Developments on the radial gauge

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Plan of the talk

Introduction

What is the radial gauge all about?

Spacetime radial gauge

Who is interested?

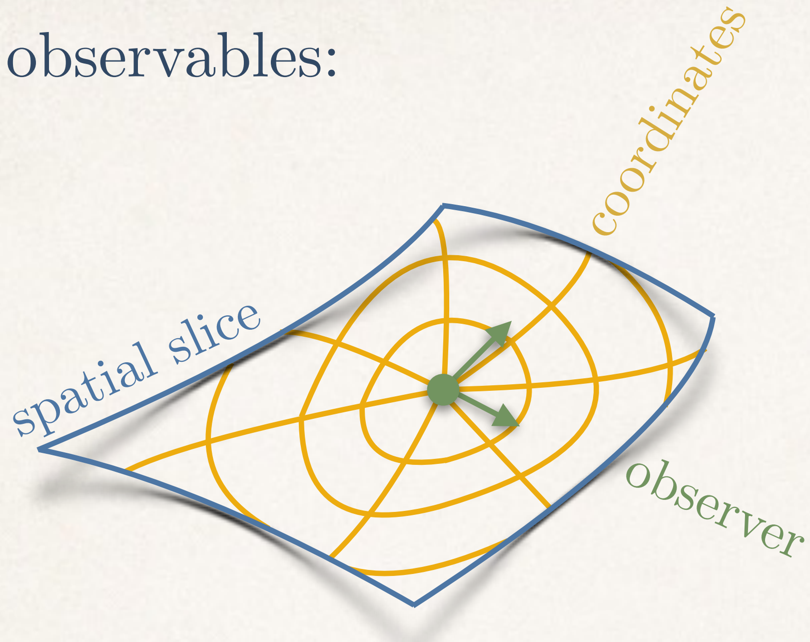
What can we tell them?

What can we learn from that?

Conclusion

Introduction

observer's observables:



1. introduce the observer
2. define coordinates adapted to the spatial metric
3. define relational observables being pull-backs of canonical fields
4. their Poisson algebra is under control

Duch, Kamiński, Lewandowski, JŚ JHEP05(2014)077, JHEP04(2015)075

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radial gauge:

1. metric has the form

$$q_{ab} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & \multicolumn{2}{c}{q_{AB}} \end{bmatrix}$$

