Developments on the radial gauge

Jędrzej Świeżewski
Faculty of Physics, University of Warsaw

in collaboration with Norbert Bodendorfer, Paweł Duch, Wojciech Kamiński and Jerzy Lewandowski

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Plan of the talk

Introduction

What is the radial gauge all about?

Spacetime radial gauge

Who is interested?

What can we tell them?

What can we learn from that?

Conclusion
observer’s observables:

1. introduce the observer
2. define coordinates adapted to the spatial metric
3. define relational observables being pull-backs of canonical fields
4. their Poisson algebra is under control

radial gauge:

2. Dirac bracket algebra is
\[ \{ q_{AB}, P^{CD} \} = \delta \delta \delta \]
\[ \{ \phi, \pi \} = \delta \]
\[ \{ \phi, P^{AB} \} = 0 \text{ etc.} \]
\[ \{ \cdot, P^{ra} \} = \text{nontrivial} \]

3. solve the vector constraint for \( P^{ra} \) and plug it into the Hamiltonian

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Duch, Kamiński, Lewandowski, JŚ JHEP05(2014)077, JHEP04(2015)075

A quantum reduction to spherical symmetry

fix radial gauge

general relativity

midisuperspace models

symmetry reduce

quantum spherically symmetric models

loop quantize

loop quantize

loop quantum gravity

loop quantize
radial lines are spatial geodesics iff: $\Gamma^a_{rr} = 0$

$$\Gamma^a_{rr} = \frac{1}{2} q^{ab} (2 q_{rb,r} - q_{rr,b})$$

so the gauge conditions are

$q_{ra} = \delta_{ra}$

radial lines are spacetime geodesics iff: $\Gamma^{\mu}_{rr} = 0$

$$\Gamma^{\mu}_{rr} = \frac{1}{N} K_{rr}$$

$$\Gamma^{a}_{rr} = -\frac{N^a}{N} K_{rr} + \frac{1}{2} q^{ab} (2 q_{rb,r} - q_{rr,b})$$

so the gauge conditions are

$q_{ra} = \delta_{ra}$

$K_{rr} = 0$
Spacetime radial gauge *

* — also known as holographic, axial or Fefferman-Graham gauge

AdS/CFT correspondence — boundary perspective:
construct operators in CFT
which mimic observables in the bulk
to deal with the gauge redundancy in the bulk
a suitable gauge is introduced
a desired feature is that at least matter fields
are commuting at spacelike separation

are they commuting in spacetime radial gauge?


1. the constraints are \( \mathcal{C}_\alpha = (H, C_r, C_A, K_{rr}, q_{rr} - 1, q_{rA}) \)

2. Dirac bracket is given by \( \{\mathcal{O}_1, \mathcal{O}_2\}_D = \{\mathcal{O}_1, \mathcal{O}_2\} - \sum_{\alpha, \beta = 1}^{8} \{\mathcal{O}_1, \mathcal{C}_\alpha\}(M^{-1})_{\alpha\beta}\{\mathcal{C}_\beta, \mathcal{O}_2\} \)

3. where \( M_{\alpha\beta}(r, \theta; \bar{r}, \bar{\theta}) = \{\mathcal{C}_\alpha(r, \theta), \mathcal{C}_\beta(\bar{r}, \bar{\theta})\} = \)

\[
\begin{bmatrix}
0 & 0 & 0 & R_{rr}^{(3)} - 2K_ArK^A_r + t^\text{matt} - \partial_r^2 & 0 & -2K_{rA} \\
0 & 0 & 0 & 2\partial_r & 0 & -2K_rA \\
0 & 0 & 0 & 2\partial_rK_{rB} & 0 & -\frac{\kappa}{\sqrt{\det q}} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}_{\alpha\beta} \delta(r, \theta; \bar{r}, \bar{\theta})
\]

4. and \( t^\text{matt}(r, \theta)\delta(r, \theta; \bar{r}, \bar{\theta}) = \frac{\kappa}{\sqrt{q}} \left( \frac{\delta h^\text{matt}(r, \theta)}{\delta q_{rr}(\bar{r}, \bar{\theta})} - q_{AB} \frac{\delta h^\text{matt}(r, \theta)}{\delta q_{AB}(\bar{r}, \bar{\theta})} - \frac{1}{2} h^\text{matt}(r, \theta)\delta(r, \theta; \bar{r}, \bar{\theta}) \right) \)
Spacetime radial gauge

\[ \mathfrak{c}_\alpha = (H, C_r, C_A, K_{rr}, q_{rr} - 1, q_{rA}) \]

