Propagation in Polymer PFT

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Non-uniqueness of Hamiltonian constraint in LQG:

- Classical Ham constr depends on local connection field A. But in LQG , no local connection operator, only holonomy operators. Cant construct Ham constr oprtr by $A \to \hat{A}$ in classical expression.

- Follow Thiemann Strategy:
  - Introduce triangulation T_δ of Cauchy slice Σ with $T_{\delta\to 0} = \Sigma$.
  - Define approximants to (A, E) in terms of holonomy -fluxes of small loops, surfaces associated with T_δ.
  -Substitute in expression for Ham constr to get approximant H_δ so that Ham constraint $= H_{\delta\to 0}$.
  - Replace classical holonomy-flux in H_δ by quantum oprtrs, get $\hat{H}_\delta$.
  - Define Ham constr oprtr as 'continuum limit' $\hat{H}_{\delta\to 0}$.

- Remarkably, limit operator can be defined. Problem is it depends on choice of hol-flux approximants at finite T_δ. **Infinitely Non-Unique!**.

- Cut down on non-uniqueness by imposing further requirements E.g. Require operator be defined on Hilbert space (Jurek,Hanno,Assanioussi....).Require non-trivial anomaly free repn of constraint algebra(Laddha,Tomlin,..).
- In this talk: try to address Smolin's propagation requirement.
- Smolin: Requires that the quantum dynamics be such that it propagates perturbations from one part of quantum geometry to another (this propagation thought of as nonpert seed of graviton propagation in semiclassical LQG).
- Argues that requirement not met in LQG due to ultralocality of Ham constr action.

Ultralocality of LQG Dynamics:

- Ham constraint acts on vertices of a spin net. Action only depends on vertex structure in infinitesmal nbrhood of vertex.
- Repeated actions of Ham constr builds a nest of structure localised at each vertex. So vertices ‘far apart’ in graph never talk to each other thru Ham constr.
- Smolin argued that it was unlikely that such quantization could describe propagating degrees of freedom between macroscopically seperated regions of quantum geometry. Arguments intuitive since not enough known about physical interpretation of states. But compelling.

In this talk..

- I shall examine this issue in LQG type `polymer' quantization of 2d Parameterised Field Theory.
- PFT is gen cov reformultn of free scalar field propagation on fixed flat spacetime. All steps of LQG program including ambiguity free defn of quantum dynamics can be completed (Laddha,MV).
- Ultralocality problem exists. Can see clearly that repeated action of Ham constr does not give long range propagation.
- But nevertheless physical states describe scalar field propagation.
- How can this be?

Plan

- Classical PFT
- Quantum Kinematics
- Unitary implementation of Finite Gauge transformations
- Physical States via Grp Avging
- Action of Quantum Hamiltonian Constraint
- Quantum states and discrete spacetime
- The ultralocality problem
- Propagation

