Schwarzschild Interior With Loop Quantum Effects

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Loop Quantum Dynamics of Schwarzschild Interior

C. Böhmer, K. Vandersloot, PRD 76, 104030 (2007), arXiv:0709.2129

Introduction

Recent advances in loop quantum cosmology (LQC) indicate replacement of big-bang with big-bounce for FRW cosmologies [Ashtekar, Pawlowski, Singh, KV, 2006-7]

Major advance was completing the quantum program of LQC: constructing physical Hilbert space, computing Dirac observables, evolution of physical semi-classical states for the simple model of FRW spacetime with massless scalar field

Additional advance: Original LQC Hamiltonian constraint operator had bad semi-classical limit. Improved Hamiltonian constraint operator constructed ("$\overline{\mu}$" quantization vs original "μ_0") - more physical results, good semi-classical limit [Ashtekar, Pawlowski, Singh, 2006]

Extending these rigorous results to more complicated scenarios challenging due to complexity of models

Would be nice to have an approximate shortcut to the physical results

Such a shortcut exists for the isotropic massless scalar results. Phenomenological effective theory incorporating holonomy features of Hamiltonian constraint operator provides explanation for bounce: Friedmann equation modified $H^2 = \frac{8\pi G}{3}\rho\,(1 - \frac{\rho}{\rho_c})$, gravity repulsive at high energies, bounce at $\rho = \rho_c$

Modified Friedmann equation approximates quantum results well [Ashtekar, Pawlowski, Singh, 2006]. Further, with $\overline{\mu}$" quantization $\rho_c \approx \rho_{PL}$, whereas μ_0 quantization ρ_c could be arbitrarily small

Introduction

What about black hole singularities?

Interior of Schwarzschild black hole can be treated as homogeneous anisotropic Kantowski-Sachs model

Initial loop quantization of Schwarzschild interior (analogous to "μ_0" quant) [Ashtekar, Bojowald, 2006]. Just construction of constraint operator. No construction of physical sector.

Analysis of phenomenological effective dynamics of this model performed [Modesto, 2006]

Constructing physical sector difficult mathematically. Even more so with the $\bar{\mu}$ quantization

This talk: analyze consequences of the improved $\bar{\mu}$ quantization using effective semi-classical equations of motion. Fate of singularity? Fate of in-falling test particle?

Schwarzschild Interior Classically

Inside Schwarzschild horizon, switching temporal and radial coordinates, metric become spatially homogeneous Kantowski-Sachs type

$$
\begin{aligned}
ds^2 &= -N^2(t)\,dt^2 + g_{xx}(t)\,dx^2 + g_{\Omega\Omega}(t)\,d\Omega^2 \\
&= -\left(\frac{2m}{t} - 1\right)^{-1} dt^2 + \left(\frac{2m}{t} - 1\right) dx^2 + t^2 d\Omega^2
\end{aligned}
$$

Metric components depend only on temporal coordinate t. Homogeneous, anisotropic type cosmology with two dynamical components of metric $g_{xx}(t)$ and $g_{\Omega\Omega}(t)$

$d\Omega^2$ ordinary unit two-sphere metric. $g_{\Omega\Omega}$ determines the *physical radius* of two-sphere

$\mathbb{R} \times \mathrm{SO}(3)$ spatial topology: $x \in \mathbb{R}$. Schwarzschild interior recovered for $t \in [0, 2m]$

Schwarzschild Interior with Connection-Triad Variables

Goal is to quantize ala loop quantum cosmology based on connection and triad variables

Basic idea: assume Kantowski-Sachs symmetry for connection A_a^i and triad E_i^a. Insert into Hamiltonian and quantize symmetry reduced degrees of freedom

Triad E_i^a determined by two dynamical quantities labeled p_b and p_c which determine two metric DOFS $g_{xx}(t)$ and $g_{\Omega\Omega}(t)$:

$$g_{xx} = \frac{p_b^2}{|p_c|} \qquad g_{\Omega\Omega} = |p_c|$$

$$ds^2 = -N^2(t)\, dt^2 + \frac{p_b^2(t)}{|p_c(t)|}\, dx^2 + |p_c(t)|\, d\Omega^2$$

