Numerical indications for a phase transition in 4D spin foam models

based on w.i.p. with Benjamin Bahr

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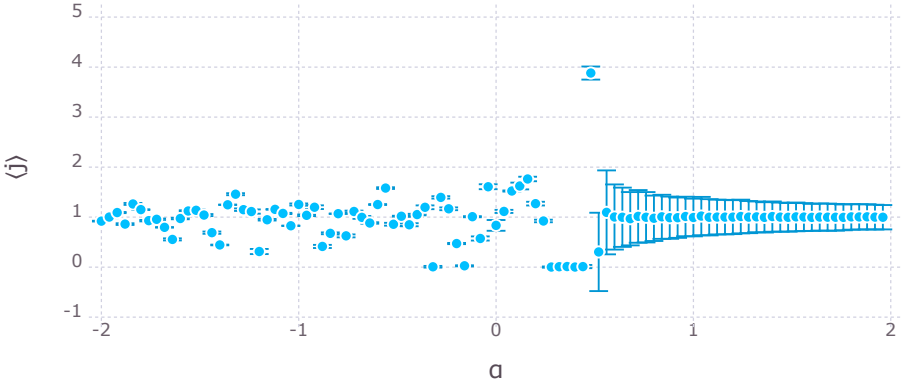
Motivation

- **Spin foams** are a **promising** path integral approach to quantum gravity
  - **Many flavours / formulations** [Barrett, Crane, Engle, Pereira, Rovelli, Livine, Freidel, Krasnov, Speziale, Wieland, Dupuis, Han, Haggard, Kamiński, Hnybida, Banburski, Chen, ...]
  - Lots of **potential** for new **development**!
- Many **ambiguities**: Are spin foams a **consistent theory**?
  - Summing à la **Group field theory**? Is the theory renormalisable? [Oriti, Carrozza, Rivasseau, Gurau, Ben Geloun, Bonzom, Lahoche,...]
  - **Refining**? How are theories related? **Consistent**? [Dittrich, Bahr, S.St., ...]
- Which configurations **dominate** in the path integral for **larger spin foams**?

- Consider **'hypercuboid' spin foams** [Bahr, S.St. '15]
  - **4D EPRL** on **hypercubic lattice + restrictions**.
  - See also Benjamin's ILQGS from September 1st 2015
  - Hypercubes / -cuboids also used in other approaches [Alesci, Cianfrani '15]

An observable of ‘fine’ geometries

- 16 ‘quantum cuboids’ + embedding maps $\sim$ **coarse grained** amplitude
  - Compute **expectation values** by **Monte Carlo techniques**

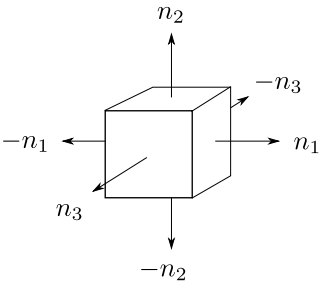


Indications for **phase transition**:
Either **‘irregular’** or **‘regular’** configurations **dominate**.

Outline

Quantum cuboid – coherent intertwiners

- **Quantum cuboids**: **4D EPRL model** [Engle, Pereira, Rovelli, Livine '08] on **hypercubic** 2-complex [Kamiński, Kisielowski, Lewandowski '10]



- **Coherent SU(2) intertwiner** [Livine, Speziale '07] for cuboids:

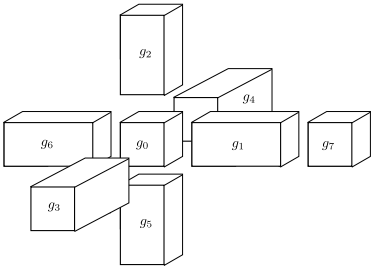
$$|\iota_{j_1,j_2,j_3}\rangle = \int_{\mathrm{SU}(2)} dg\; g \triangleright \bigotimes_{i=1}^{3} |j_i, e_i\rangle \otimes |j_i, -e_i\rangle$$

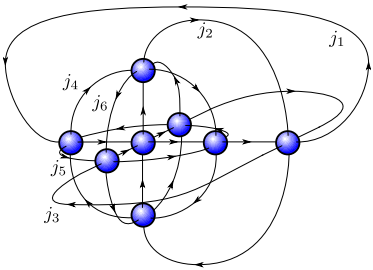
  - e_i: normal unit vectors in $\mathbb{R}^3$.
  - $e_1 = \exp(-i\pi\,\sigma_2/4) \triangleright e_3$ and $e_2 = \exp(i\pi\,\sigma_1/4) \triangleright e_3$.