2. Dirac bracket algebra is

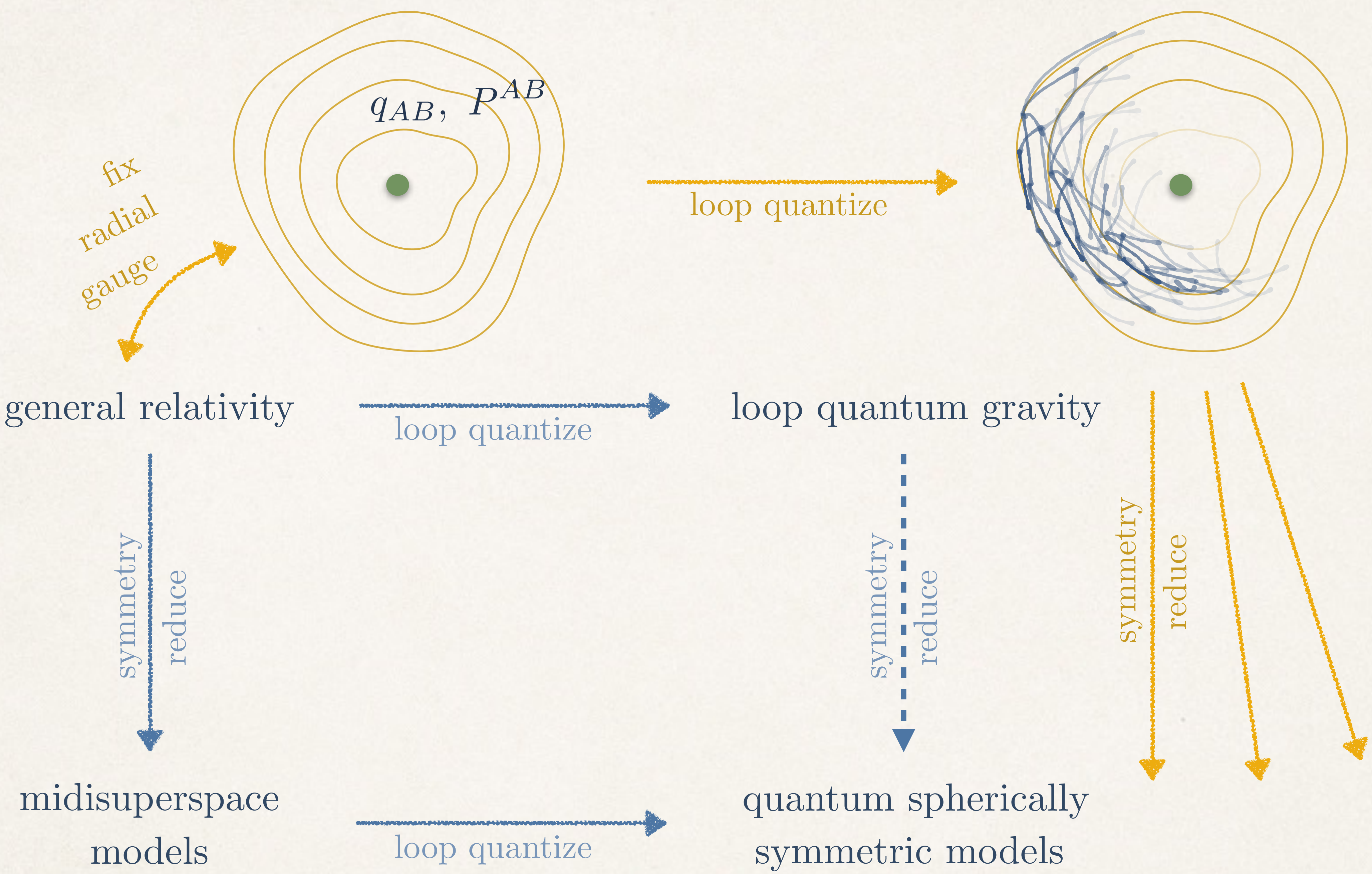
$$\{q_{AB},\ P^{CD}\} = \delta\delta\delta$$
$$\{\phi,\ \pi\} = \delta$$
$$\{\phi,\ P^{AB}\} = 0 \text{ etc.}$$

$$\{\cdot,\ P^{ra}\} = \text{nontrivial}$$

3. solve the vector constraint for P^{ra} and plug it into the Hamiltonian

Bodendorfer, Lewandowski, JŚ Phys.Rev. D92 (2015) 8, 084041

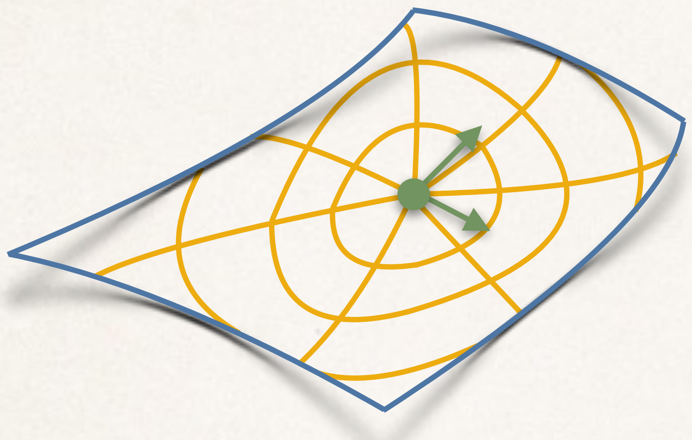
A quantum reduction to spherical symmetry