Classical PFT

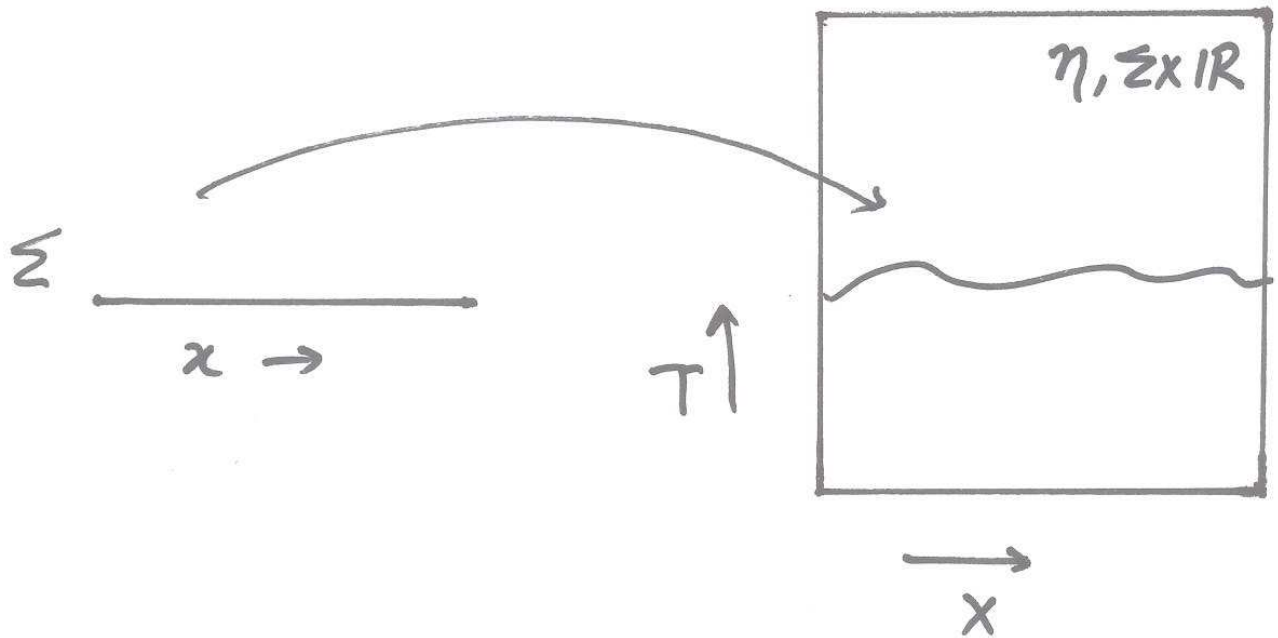
- Free Scalar Field Action: $\mathbf{S}_0[f] = -\frac{1}{2}\int d^2X \eta^{AB}\partial_A f \partial_B f$
- Parametrize $X^A = (T, X) \to X^A(x^\alpha) = (T(x,t), X(x,t))$.
  $\Rightarrow \mathbf{S}_0[f] = -\frac{1}{2}\int d^2x \sqrt{\eta}\eta^{\alpha\beta}\partial_\alpha f \partial_\beta f$,
  $\eta_{\alpha\beta} = \eta_{AB}\partial_\alpha X^A \partial_\beta X^B$.
- Vary this action w.r.to f and 2 new scalar fields X^A:
  $S_{PFT}[f, X^A] = -\frac{1}{2}\int d^2x \sqrt{\eta(X)}\eta^{\alpha\beta}(X)\partial_\alpha f \partial_\beta f$
  δf: $\partial_\alpha(\sqrt{\eta}\eta^{\alpha\beta}\partial_\beta f) = 0 \equiv \eta^{AB}\partial_A\partial_B f = 0$
  δX^A: no new equations, $\Rightarrow X^A$ are undetermined functions of x, t, so 2 functions worth of Gauge!
- x, t arbitrary $\equiv$ general covariance
- So Hamiltonian theory has 2 constraints.
- **Remark:**Free sclar field solns are $f = f_+(T+X) + f_-(T-X)$
  Use split into "left movers + right movers" in Hamiltonian theory.

Hamiltonian description

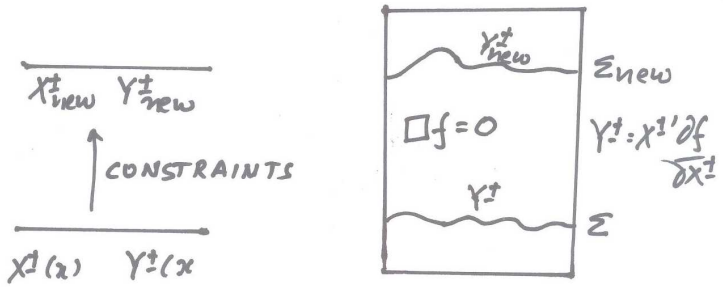
- $T(x), X(x)$ become canonical variables. Use light cone variables $T(x) \pm X(x) := X^{\pm}(x)$.
- Phase space: $(f, \pi_f), (X^+, \Pi_+), (X^-, \Pi_-)$
- Constraints: $H_{\pm}(x) \; = \; [\, \Pi_{\pm}(x) X^{\pm'}(x) \; \pm \; \frac{1}{4}(\pi_f \pm f')^2 \,]$
- Gauge fix: $X^{\pm} = t \pm x$ "deparameterize".
  get back standard flat spacetime free scalar field action.
- Define: $Y^{\pm} = \pi_f \pm f'$
  $\{Y^+, Y^-\} = 0$, $\{Y^{\pm}(x), Y^{\pm}(y)\}$= derivative of delta function
- $\{H_+, H_-\} = 0$. P.B. algebra between smeared H_+'s isomorphic to Lie algebra of diffeomorphisms of Cauchy slice. Same for H_- algebra.
- $H_{\pm}$ generate spatial diffeomorphisms of $\pm$ fields.
  Finite Evoltn $\equiv$ 2 independent diffeomorphisms (Φ_+, Φ_-)!
  Φ_+ moves only `+' fields, $\Phi_-$ moves only `-' fields.