- **Restriction**: same spin j for 'opposite' faces.

- Boost map + SU(2) × SU(2) group averaging: $|\iota_{j_1,j_2,j_3}\rangle \to |\Phi\,\iota_{j_1,j_2,j_3}\rangle$.
  - $j \to (j^+, j^-) = (\frac{1}{2}\,(1+\gamma)\,j, \frac{1}{2}\,(1-\gamma)\,j)$

Quantum cuboids – amplitudes





- **Vertex amplitude** $\mathcal{A}_v$: contract 8 coherent intertwiners
- $\mathcal{A}_v = \mathcal{A}_v^+ \mathcal{A}_v^-$ for $\gamma < 1$.

$$\mathcal{A}_v^\pm = \int_{\mathrm{SU}(2)^8} dg_a \; e^{S^\pm[g_a]} \; , \text{ with}$$

$$S^\pm[g_a] = \frac{1 \pm \gamma}{2} \sum_l 2j_l \; \ln \langle -\vec{n}_{ab} | \, g_a^{-1} g_b \, | \vec{n}_{ba} \rangle$$

- **Edge amplitude** $\mathcal{A}_e$:

$$\mathcal{A}_e = \| \Phi \iota_{j_1, j_2, j_3} \|^{-2}$$

- **Face amplitude** $\mathcal{A}_f$:

$$\mathcal{A}_f = ((2j_f^+ + 1)(2j_f^- + 1))^\alpha$$

Quantum cuboids – Large j asymptotics

- **Large j asymptotics** of partition function [Freidel, Conrady '08, Barrett, Dowdall, Fairbairn, Gomes, Hellmann '09] (to leading order):

$$
\begin{aligned}
Z &= \sum_{\{j_f\}} \prod_f \mathcal{A}_f \prod_e \mathcal{A}_e \prod_v \mathcal{A}_v \\
&\sim \left(\frac{1-\gamma^2}{4}\right)^{\alpha F - \frac{3}{2}E + \frac{21}{2}V} \sum_{\{j_f\}} \prod_f j_f^{2\alpha} \prod_e (j_1+j_2)(j_1+j_3)(j_2+j_3) \prod_v \tilde{\mathcal{A}}_v^2 \\
&=: \left(\frac{1-\gamma^2}{4}\right)^{(6\alpha - \frac{9}{2})V} \sum_{\{j_f\}} \prod_v \hat{\mathcal{A}}_v(\alpha, \{j_f\})
\end{aligned}
$$

- $\tilde{\mathcal{A}}_v := \left(\frac{1+\gamma}{2}\right)^{\frac{21}{2}} \mathcal{A}_v^+ = \left(\frac{1-\gamma}{2}\right)^{\frac{21}{2}} \mathcal{A}_v^-$.
- In detail $\tilde{\mathcal{A}}_v$ is of the form:

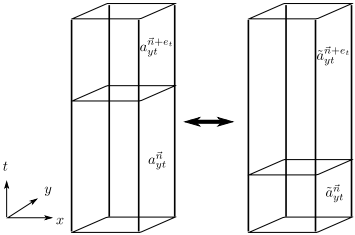
$$
\tilde{\mathcal{A}}_v \sim \sum_c \sqrt{\frac{1}{-\det(H(\vec{g}_c))}}\, e^{S[g_c]} \quad \text{with } S[g_c] = 0
$$

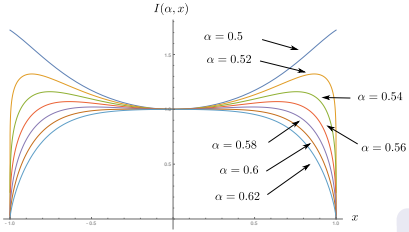
On the amplitude $\hat{\mathcal{A}}_v$

- $\hat{\mathcal{A}}_v(\alpha, \{j_f\})$ contains **all important ingredients** of the model.
- $\hat{\mathcal{A}}_v(\alpha)$ via face amplitude $\mathcal{A}_f(\alpha) \sim j_f^{2\alpha}$
  - $\alpha \sim$ **coupling constant**
- $\hat{\mathcal{A}}_v(j_1, j_2, j_3, j_4, j_5, j_6)$ depends on 6 spins $\{j\}$
  - Normally hypercuboid has 24 areas, 4 per pair of directions
  - **'Cuboid symmetry': assign same spin to these areas**
- $\hat{\mathcal{A}}_v(\alpha, \{\lambda j\}) = \lambda^{12\alpha-9} \hat{\mathcal{A}}_v(\alpha, \{j\})$