Bodendorfer, Lewandowski, JŚ Phys.Lett. B747 (2015) 18-21

Spacetime radial gauge

radial gauge



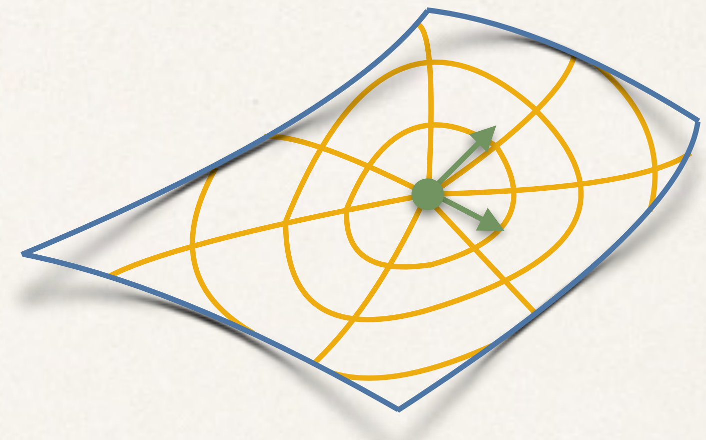
radial lines are spatial geodesics iff: $\overset{(3)}{\Gamma}{}^{a}_{rr} = 0$

$$\overset{(3)}{\Gamma}{}^{a}_{rr} = \frac{1}{2} q^{ab} (2 q_{rb,r} - q_{rr,b})$$

so the gauge conditions are

$$q_{ra} = \delta_{ra}$$

spacetime radial gauge



radial lines are spacetime geodesics iff: $\overset{(4)}{\Gamma}{}^{\mu}_{rr} = 0$

$$\overset{(4)}{\Gamma}{}^{t}_{rr} = \frac{1}{N} K_{rr}$$

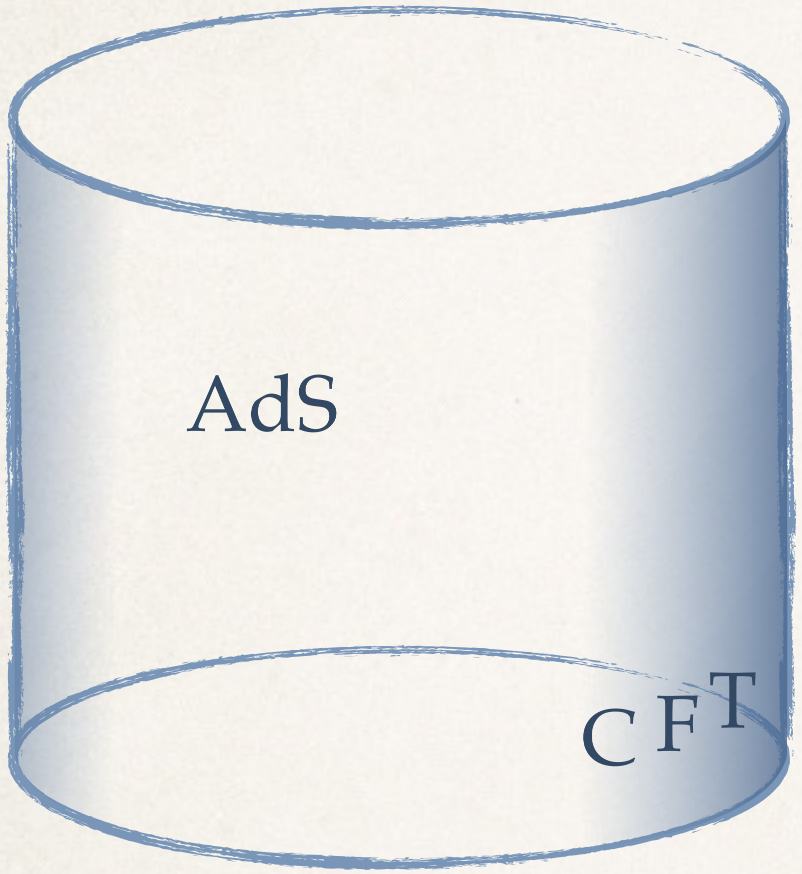
$$\overset{(4)}{\Gamma}{}^{a}_{rr} = -\frac{N^a}{N} K_{rr} + \frac{1}{2} q^{ab} (2 q_{rb,r} - q_{rr,b})$$

so the gauge conditions are

$$q_{ra} = \delta_{ra}$$
$$K_{rr} = 0$$

Spacetime radial gauge *

* — also known as holographic, axial or Fefferman-Graham gauge



AdS/CFT correspondence — boundary perspective:

construct operators in CFT
which mimic observables in the bulk

to deal with the gauge redundancy in the bulk
a suitable gauge is introduced

a desired feature is that at least matter fields
are commuting at spacelike separation

are they commuting in spacetime radial gauge?