Spacetime interpretation of canonical data:

Embeddings: $X(x), T(x) \equiv X^+(x), X^-(x)$

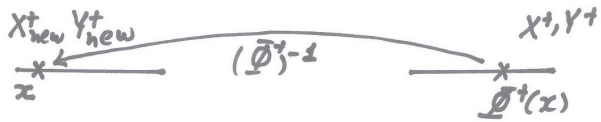


ACTION OF CONSTRAINTS:

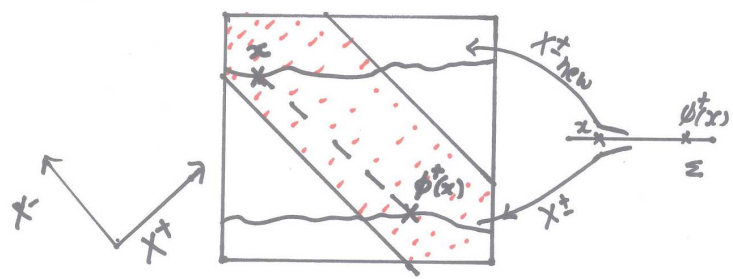


Y^{+}_{new}, X^{+}_{new} : Action of diffeo Φ^{+} on Y^{+}, X^{+}

Y^{-}_{new}, X^{-}_{new} : Action of diff-eo Φ^{-} on Y^{-}, X^{-}.



PLAUSIBLE... $f = f_{+}(x^{+}) + f_{-}(x^{-})$



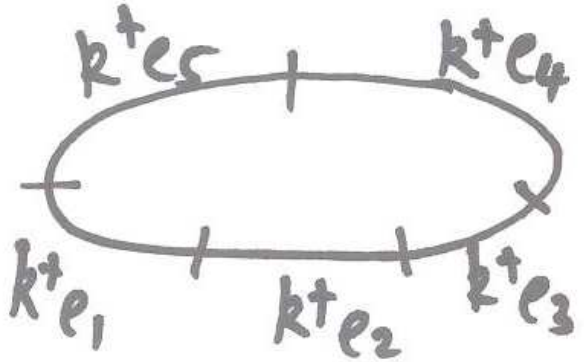
Note:

- Not much known re:Polymer repn on non-compact space. Hence set Σ= circle so that Spacetime Topology = $S^1 \times R$.
- There are complications coming from using "single angular coordinate chart" x on Σ and single spatial angular inertial coordinate X on the flat spacetime. Identifications of x and X"mod 2π" are needed. These can be taken care of. Will mention subtelities when necessary.

Quantum Kinematics: Embedding Sector

- Focus on `+' sector. Canonical coordinates:
  $(\Pi^+(x), X^+(x)) \sim (A, E)_{\rm LQG}$
- Holonomies of Π^+:
  Graph $\sim$ set of edges which cover the circle.
  Spins $\sim$ an integer label k_e for each edge e.
  Holonomies $\sim e^{i\sum_e k_e \int_e \Pi_+}$
- Electric Field $\sim X^+(x)$
- Poisson Brkts: $\{X^+(x), e^{i\sum_e k_e \int_e \Pi_+}\} = ik_e e^{i\sum_e k_e \int_e \Pi_+}$
  (for x inside e)

- Charge Networks: $|\gamma, \vec{k}^+\rangle$



$\hat{X}^+(x)|\gamma, \vec{k}^+\rangle = \hbar k_e^+ |\gamma, \vec{k}^+\rangle$, x inside e.

$e^{i\sum_e \widehat{k_e'^+} \int_e \Pi_+} \; |\gamma, \vec{k}^+\rangle \; = \; |\gamma, \vec{k}^+ + \vec{k}'^+\rangle$

Inner Product: $\langle \gamma', \vec{k}'^+ | \gamma, \vec{k}^+\rangle = \delta_{\gamma',\gamma}\delta_{\vec{k}^+,\vec{k}'^+}$

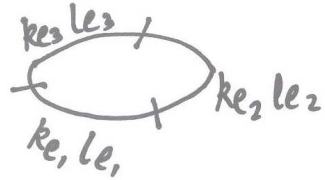
- Range of k_e: $\hbar k_e \in \mathbf{Z}a$, a is a Barbero- Immirzi parameter with dimensions of length.
- Similarly for $-$ sector.