- Quantum cuboids are a **restriction** of the EPRL model
  - But they are **contained** in the EPRL model.
- Quantum cuboids **cannot** capture **curvature** degrees of freedom.
  - Quantum cuboids can be **non-geometric**: cuboid not described by 4 edge lengths $\rightarrow$ **twisted geometries** [Freidel, Speziale '10, Freidel, Ziprick '14]
- We only discuss the **large j limit** today – already interesting!

‘Diffeomorphism’ symmetry? – Hints for a phase transition





- Consider two hypercuboids glued along a 3D cube in (xyz) direction.
- Spins in (xyz) direction set to j, in t-direction to $j(1+x)$ / $j(1-x)$, $x \in [-1, 1]$.

$$I(\alpha, x) = \hat{\mathcal{A}}_v(\alpha, \{j\}, \{j(1+x)\}) \times \hat{\mathcal{A}}_v(\alpha, \{j\}, \{j(1-x)\})$$

- $\alpha < 0.6$: **irregular** configuration preferred
- $\alpha > 0.6$: **regular** configuration preferred

Face amplitudes suppress irregular configurations. Cause?
No free variations, we keep the total area fixed.

Outline

Spin foams as a theory

- What does 'summing over all spin foams' mean?
- Two conceptually different approaches tackling these questions:

- **Group field theory approach**: Summing over all triangulations and topologies [Oriti, Carrozza, Rivasseau, Ben Geloun, Gurau, Bonzom, Lahoche,...]
  - GFT generates spin foam amplitudes as Feynman diagrams
  - **Renormalisable (as a QFT)?**
  - Spin foam vertices $\sim$ 'atoms of spacetime'

- **Refining approach**: renormalize the spin foam amplitudes by coarse graining [Dittrich, Bahr, Martin-Benito, S.St., Mizera,...]
  - How is a coarse spin foam related to a fine spin foam? Boundary data?
  - **Results consistent? Discretization dependent?**
  - 2-complex is a regulator.

Complementary approaches, not opposing!

Refinement and embedding maps

- Suppose $\psi_b \in \mathcal{H}_b$ and $\phi_{b'} \in \mathcal{H}_{b'}$.
  - b and b' can be boundaries of **different complexity**.
  - $\mathcal{H}_b$ and $\mathcal{H}_{b'}$ are **different boundary Hilbert spaces** [Oeckl '03].
- Need **embedding maps** $\iota_{b,b''} : \mathcal{H}_b \hookrightarrow \mathcal{H}_{b''}$ and $\iota_{b',b''} : \mathcal{H}_{b'} \hookrightarrow \mathcal{H}_{b''}$.
  - $b \prec b''$ and $b' \prec b''$, b'' **common refinement** of b and b'.
- **Cylindrical consistency conditions**: $\forall b \prec b' \prec b''$

$$\iota_{b',b''} \circ \iota_{b,b'} = \iota_{b,b''}$$

- Compare states in different Hilbert spaces and define **equivalence classes**:

$$\psi_b \sim \phi_{b'} \iff \iota_{b,b''}(\psi_b) = \iota_{b',b''}(\phi_{b''})$$

- **Inductive limit construction**: $\mathcal{H} := \cup_b \mathcal{H}_b / \sim$

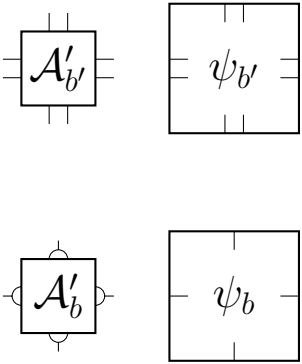
Embedding maps and cylindrical consistency

- Different embedding maps in **loop quantum gravity**
- **Ashtekar–Lewandowski vacuum** [Ashtekar, Isham '92, Ashtekar, Lewandowski '95]:
  - Arbitrary refinement of graphs
  - New edges carry spin $j = 0$.
- **'New' BF-vacuum** [Dittrich, Geiller '15, Bahr, Dittrich, Geiller '15]:
  - Refinement of triangulations by Pachner / Alexander moves
  - New closed holonomies are flat.
- Cylindrical consistency: how to **refine boundaries** and how to **add degrees of freedom in vacuum state**.
  - Vacuum refers to representing the same state (w.r.t. an inner product) on a finer boundary.
- Consider as well variational discrete systems [Dittrich, Höhn '12, '13, Höhn '14].