yes | 1 | Kabat, Lifschytz, *Decoding the hologram: Scalar fields interacting with gravity*, Physical Review D 89 (2014) 066010

no | 2 | Donnelly, Giddings, *Diffeomorphism-invariant observables and their nonlocal algebra* arXiv:1507.07921 [hep-th]

Spacetime radial gauge

$$H[N] = \int d^3x N \left(\frac{2\kappa}{\sqrt{q}} \left(P^{ab}P_{ab} - \frac{1}{2}P^2 \right) - \frac{\sqrt{q}}{2\kappa} R^{(3)} + h^{\mathrm{matt}} \right)$$

$$C_a[N^a] = \int d^3x N^a \left(-2\nabla_b P^b{}_a + P_\phi \partial_a \phi \right)$$

1. the constraints are $\mathfrak{C}_\alpha = (H, C_r, C_A, K_{rr}, q_{rr} - 1, q_{rA})$

2. Dirac bracket is given by $\{\mathcal{O}_1,\ \mathcal{O}_2\}_D = \{\mathcal{O}_1,\ \mathcal{O}_2\} - \sum_{\alpha,\beta=1}^{8} \{\mathcal{O}_1,\ \mathfrak{C}_\alpha\}(M^{-1})_{\alpha\beta}\{\mathfrak{C}_\beta,\ \mathcal{O}_2\}$

3. where $M_{\alpha\beta}(r,\theta;\bar{r},\bar{\theta}) = \{\mathfrak{C}_\alpha(r,\theta), \mathfrak{C}_\beta(\bar{r},\bar{\theta})\} =$

$$= \begin{bmatrix} 0 & 0 & 0 & R^{(3)}_{rr} - 2K_{Ar}K^{Ar} + \mathfrak{t}^{\mathrm{matt}} - \partial_r^2 & 0 & -2K_{rA} \\ & 0 & 0 & 0 & 2\partial_r & \partial_B \\ & & 0 & 2\partial_r K_{rB} & 0 & \partial_r q_{AB} \\ & & & 0 & -\frac{\kappa}{\sqrt{\det q}} & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{bmatrix}_{\alpha\beta} \delta(r,\theta;\bar{r},\bar{\theta})$$

4. and $\mathfrak{t}^{\mathrm{matt}}(r,\theta)\delta(r,\theta;\bar{r},\bar{\theta}) = \frac{\kappa}{\sqrt{q}} \left(\frac{\delta h^{\mathrm{matt}}(r,\theta)}{\delta q_{rr}(\bar{r},\bar{\theta})} - q_{AB} \frac{\delta h^{\mathrm{matt}}(r,\theta)}{\delta q_{AB}(\bar{r},\bar{\theta})} - \frac{1}{2} h^{\mathrm{matt}}(r,\theta)\delta(r,\theta;\bar{r},\bar{\theta}) \right)$

Spacetime radial gauge

$$\mathfrak{C}_\alpha = (H, C_r, C_A, K_{rr}, q_{rr}-1, q_{rA})$$

1. $$\begin{bmatrix} 0 & 0 & 0 & R^{(3)}_{rr} - 2K_{Ar}K^{Ar} + \mathfrak{t}^{\mathrm{matt}} - \partial_r^2 & 0 & -2K_{rA} \\ & 0 & 0 & 0 & 2\partial_r & \partial_B \\ & & 0 & 2\partial_r K_{rB} & 0 & \partial_r q_{AB} \\ & & & 0 & -\frac{\kappa}{\sqrt{\det q}} & 0 \\ & & & & 0 & 0 \\ & & & & & 0 \end{bmatrix} = \begin{bmatrix} 0 & F \\ -F^T & G \end{bmatrix}$$