Quantum Kinematics: Matter Sector and $\mathcal{H}_{kin}$

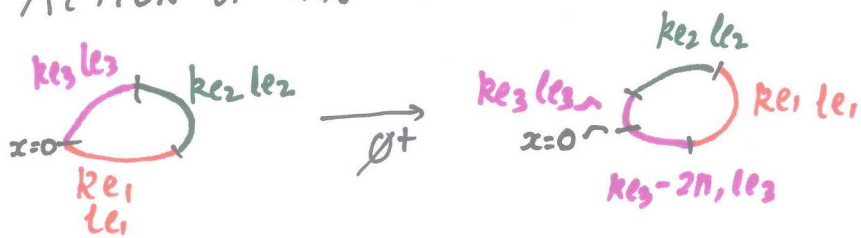
- Can define holonomies and repn on matter charge networks.
- Charge network: $|\gamma, \vec{l}\rangle$.
- $\mathcal{H}_{kin}$ obtained as product of $+, -$ embedding and matter charge nets.

- $\mathcal{H}_{kin} = \mathcal{H}^{+}_{kin} \otimes \mathcal{H}^{-}_{kin}$

  $\mathcal{H}^{\pm}_{kin} : |\gamma^{\pm}, \vec{k}^{\pm}, \vec{l}^{\pm}\rangle$



- ACTION OF GGE TRANSF: $\hat{U}(\phi^{+}\gamma)|\gamma^{+}, \vec{k}^{+}, \vec{l}^{+}\rangle$



ϕ^{+} = diffeo + winding

l's move by diffeo along with edges

k's move with edges by diffeo but are augmented by "$2m\pi$"

Physical Hilbert Space by Grp Avging

- Can construct physical states via grp avging w.r.to gge transformations just as we do for spatial diffeos in LQG.
- Grp Avg of a chrge net $|\gamma^+, \vec{k}^+, \vec{l}^+\rangle \otimes |\gamma^-, \vec{k}^-, \vec{l}^-\rangle$ is the distribution, $\sum \langle \gamma^+_{\phi^+}, \vec{k}^+_{\phi^+}, \vec{l}^+_{\phi^+}| \langle \gamma^-_{\phi^-}, \vec{k}^-_{\phi^-}, \vec{l}^-_{\phi^-}|$, where the sum is over all distinct gge related chrge nets.
- There is a nice geometrical interpretation for a chrge net and for physical states obtained by grp avging.

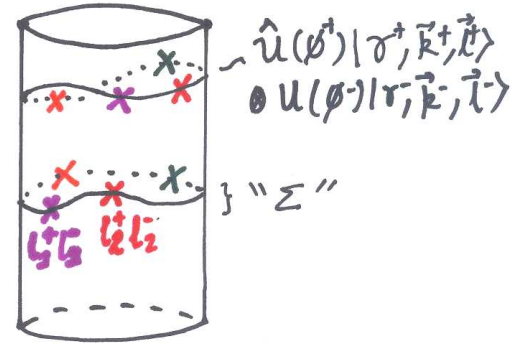
Geometrical picture:

- First, go to fine enough graph so as to write $|\gamma^+, \vec{k}^+, \vec{l}^+\rangle \otimes |\gamma^-, \vec{k}^-, \vec{l}^-\rangle \equiv |\gamma, \vec{k}^+, \vec{k}^-, \vec{l}^+, \vec{l}^-\rangle$ so that each edge of γ has pair of embedding chrge labels k^+, k^- and pair of matter labels l^+, l^-.
- $k^\pm$ are eigen values of $\hat{X}^\pm$. So (k^+, k^-) defines (X^+, X^-) coordinate of point in flat sptime! Associate matter chrge pair (l^+, l^-) with this point.
  Doing this for all edges of chargenet, we get a discrete Cauchy slice with quantum matter.
- Ggge transf $\Phi_\pm$ move `+', `-' edges independently. Gge transformed chrgnet has different pairs of `+-' embeddning and matter chrges. Defines new slice with new matter data. Thus quantum matter data propagates from 1 slice to another. Turns out that a (grp avged) physical state then defines discrete sptime with matter propagating on it. So quantization by grp avging encodes propagation.

POLYMER STATES AND DISCRETE SPTIME:

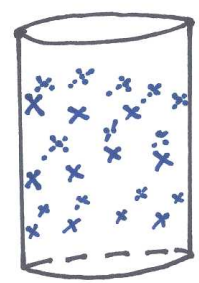
$$|\gamma^+, \vec{k}^+, \vec{l}^+\rangle \otimes |\gamma^-, \vec{k}^-, \vec{l}^-\rangle$$

$$(X^+, X^-) = (k^+, k^-)$$



GRP AVG : $\sum \langle \gamma^+_{\phi^+}, \vec{k}^+_{\phi^+}, \vec{l}^+_{\phi^+}| \otimes \langle \gamma^-_{\phi^-}, \vec{k}^-_{\phi^-}, \vec{l}^-_{\phi^-}|$