These concepts work well in the **kinematical** setting.

Can they be extended to the dynamical / physical setting?

Spin foam amplitudes and embedding maps



- The question of cylindrical consistency also arises for spin foam models.
- Consider two spin foam amplitudes, where $b \prec b'$:
  - $\mathcal{A}_b : \mathcal{H}_b \to \mathbb{C}$
  - $\mathcal{A}'_{b'} : \mathcal{H}_{b'} \to \mathbb{C}$
- Relate / renormalize spin foam amplitudes:

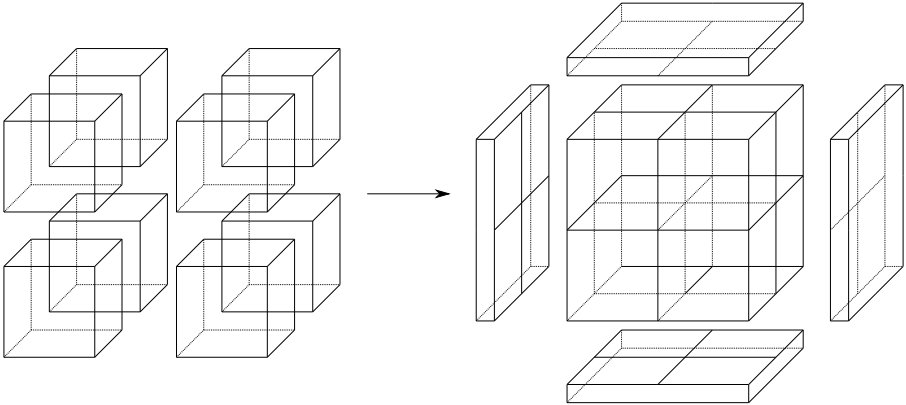
$$\mathcal{A}'_{b'}(\underbrace{\iota_{b,b'}(\psi_b)}_{=\psi_{b'}}) := \mathcal{A}'_b(\psi_b) \stackrel{?}{=} \mathcal{A}_b(\psi_b)$$

- $\iota_{b,b'}$ relates amplitudes $\mathcal{A}_b$ and $\mathcal{A}'_{b'}$, but **need not be dynamical**.
- **Dynamical** embedding maps [Dittrich '12, '14] from **spin foam amplitudes** [Dittrich, S.St. '14]
  - Realized for **analogue spin foam models** [Dittrich, Eckert, Martin-Benito '12, Dittrich, Martin-Benito, Schnetter '13, Dittrich, Martin-Benito, S.St. '14, S.St. '15] and LGT [Dittrich, Mizera, S.St. '14] via **tensor network renormalization** [Levin, Nave '07, Gu, Wen '09, Vidal, Evenbly '15].

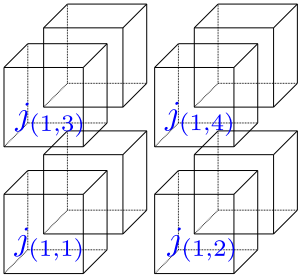
Outline

Quantum cuboid renormalization: Strategy

- Renormalize **quantum cuboid amplitude** $\hat{\mathcal{A}}_v$
- Combine several quantum cuboids (in most cases: 16): $\hat{\mathcal{A}}_v^{(16)}$
- Employ **embedding maps** Υ to $\hat{\mathcal{A}}_v^{(16)}$ to get $\hat{\mathcal{A}}'_v$



Before employing embedding maps



- Consider the product of 16 hypercuboids:

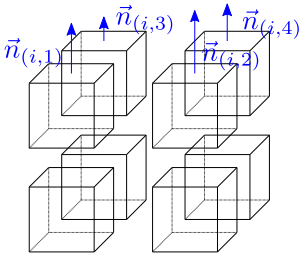
$$\hat{\mathcal{A}}_v^{(16)}(\{j_{i,\beta}\}) := \prod_{k=1}^{16} \hat{\mathcal{A}}_v(\{j_{i,\beta}\})$$

  - 'Coarse' face subdivided into four 'fine' faces
  - 4 'fine' spins $j_{i,\beta}$, $\beta = 1, \ldots, 4$, for each $i = 1, \ldots, 6$

- **'Cuboid symmetry'**: all spins $\{j_{i,\beta}\}$ are fixed as boundary data
- Compared to original cuboid: **More boundary data**
  - 24 spins $\{j_{i,\beta}\}$ compared to 6 spins $\{j_i\}$
  - More complicated 'boundary graph': 8 'subdivided' 3D cuboids / intertwiners instead of 8 cuboids / intertwiners.