Connection also determined by two dynamical quantities b, c

Poisson brackets between connection and triad variables

$$\{b, p_b\} = G\gamma \qquad \{c, p_c\} = 2G\gamma$$

Dynamics determined from Hamiltonian

$$H = \frac{-N}{2G\gamma^2}\left[2\, b\, c\, \sqrt{p_c} + (b^2 + \gamma^2)\frac{p_b}{\sqrt{p_c}}\right]$$

Schwarzschild Solution

Equations of motion

$$\begin{aligned}\dot{p}_b &= \{p_b, H\} = -G\gamma\frac{\partial H}{\partial b} \quad \text{etc} \\ H &= 0\end{aligned}$$

How to identify horizon and singularity in terms of triad variables?

Horizon: $g_{\Omega\Omega} = 4m^2$, $g_{xx} = 0 \quad \Rightarrow \quad p_b = 0,\ p_c = 4m^2$

Singularity: $g_{\Omega\Omega} = 0$, $g_{xx} = 0 \quad \Rightarrow \quad p_b = 0,\ p_c = 0$

At horizon two-sphere radius $\propto m$ ($p_c \propto m^2$), at singularity two-sphere radius vanishes ($p_c = 0$)

Equations of motion can be solved starting with boundary values at horizon. Schwarzschild solution given by

$$p_b(t) = t\sqrt{2m/t - 1} \qquad p_c(t) = t^2 \qquad b(t) = \ldots$$

Useful for interpreting quantum dynamics:

p_c component directly determines two-sphere radius

Radial geodesics for in-falling test particle (τ is proper time, $\mathcal{E}$ energy at ∞):

$$\left(\frac{dt}{d\tau}\right)^2 = \left(\frac{p_c}{p_b^2}\mathcal{E}^2 + 2\right)\frac{1}{N^2}$$

To interpret effective dynamics, calculate $p_c(\tau)$ along in-falling geodesic

Quantum Dynamics

Homogeneous model quantized ala loop quantum cosmology [Ashtekar, Bojowald, 2006]. Leads to discrete difference equation with finite steps in p_b and p_c

Ideally would like to construct semi-classical states, physical inner product, calculate expectation values, but difficult esp with a $\bar{\mu}$ like quantization (discrete structure of diff equation much more complicated)

Instead as simplification, incorporate holonomy effects of Hamiltonian constraint operator into modified classical Hamiltonian, derive and analyze modified equations of motion

Holonomies roughly exponentials of connection components $b \to e^{ib\delta_b}$ etc. δ_b, δ_c proportional to holonomy edge length and measure discreteness in difference equation

[Ashtekar, Bojowald, 2006]: δ_b, δ_c were constants (analogous to μ_0 parameter of LQC)

More recent work of LQC has length parameters dependant on triad components - better semi-classical limit, more physical results

Quantum Dynamics

Holonomy effects incorporated in form of effective Hamiltonian with connection components replaced by holonomy equivalents

$$H_{cl} = \frac{-N}{2G\gamma^2}\Big[2\,b\,c\,\sqrt{p_c} + (b^2+\gamma^2)\frac{p_b}{\sqrt{p_c}}\Big]$$

$$H_{eff} = -\frac{N}{2G\gamma^2}\Big[2\frac{\sin b\delta_b}{\delta_b}\frac{\sin \delta_c c}{\delta_c}\sqrt{p_c} + \big(\frac{\sin^2 b\delta_b}{\delta_b^2} + \gamma^2\big)\frac{p_b}{\sqrt{p_c}}\Big]$$

Caveats of effective Hamiltonian:

First step in analyzing quantum corrections - not rigorous derivation of effects, possible additional corrections