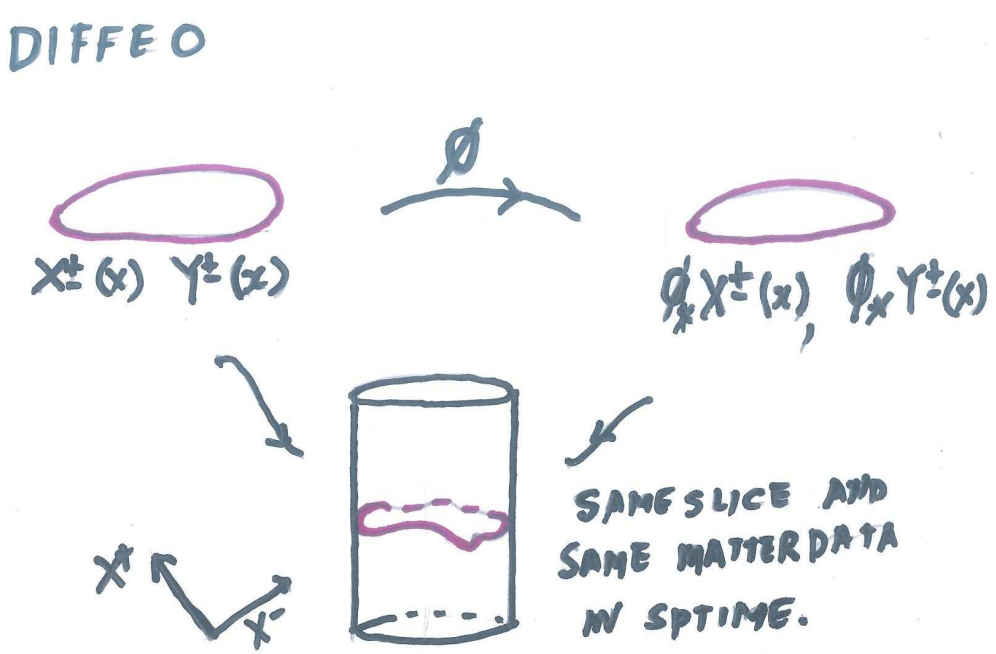
$$\{(k_1^+, k_1^-, l_1^+, l_1^-), (k_2^+, k_2^-, l_2^+, l_2^-) \ldots\}$$

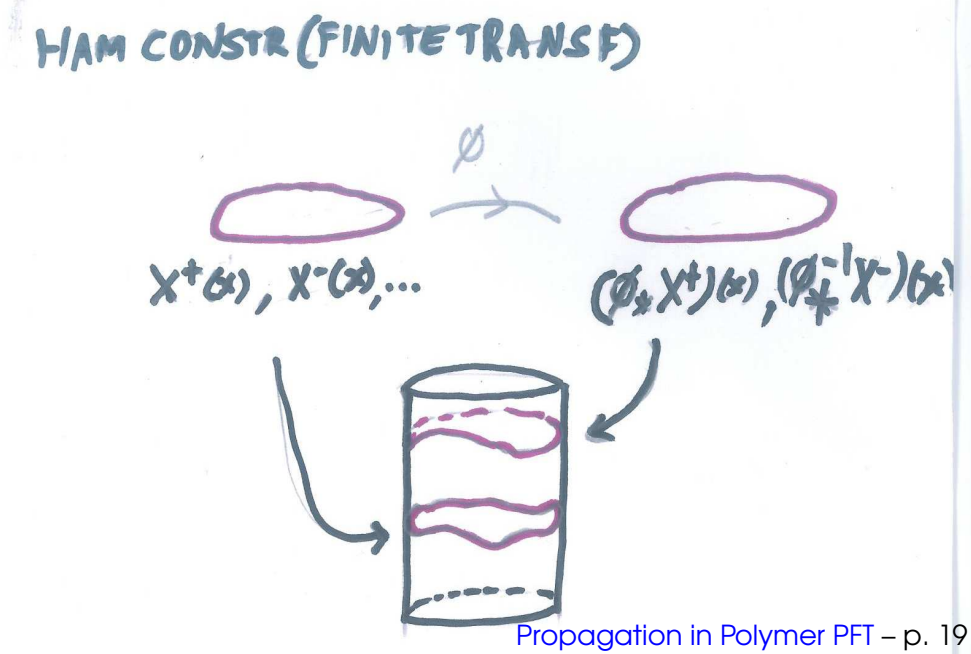


DISCRETE SPTIME, each pt. $(X^+, X^-) = (k^+_i, k^-_i)$ is labelled by (l^+_i, l^-_i)