Construct an **embedding map** relating 8 'fine' (coherent) intertwiners to one 'coarse' (coherent) intertwiner by relating 'fine' spins to 'coarse' spins.

A geometric embedding map I



- Recall **'coherent cuboid'** construction:
  - SU(2) coherent states labelled $\vec{n}_i = j_i e_i$
  - $\vec{n}_i \cdot \vec{n}_j = 0$ if i, j neighbours
- **Glued hypercuboids** in large j limit:
  - Glued along a shared 3D cuboid
  - Hypercuboid is flat
- **Geometry highly peaked on cuboids**

- Geometric **intuition** in large j limit: 'Coarse' cuboids from 'fine' cuboids
  - Normal vectors $\vec{n}_i$ in the same subdivided face point in the same direction.

- **Embed** $|\Phi\iota_{J_1,J_2,J_3}\rangle \hookrightarrow \bigotimes_{k=1}^{8} \left|\Phi\iota_{\{j_{(i,\beta)}\}_{(i,\beta)\in k}}\right\rangle$
  - J_i, $i = 1, \ldots, 3$ label 'coarse' spins.
  - $j_{(i,\beta)}$, $\beta = 1, \ldots, 4$, label fine spins in 'coarse' face i.
- **3 conditions** $J_i = \sum_{\beta=1}^{4} j_{(i,\beta)}$.

A geometric embedding map II

- $J_i = \sum_{\beta=1}^{4} j_{(i,\beta)}$: 'fine areas sum up to coarse area'
- **Choice! Embedding map non-dynamical!**

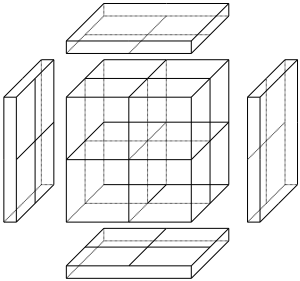
$$\Upsilon : \mathrm{Inv}(V_{J_1} \otimes V_{J_2} \otimes V_{J_3}) \hookrightarrow \bigotimes_{k=1}^{8} \mathrm{Inv}\left(\bigotimes_{(i,\beta)\in k} V_{j_{(i,\beta)}} \right)$$

$$|\Phi\iota_{J_1,J_2,J_3}\rangle \mapsto \sum_{\{j_{(i,\beta)}\}}^{\sum_\beta j_{(i,\beta)} \overset{!}{=} J_i} c^{J_1,J_2,J_3}_{\{j_{(i,\beta)}\}} \bigotimes_{k=1}^{8} \left| \Phi\iota_{\{j_{(i,\beta)}\}_{(i,\beta)\in k}} \right\rangle$$

- $c^{J_1,J_2,J_3}_{\{j_{(i,\beta)}\}}$ fixed by $\|\Phi\iota_{J_1,J_2,J_3}\| \overset{!}{=} 1$
  - $\left| \Phi\iota_{\{j_{(i,\beta)}\}_{(i,\beta)\in k}} \right\rangle$ **already normalized** (edge amplitude!)
- $\left(c^{J_1,J_2,J_3}_{\{j_{(i,\beta)}\}} \right)^{-1} = \sqrt{\prod_{i=1}^{3} N_{J_i}} = \sqrt{\prod_{i=1}^{3} \binom{2J_i-1}{4-1}} \sim \left(\frac{4}{3} J_1 J_2 J_3 \right)^{\frac{3}{2}}$

The renormalized amplitude

- Conversely, we use the embedding map to **project on coarse boundary data**.



- Sum over 'fine' configurations $\{j_{(i,\beta)}\}$ contributing to the 'coarse' configuration $\{J_i\}$.
- Relation 'fine' $\to$ 'coarse' essentially captured in factors $c^{J_1,J_2,J_3}_{\{j_{(i,\beta)}\}}$.
- Large j limit: $\sum_{\{j_{(i,\beta)}\}} \sim \int \prod_{(i,\beta)} dj_{(i,\beta)}$

$$\hat{\mathcal{A}}'_v(\alpha, J_1, J_2, J_3, J_4, J_5, J_6) :=$$
$$\int_0^{J_i} \prod_{(i,\beta)} dj_{(i,\beta)} \prod_{i=1}^{6} \delta(\sum_{\beta=1}^{4} j_{(i,\beta)} - J_i) \prod_{k=1}^{8} c^{(k),\{J_i\}}_{\{j_{(i,\beta)}\}} \hat{\mathcal{A}}^{(16)}_v(\alpha, \{j_{(i,\beta)}\})$$

Outline

What can we learn about $\hat{\mathcal{A}}'_v$?