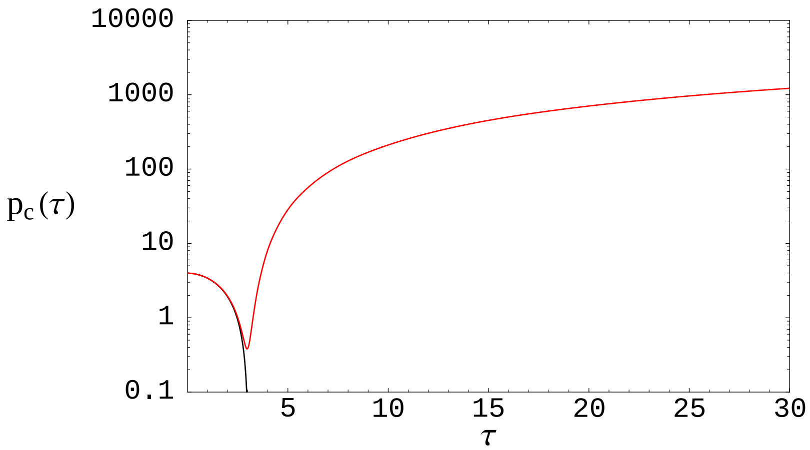
Has provided excellent accounting of bounce results of LQC for massless scalar field with Λ

Interested in phenomenological effects of these corrections, not necessarily final word

Effective Hamiltonian for $\delta_b, \delta_c = \text{const}$ results [Modesto, 2006]:

Singularity avoided, bounce in two-sphere radius: $p_c \geq \gamma\delta m$

Solution classical before bounce, connects to another classical solution with different mass in general (can be made symmetric)



Improved Effective Dynamics

Would like to consider case with non-constant δ_b, δ_c parameters (analogue to "$\bar{\mu}$" in LQC)

In LQC, more physical results when $\overline{\mu} \propto \frac{\Delta}{p^{1/2}}$ [Ashtekar, Pawlowski, Singh, 2006]

Constrain by shrinking loop of Hamiltonian constraint to have minimum LQG area: $\Delta = A_{min}$

Schwarzschild interior anisotropic, so more possible ways to implement the δ_c, δ_b parameters

Two possible schemes:

A) More geometric approach - constrain classical area of holonomy loops to have minimum area

End result

$$\delta_b = \frac{\sqrt{\Delta}}{\sqrt{p_c}} \qquad \delta_c = \sqrt{\Delta}\frac{\sqrt{p_c}}{p_b}$$

B) Alternative approach - loop area dependent on transversal holonomy

$$\delta_b = \frac{\sqrt{\Delta}}{\sqrt{p_b}} \qquad \delta_c = \frac{\sqrt{\Delta}}{\sqrt{p_c}}$$

A favored by stability analysis of quantum difference equation [Bojowald, Cartin, Khanna, 2007]

B applied to Bianchi I model [Chiou, 2006], but suffers from gauge dependance issues [Chiou, 2007]

B gives similar results to the constant δ case

Improved Effective Dynamics

Our work has focused on scheme A:

$$\delta_b = \frac{\sqrt{\Delta}}{\sqrt{p_c}} \qquad \delta_c = \sqrt{\Delta}\frac{\sqrt{p_c}}{p_b}$$

The effective Hamiltonian:

$$H_{eff} = -\frac{N}{2G\gamma^2}\Big[2\frac{\sin b\delta_b}{\delta_b}\frac{\sin \delta_c c}{\delta_c}\sqrt{p_c} + \big(\frac{\sin^2 b\delta_b}{\delta_b^2} + \gamma^2\big)\frac{p_b}{\sqrt{p_c}}\Big]$$

Equations of motion determined in similar way to classical case e.g. $\dot{p_b} \propto \partial H_{eff}/\partial b$ etc.