2. inverse is given by $$\begin{bmatrix} 0 & F \\ -F^T & G \end{bmatrix}^{-1} = \begin{bmatrix} (F^{-1})^T G F^{-1} & -(F^{-1})^T \\ F^{-1} & 0 \end{bmatrix}$$

3. to find F^{-1} we need to solve $$\sum_{\beta=1}^{4} \int d\bar{r} d^2\bar{\theta}\, F_{\alpha\beta}(r,\theta;\bar{r},\bar{\theta}) N_\beta(\bar{r},\bar{\theta}) = M_\alpha(r,\theta)$$

4. that is

$$\begin{aligned} \partial_r^2 N + (-R^{(3)}_{rr} + 2K_{Ar}K^{Ar} - \mathfrak{t}^{\mathrm{matt}})N + 2K_{rA}N^A &= -M \\ 2\partial_r N^r + \partial_B N^B &= M_r \\ \partial_r N_A + 2\partial_r (K_{rA} N) &= M_A \end{aligned}$$

Spacetime radial gauge

$$\partial_r^2 N + (-R^{(3)}_{rr} + 2K_{Ar}K^{Ar} - \mathfrak{t}^{\mathrm{matt}})N + 2K_{rA}N^A = -M$$

$$2\partial_r N^r + \partial_B N^B = M_r$$

$$\partial_r N_A + 2\partial_r(K_{rA}N) = M_A$$

$$N_A(r,\theta) + 2K_{rA}N(r,\theta) = \ldots$$

$$N_{[M_\alpha]}$$

$$N^A_{[M_\alpha]}$$

$$N^r_{[M_\alpha]}$$

$$N_{\beta[M_\alpha]} = (F^{-1})_{\beta\alpha}M_\alpha$$

Spacetime radial gauge

$$\mathfrak{C}_\alpha = (H, C_r, C_A, K_{rr}, q_{rr} - 1, q_{rA})$$

1. $$\{\mathcal{O}_1,\ \mathcal{O}_2\}_D = \{\mathcal{O}_1,\ \mathcal{O}_2\} - \sum_{\alpha,\beta=1}^{8} \{\mathcal{O}_1,\ \mathfrak{C}_\alpha\} \begin{bmatrix} (F^{-1})^T G F^{-1} & -(F^{-1})^T \\ F^{-1} & 0 \end{bmatrix}_{\alpha\beta} \{\mathfrak{C}_\beta,\ \mathcal{O}_2\}$$

2. $$\{\phi(r_1,\theta_1),\ \phi(r_2,\theta_2)\}_D = \int drd^2\theta\ N^r_{[\{\phi(r_1,\theta_1),\ \mathfrak{C}_\alpha\}]}(r,\theta)\ \frac{\kappa}{\sqrt{\det q(r,\theta)}}\ N_{[\{\mathfrak{C}_\alpha,\ \phi(r_2,\theta_2)\}]}(r,\theta)$$
$$- \int drd^2\theta\ N_{[\{\phi(r_1,\theta_1),\ \mathfrak{C}_\alpha\}]}(r,\theta)\ \frac{\kappa}{\sqrt{\det q(r,\theta)}}\ N^r_{[\{\mathfrak{C}_\alpha,\ \phi(r_2,\theta_2)\}]}(r,\theta)$$

3. it leads to the following conclusions:

   a) in spacetime radial gauge matter fields do not commute

   b) in spacetime radial gauge matter fields have non-local brackets

   c) the non-localities vanish in weak-gravity limit since $\kappa \sim G_{\mathrm{Newton}}$

   d) to guarantee simple matter sector, the gravitational gauge fixing conditions should be mutually commuting

Conclusion

Radial gauge

is a useful tool for dealing with spatial diffeomorphisms in canonical GR

leads to a novel definition of spherical symmetry on the quantum level

Spacetime radial gauge

is of interest for AdS/CFT correspondence

has non-local Dirac brackets (also in the matter sector)

commutativity of gravitational gauge fixings is crucial for commutation properties of matter sector