- $\hat{\mathcal{A}}'_v$: 18-dim. integral – use **numerical methods**
- **Monte Carlo integration**:
  - **Sample** N points in 18-dim. configuration space (w.r.t. a probability distribution) to **approximate** integral.
  - **Error drops off** with $1/\sqrt{N}$.
- **Possible to compute** $\hat{\mathcal{A}}'_v$ in such a way, but **not naively**.
  - Sampling randomly over configuration space **converges very slowly**. Advanced techniques necessary, e.g. dividing into smaller volumes.
  - **Mathematica quote**: "Catastrophic loss of precision."

Instead, we evaluate **expectation values of observables** $\mathcal{O}$:

$$\langle \mathcal{O} \rangle = \frac{\int_{(\sum_{\beta=1}^{4} j_{(i,\beta)} = J_i)} \prod_{(i,\beta)} dj_{(i,\beta)} \; \mathcal{O}(J_i, j_i) \, \mathcal{I}(j_i, J_i)}{\int_{(\sum_{\beta=1}^{4} j_{(i,\beta)} = J_i)} \prod_{(i,\beta)} dj_{(i,\beta)} \; \mathcal{I}(j_i, J_i)} \quad ,$$

with $\mathcal{I}(j_i, J_i) := \prod_{k=1}^{8} c^{(k),\{J_i\}}_{\{j_{(i,\beta)}\}} \; \hat{\mathcal{A}}_v^{(16)}(\alpha, \{j_{(i,\beta)}\})$.

Sampling w.r.t. $\mathcal{I}$ and observables

- Compute $\langle \mathcal{O} \rangle$ with **Monte Carlo methods**.
- **Random walk** sampling w.r.t. $\mathcal{I}(j_i, J_i)$:
  - $\{j_{(i,\beta)}\}_n \to \{j_{(i,\beta)}\}_{n+1}$ randomly generated
  - Define $p_{n\to n+1} := \left|\mathcal{I}(\{j_{(i,\beta)}\}_{n+1})\right| / \left|\mathcal{I}(\{j_{(i,\beta)}\}_n)\right|$
  - Generate random number $0 < r < 1$.
  - Accept $\{j_{(i,\beta)}\}_{n+1}$ if $r < \min(p_{n\to n+1}, 1)$.
- **Information** on $\mathcal{I}$ stored in samples $\{j_{(i,\beta)}\}_n$.

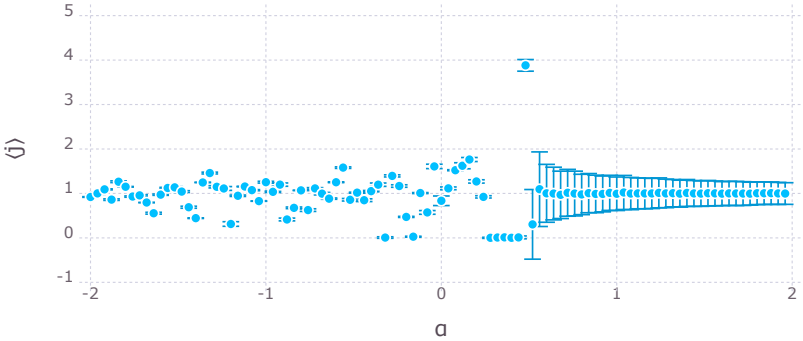
$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(J_i, \{j_{(i,\beta)}\}_n)$$

  - Note that $\mathcal{I}$ does not need to be normalized.

