta)\}_n)|$
  - Generate random number $0 < r < 1$.
  - Accept $\{j(i,\beta)\}_{n+1}$ if $r < \min(p_{n\rightarrow n+1}, 1)$.

- **Information** on $\mathcal{I}$ stored in samples $\{j(i,\beta)\}_n$.

  $$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(J_i, \{j(i,\beta)\}_n)$$

  - Note that $\mathcal{I}$ does not need to be normalized.

- **Interesting observables**:
  - $\langle j(i,\beta) \rangle$: ‘fine’ spin
  - $\langle V^{(\text{fine})} - V^{(\text{coarse})} \rangle$: ‘fine’ volume - ‘coarse’ volume
  - $\langle j(i,\beta)j(i',\beta') \rangle - \langle j(i,\beta) \rangle \langle j(i',\beta') \rangle$: correlations between spins
Observable: fine spin $\langle j \rangle$

- Expectation value of $j$ depending on $\alpha$ (for all $J = 4$):

  For $\alpha > 0.52$: $\langle j \rangle = \frac{J}{4}$ – coarse cube subdivided into 16 ‘regular’ cuboids.
  For $\alpha < 0.52$: $\langle j \rangle$ fluctuates between 0 and $J$. One large and 15 small cuboids – ‘irregular’
  Variance $\Delta j$ grows closer to the transition.
The transition in pictures

- Plots show $\mathcal{I}(\alpha, \{j_{(i,\beta)}\})$ for solved constraints $\sum_{\beta} j_{(i,\beta)} = J_i$ and many $j_{(i,\beta)}$ chosen to be equal. Normalized to regular subdivisions.
Observable: $V^{(\text{fine})} - V^{(\text{coarse})}$

- Volume of cuboids given as a function of spins
  
  \[ V = \left( \prod_{i=1}^{6} j_i \right)^{\frac{1}{3}} \]

- Compare the volume of fine cuboids and the volume of the coarse cuboid.
- Indication for (non-)geometric contributions.
Observable: correlations

- Study correlations among spins, here e.g. spins $j_1$ and $j_6$ in different cuboids.
  \[
  \frac{\langle j_{(1,\beta)} j_{(6,\beta')} \rangle - \langle j_{(1,\beta)} \rangle \langle j_{(6,\beta')} \rangle}{\sigma_{j_{(1,\beta)}} \sigma_{j_{(6,\beta')}}}
  \]

- No correlation away from transition, tentatively some correlations between spins close to transition.
Renormalization prescription - scaling

- Can we draw conclusions on \( \hat{A}'_v \) without calculating it explicitly?
  - \( \hat{A}'_v \) constructed to be of the similar form as \( \hat{A}_v \).
- Can we determine \( \hat{A}'_v(\alpha, \{ J \}) \sim \hat{A}_v(\alpha', \{ J \}) \)?
- Scaling argument: Recall \( \hat{A}_v \) is a homogeneous function.

\[
\hat{A}_v(\alpha, \{ \lambda j \}) = \lambda^{12\alpha - 9} \hat{A}_v(\alpha, \{ j \})
\]

- By construction, \( \hat{A}'_v \) is also a homogeneous function.

\[
\hat{A}'_v(\alpha, \lambda \{ J \}) = \lambda^{16(12\alpha - 9) + 24 - 6 - 18} \hat{A}'_v(\alpha, \{ J \}) \overset{!}{=} \lambda^{12\alpha' - 9} \hat{A}'_v(\alpha, \{ J \})
\]

Same scaling behaviour if

\[
\alpha' = 16\alpha - \frac{45}{4},
\]

which indicates an unstable fixed point \( \alpha^* = 0.75 \).
Renormalization prescription - expectation values

- Uniform scaling rare in our simulations!
- Two glued geometric (!) hypercuboids, once given by $\hat{A}_v$, once by $\hat{A}_v'$.
- Total volume fixed, but vary in glueing direction.
- Compare Variance $\Delta V^2$ of the volume of:

$$\langle \Delta V^2 \rangle_{\hat{A}_v(\alpha')} \approx \langle \Delta V^2 \rangle_{\hat{A}_v'(\alpha)}$$

- Determine RG flow for $\alpha$, i.e. $\alpha \to \alpha'$.

Unstable fixed point around $\alpha^* \approx 0.64$. 
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Summary

- **Renormalization scheme for quantum cuboid spin foams**
  - Product of 16 vertex amplitudes
  - **Geometric embedding**: ‘fine cuboids ~ coarse cuboids’
  - Fine spins sum up to coarse spins: $J_i = \sum_{(i,\beta)} j_{(i,\beta)}$

- Studied **expectation values** with **Monte Carlo integration**

- **Indications for phase transition**: ‘irregular’ vs. ‘regular’ configurations
  - **Embedding maps**: face amplitudes peaked on regular subdivisions
  - **Renormalization prescription** from observables.

- **Restoration of diffeomorphism symmetry** at transition?
  - Close to transition: Variance of $\langle j \rangle$ grows, possibly correlations among spins (in different cuboids), but no divergencies.
  - Spins $j$ are bound by $J$ due to embedding map and symmetry. ‘Spikes’ forbidden, no orientation change. [Christodoulou, Långvik, Riello, Röken, Rovelli ’13]

- **Divergencies?** [Freidel, Louapre ’03, Bonzom, Smerlak ’12, Riello ’13, Bonzom, Dittrich ’13, Chen ’15] Orientation change necessary to reobtain symmetry?

- **No toy model**: Quantum cuboids included in EPRL path integral.
Outlook

- **Interesting** results, but **caveats:**
  - Quantum cuboids do **not** capture **curvature** degrees of freedom!
  - EPRL path integral allows many more geometries.
  - Calculation in large $j$ limit
  - Careful in concluding phase transition from ‘fine’ observables

- **Numerical methods** provide **new ways** to study spin foams!
  - Intuition on which geometries dominate
  - Conversely, stimulate analytical advances

- Indications for a **phase transition in more general setting**?
  - Go beyond large $j$ limit
  - Go beyond quantum cuboids

- **Implications** of possible phase transition? Does **diffeomorphism symmetry** get restored?

- Cosmology: Frustums instead of cuboids [Bahr, Klöser, Rabuffo w.i.p.]
- Yang Mills on spin foams [S.St. ’15, Bahr, S.St. w.i.p.]

Thank you for your attention!