Too complicated for analytical solution, solve numerically. Get $p_b(t), p_c(t)$ etc, analyze resulting metric, geodesics etc

Improved Effective Dynamics Results

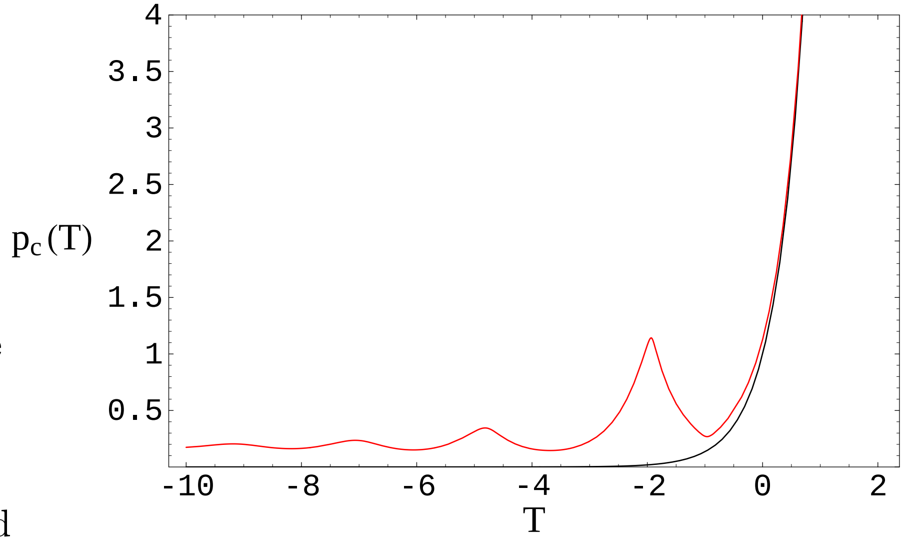
Plot of $p_c(T)$ shown at right. Bounded from below and asymptotes to fixed value after undergoing damped oscillations.

Singularity thus absent, resolved dynamically!

The asymptotic behavior matches that of Nariai universe

Nariai universe special spherically symmetric solution of classical GR: two-sphere radius is constant (const p_c) and requires a cosmological constant Λ

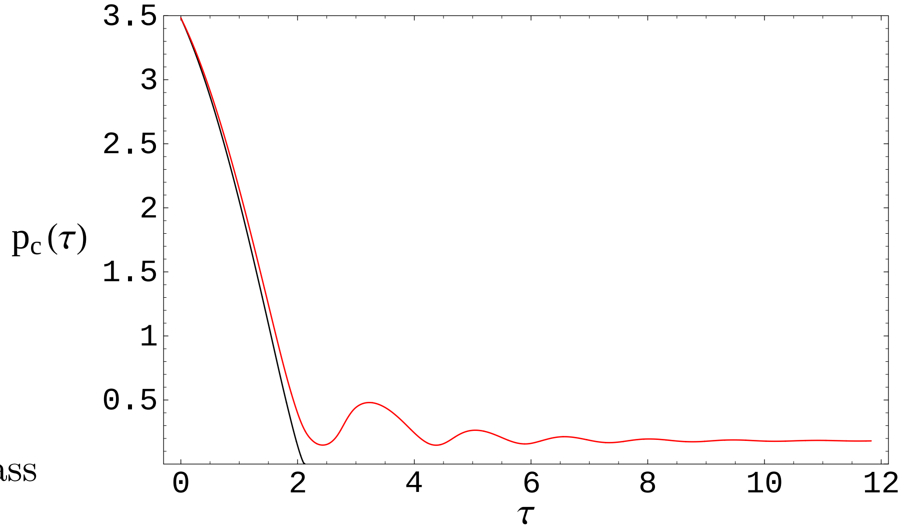
$$ds^2 = -dt^2 + A\cosh^2(\sqrt{\Lambda}t)dx^2 + 1/\Lambda\, d\Omega^2$$