1. 
\[
\begin{bmatrix}
0 & 0 & 0 & R_{rr}^{(3)} - 2K_{Ar}K^{Ar} + t^{\text{matt}} - \partial_r^2 & 0 & -2K_{rA} \\
0 & 0 & 0 & 2\partial_r K_{rB} & 0 & \partial_B \\
0 & 2\partial_r K_{rB} & 0 & -\frac{\kappa}{\sqrt{\det q}} & 0 & \partial_r q_{AB} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[ = \begin{bmatrix} 0 & F \\ -F^T & G \end{bmatrix} \]

2. inverse is given by
\[
\begin{bmatrix} 0 & F \\ -F^T & G \end{bmatrix}^{-1} = \begin{bmatrix} (F^{-1})^T GF^{-1} & -(F^{-1})^T \\ F^{-1} & 0 \end{bmatrix}
\]

3. to find \( F^{-1} \) we need to solve
\[
\sum_{\beta=1}^{4} \int d\bar{r}d^2\bar{\theta} \ F_{\alpha\beta}(r, \theta; \bar{r}, \bar{\theta}) N_{\beta}(\bar{r}, \bar{\theta}) = M_\alpha(r, \theta)
\]

4. that is
\[
\partial_r^2 N + (-R_{rr}^{(3)} + 2K_{Ar}K^{Ar} - t^{\text{matt}}) N + 2K_{rA} N^A = -M
\]
\[
2\partial_r N^r + \partial_B N^B = M_r
\]
\[
\partial_r N_A + 2\partial_r (K_{rA} N) = M_A
\]
Spacetime radial gauge

\[
\partial_r^2 N + ( - R_{rr}^{(3)} + 2K_A K^Ar - t^{\text{matt}} ) N + 2K_A N^A = -M \\
2\partial_r N^r + \partial_B N^B = M_r \\
\partial_r N_A + 2\partial_r (K_r A N) = M_A
\]

\[
N_A(r, \theta) + 2K_r A N(r, \theta) = \ldots
\]

\[
N_{\beta[M_\alpha]} = (F^{-1})_{\beta\alpha} M_\alpha
\]
Spacetime radial gauge

\[ \mathbf{c}_\alpha = (H, C_r, C_A, K_{rr}, q_{rr} - 1, q_{rA}) \]

1. \( \{\mathcal{O}_1, \mathcal{O}_2\}_D = \{\mathcal{O}_1, \mathcal{O}_2\} - \sum_{\alpha, \beta = 1}^{8} \{\mathcal{O}_1, \mathbf{c}_\alpha\} \left[ \begin{array}{cc} (F^{-1})^T G F^{-1} & -(F^{-1})^T \\ F^{-1} & 0 \end{array} \right]_{\alpha\beta} \{\mathbf{c}_\beta, \mathcal{O}_2\} \)

2. \( \{\phi(r_1, \theta_1), \phi(r_2, \theta_2)\}_D = \int dr d^2 \theta \ N_r^r[\{\phi(r_1, \theta_1), \mathbf{c}_\alpha\}](r, \theta) \frac{\kappa}{\sqrt{\det q(r, \theta)}} N_r^r[\{\mathbf{c}_\alpha, \phi(r_2, \theta_2)\}](r, \theta) \)
\[ - \int dr d^2 \theta \ N_r^r[\{\phi(r_1, \theta_1), \mathbf{c}_\alpha\}](r, \theta) \frac{\kappa}{\sqrt{\det q(r, \theta)}} N_r^r[\{\mathbf{c}_\alpha, \phi(r_2, \theta_2)\}](r, \theta) \)

3. it leads to the following conclusions:

a) in spacetime radial gauge matter fields do not commute

b) in spacetime radial gauge matter fields have non-local brackets

c) the non-localities vanish in weak-gravity limit since \( \kappa \sim G_{\text{Newton}} \)

d) to guarantee simple matter sector, the gravitational gauge fixing conditions should be mutually commuting
Conclusion

Radial gauge

is a useful tool for dealing with spatial diffeomorphisms in canonical GR

leads to a novel definition of spherical symmetry on the quantum level

Spacetime radial gauge

is of interest for AdS/CFT correspondence

has non-local Dirac brackets (also in the matter sector)

commutativity of gravitational gauge fixings is crucial for commutation properties of matter sector