Classical Diffeo, Ham constraints

- Recall: $H_\pm(x) \;=\; [\,\Pi_\pm(x)X^{\pm\prime}(x) \;\pm\; \frac{1}{4}(\pi_f \pm f')^2\,]$
- $C_{diff} := H_+ + H_- = \Pi_+(x)X^{+\prime}(x) + \Pi_-(x)X^{-\prime}(x) + \pi_f f'$.
  Generates spatial diffeo i.e. "evoltn along slice".
  In terms of $\Phi^\pm$: Sptl diffeo = transf in which $\Phi^+ = \Phi^-$.
- $C_{ham} = H_+ - H_-$ (density weight 2).
  Generates motion along timelike normal to slice (normal defined wrto flat spacetime metric).
  Finite transf corresponds to choice $\Phi^+ = (\Phi^-)^{-1}$.

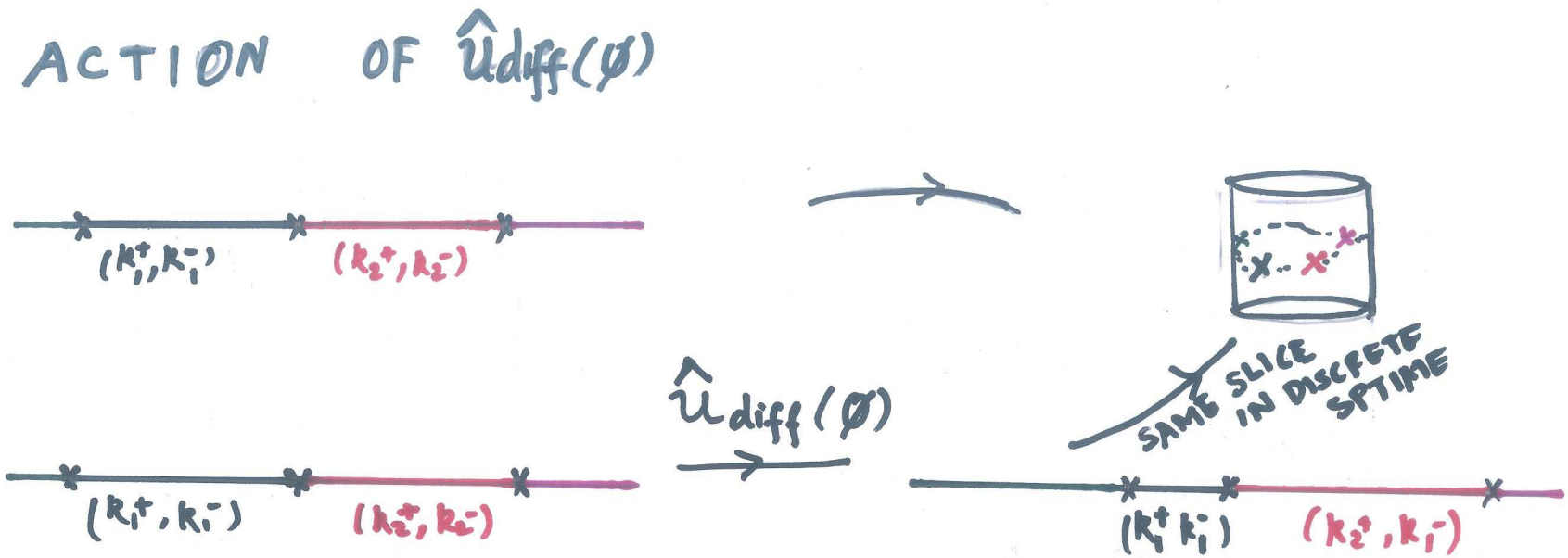


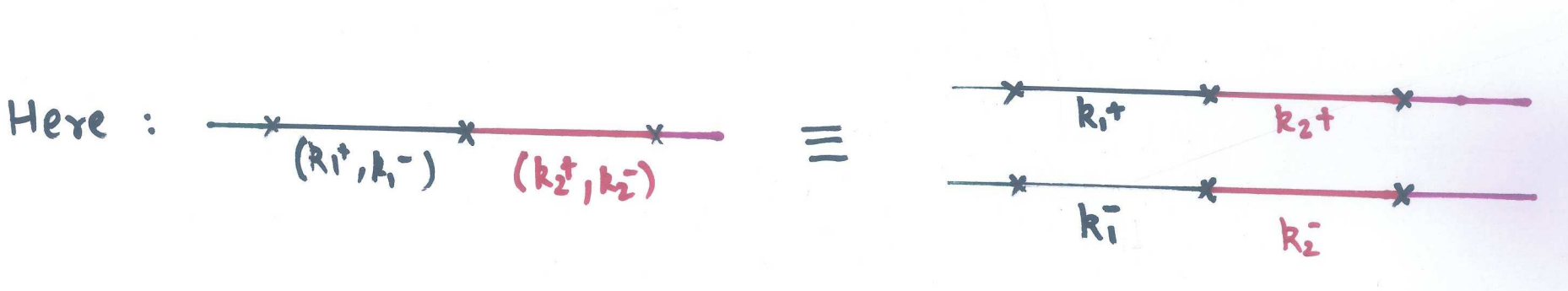


Quantum Constraints a la LQG:

Since for diffeos, $\Phi^+ = \Phi^- \equiv \Phi$

$\hat{U}_{diff}(\Phi) = \hat{U}_+(\Phi_+ = \Phi)\hat{U}_-(\Phi_- = \Phi)$. Can solve by grp avging.





Theimann type Ham Constr:

- Can construct finite triangulation constraint which acts as infinitesmal version of '$\Phi^+ = (\Phi^-)^{-1}$'.
- Let $\phi_{\delta,v}$ be diffeo which is identity outside small nbrhood of v and which 'pulls' point v to right.
- $\hat{C}^{(\delta)}_{ham}$ pulls '+' edges at v to right by $\phi_{\delta,v}$,
  pulls '-' edges at v to left by $\phi^{-1}_{\delta,v}$.

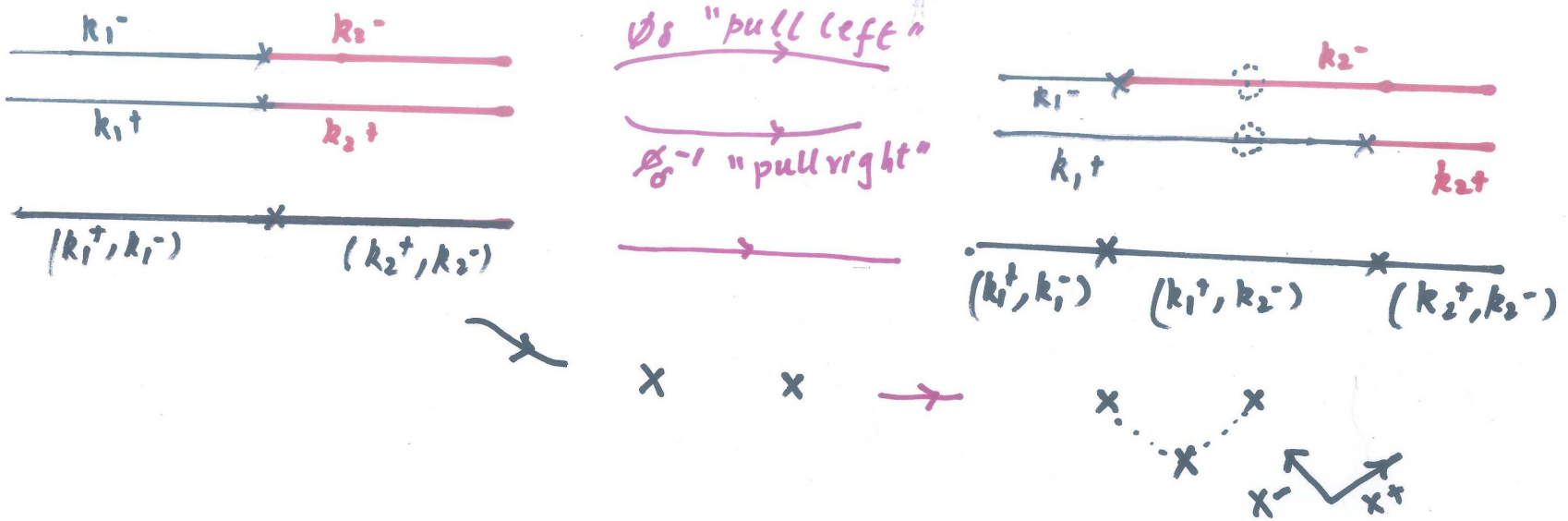
  $\hat{C}^{(\delta)}_{ham}|\Psi\rangle \sim \sum_v N(v) \frac{\left(\hat{U}_+(\phi_{\delta,v})\hat{U}_-(\phi^{-1}_{\delta,v}) - \mathbf{1}\right)}{\delta}|\Psi\rangle$

  $\hat{U}_+(\phi_{\delta,v})\hat{U}_-(\phi^{-1}_{\delta,v}) \equiv \hat{U}_{ham,\delta,v}$.
- Equally possible to make choices in which oprtr pulls '+' *left*, '-' *right* i.e. in which we replace $\hat{U}_{ham,\delta,v}$ by its adjoint.
- We impose constraint and its kinematic adjoint in quantum theory.