Improved Effective Dynamics Results

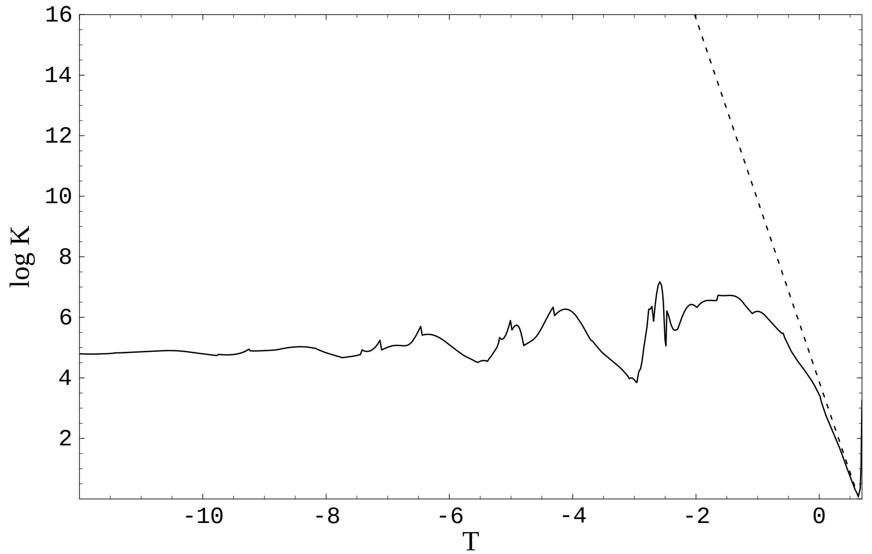
Radial in-falling geodesic:

- Decrease in two-sphere radius as classical singularity approached
- Damped oscillations in radius
- particle settles in at finite radius dependant only on Δ and γ
- For $\Delta = A_{min}$, $p_c \approx .2 l_p^2$
- Final radius Planckian, independent of black-hole mass
- Interpret as repulsive gravity, similar to bouncing results of LQC
- Particle remains there for infinite proper time



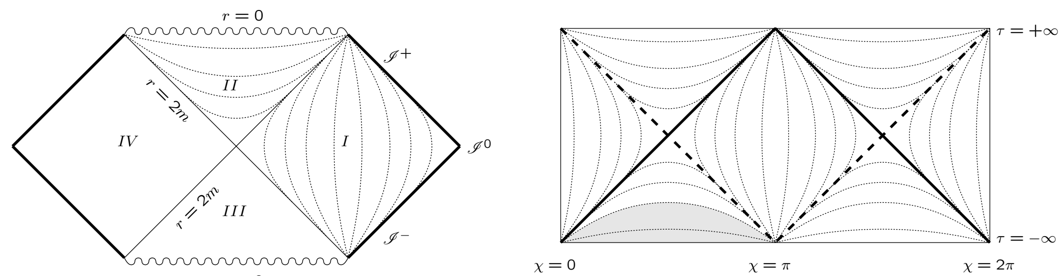
Curvature when quantum effects appreciable?

- Kretschmann invariant does not blow up
- Asymptotes to fixed value
- From numerics $K \approx 124.35\, m_p^4$
- Quantum effects at Planckian curvature

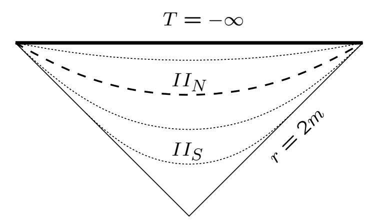


Improved Effective Dynamics Results

Penrose diagram for Schwarzschild (left) and Nariai (right) spacetimes.



Shaded part of Nariai diagram glued onto upper part of Schwarzschild diagram for interior region II



Caveat of the results - effective Hamiltonian also predicts deviations from classical behavior near classical horizon.

Not clear if problem with quantization scheme, or Kantowski-Sachs approximation not to be trusted there, or boundary matching more complicated

Conclusion/Outlook

Each phenomenological study indicates singularity resolution of Schwarzschild black hole analogous to LQC results

Detailed consequences dependant on quantization scheme

Two-interesting outcomes - First (constant δ_b, δ_c scheme) reminiscent of wormhole like solution connecting two black holes of possibly different mass

Improved quantization scheme, interior asymptotes to Nariai type space-time - in-falling particle trapped at finite Planckian radius and curvature

Results are suggestive but arise from simple effective theory - ideally require more justification by analyzing semi-classical states in quantum theory, though not clear how to proceed on this front

Future - apply results to inhomogeneous spherically symmetric models for instance work of Campiglia, Gambini, Pullin. Ideal to investigate true collapse models.