- Interesting observables:
  - $\langle j_{(i,\beta)} \rangle$: 'fine' spin
  - $\langle V^{(\text{fine})} - V^{(\text{coarse})} \rangle$: 'fine' volume - 'coarse' volume
  - $\langle j_{(i,\beta)} j_{(i',\beta')} \rangle - \langle j_{(i,\beta)} \rangle \langle j_{(i',\beta')} \rangle$: correlations between spins

Observable: fine spin $\langle j \rangle$

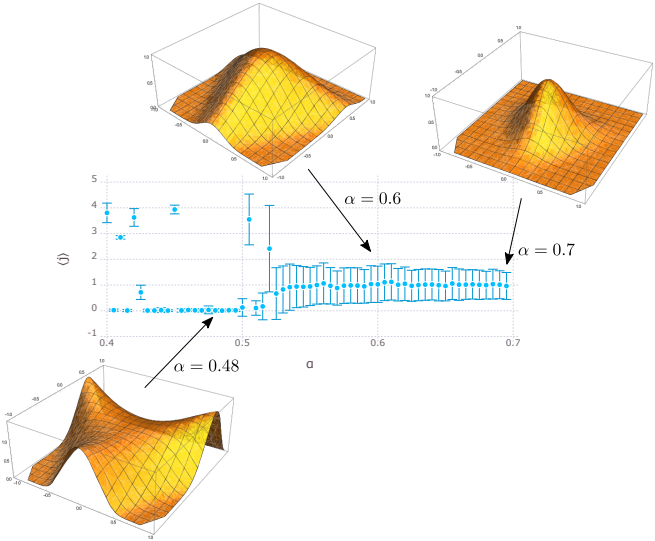
- Expectation value of j depending on α (for all $J = 4$):



- For $\alpha > 0.52$: $\langle j \rangle = \frac{J}{4}$ – coarse cube subdivided into 16 'regular' cuboids.
- For $\alpha < 0.52$: $\langle j \rangle$ fluctuates between 0 and J. One large and 15 small cuboids – 'irregular'
- Variance Δj grows closer to the transition.

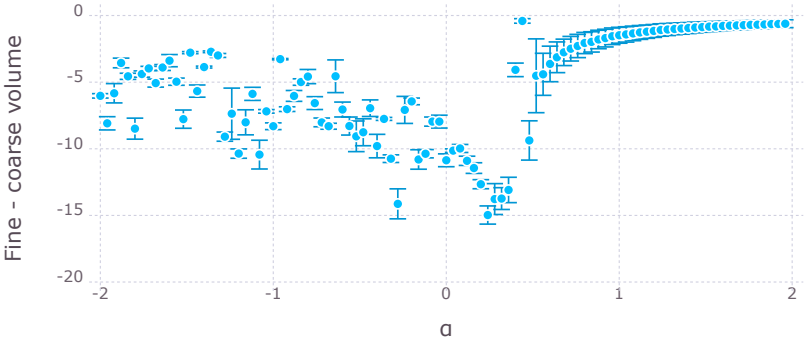
The transition in pictures

- Plots show $\mathcal{I}(\alpha, \{j_{(i,\beta)}\})$ for solved constraints $\sum_\beta j_{(i,\beta)} = J_i$ and many $j_{(i,\beta)}$ chosen to be equal. Normalized to regular subdivisions.



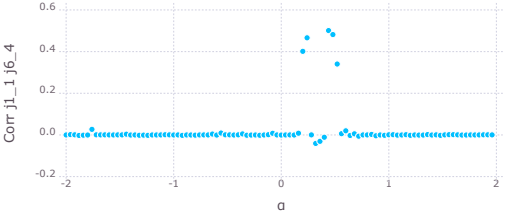
Observable: $V^{(\text{fine})} - V^{(\text{coarse})}$

- Volume of cuboids given as a function of spins
  - $V = \left(\prod_{i=1}^{6} j_i\right)^{\frac{1}{3}}$
- Compare the volume of fine cuboids and the volume of the coarse cuboid.
- Indication for (non–)geometric contributions.



Observable: correlations

- Study correlations among spins, here e.g. spins j_1 and j_6 in different cuboids.
  - $\left(\langle j_{(1,\beta)} j_{(6,\beta')}\rangle - \langle j_{(1,\beta)}\rangle\langle j_{(6,\beta')}\rangle\right) / (\sigma_{j_{(1,\beta)}}\sigma_{j_{(6,\beta')}})$



- No correlation away from transition, **tentatively** some correlations between spins close to transition.

Renormalization prescription - scaling

- Can we draw conclusions on $\hat{\mathcal{A}}'_v$ without calculting it explicitly?
  - $\hat{\mathcal{A}}'_v$ constructed to be of the similar form as $\hat{\mathcal{A}}_v$.
- Can we determine $\hat{\mathcal{A}}'_v(\alpha, \{J\}) \sim \hat{\mathcal{A}}_v(\alpha', \{J\})$?
- Scaling argument: Recall $\hat{\mathcal{A}}_v$ is a homogeneous function.

$$\hat{\mathcal{A}}_v(\alpha, \{\lambda j\}) = \lambda^{12\alpha-9}\hat{\mathcal{A}}_v(\alpha, \{j\})$$

- By construction, $\hat{\mathcal{A}}'_v$ is also a homogeneous function.

$$\hat{\mathcal{A}}'_v(\alpha, \lambda\{J\}) = \lambda^{16(12\alpha-9)+24-6-18}\hat{\mathcal{A}}'_v(\alpha, \{J\}) \stackrel{!}{=} \lambda^{12\alpha'-9}\hat{\mathcal{A}}'_v(\alpha, \{J\})$$

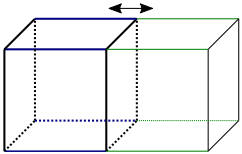
Same scaling behaviour if

$$\alpha' = 16\alpha - \frac{45}{4} \quad ,$$

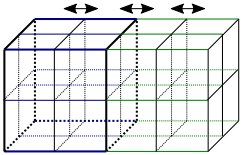
which indicates an unstable **fixed point** $\alpha^* = 0.75$.

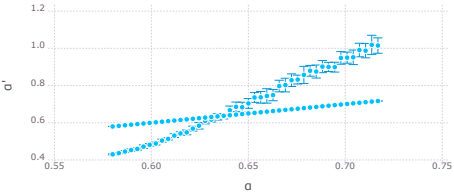
Renormalization prescription - expectation values

$\langle \Delta V^2 \rangle_{\hat{\mathcal{A}}_v}$



$\langle \Delta V^2 \rangle_{\hat{\mathcal{A}}'_v}$





- Uniform scaling rare in our simulations!
- Two glued geometric (!) hypercuboids, once given by $\hat{\mathcal{A}}_v$, once by $\hat{\mathcal{A}}'_v$.
- Total volume fixed, but vary in glueing direction.
- Compare Variance ΔV^2 of the volume of :

$$\left\langle \Delta V^2 \right\rangle_{\hat{\mathcal{A}}_v(\alpha')} \approx \left\langle \Delta V^2 \right\rangle_{\hat{\mathcal{A}}'_v(\alpha)}$$

- Determine **RG flow** for α, i.e. $\alpha \to \alpha'$.

Unstable **fixed point** around $\alpha^* \approx 0.64$.

Outline

Summary

- **Renormalization scheme for quantum cuboid spin foams**
  - Product of 16 vertex amplitudes
  - **Geometric embedding**: 'fine cuboids $\sim$ coarse cuboids'
  - Fine spins sum up to coarse spins: $J_i = \sum_{(i,\beta)} j_{(i,\beta)}$
- Studied **expectation values** with **Monte Carlo integration**
- **Indications** for **phase transition**: 'irregular' vs. 'regular' configurations
  - **Embedding maps**: face amplitudes peaked on regular subdivisions
  - **Renormalization prescription** from observables.
- **Restoration of diffeomorphism symmetry** at transition?
  - Close to transition: Variance of $\langle j \rangle$ grows, possibly correlations among spins (in different cuboids), but no divergencies.
  - Spins j are bound by J due to embedding map and symmetry. 'Spikes' forbidden, no orientation change. [Christodoulou, Långvik, Riello, Röken, Rovelli '13]
  - **Divergencies?** [Freidel, Louapre '03, Bonzom, Smerlak '12, Riello '13, Bonzom, Dittrich '13, Chen '15] Orientation change necessary to reobtain symmetry?
- **No toy model**: Quantum cuboids included in EPRL path integral.

Outlook

- **Interesting** results, but **caveats**:
  - Quantum cuboids do **not** capture **curvature** degrees of freedom!
  - EPRL path integral allows many more geometries.
  - Calculation in large j limit
  - **Careful** in concluding phase transition from 'fine' observables
- **Numerical methods** provide **new ways** to study spin foams!
  - Intuition on which geometries dominate
  - Conversely, stimulate analytical advances
- Indications for a **phase transition** in **more general setting**?
  - Go beyond large j limit
  - Go beyond quantum cuboids
- **Implications** of possible phase transition? Does **diffeomorphism symmetry** get restored?

- Cosmology: Frustums instead of cuboids [Bahr, Klöser, Rabuffo w.i.p.]
- Yang Mills on spin foams [S.St. '15, Bahr, S.St. w.i.p.]

Thank you for your attention!