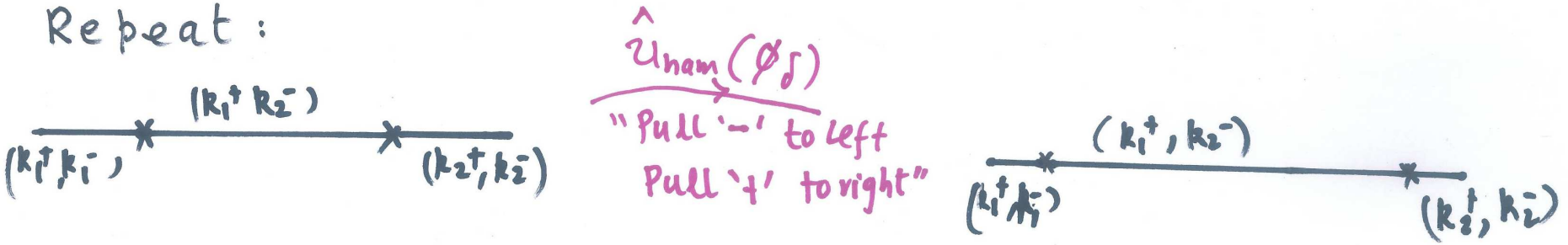
  NOTE: We are interested in action of $\hat{U}_{ham,\delta}$ for suff small δ.

Action of $\hat{U}_{ham,\delta,v}$

$\hat{U}_{ham}(\phi_\delta)$: $\phi_\delta \neq 1$ only in nbrhood of vertex:



Repeat:

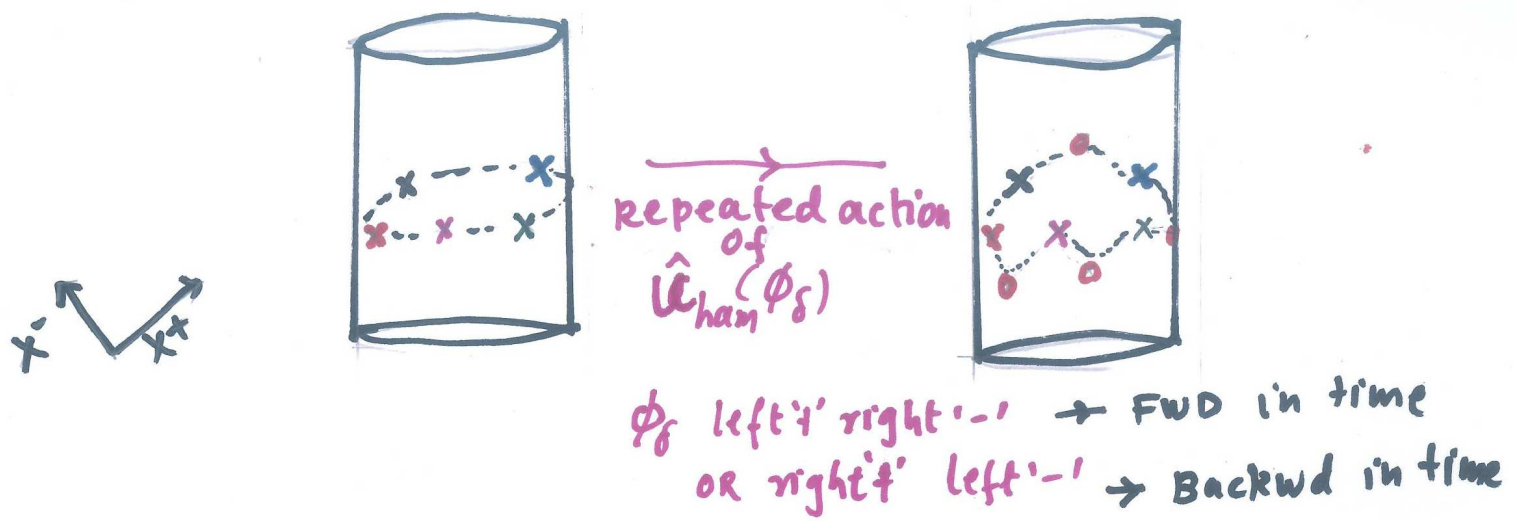


STUCK AT  NO PROPAGATION

NOTE: Recall 'δ' is to be thought of as arbitrarily small since interested in limiting behaviour as $\delta \to 0$

Summary:

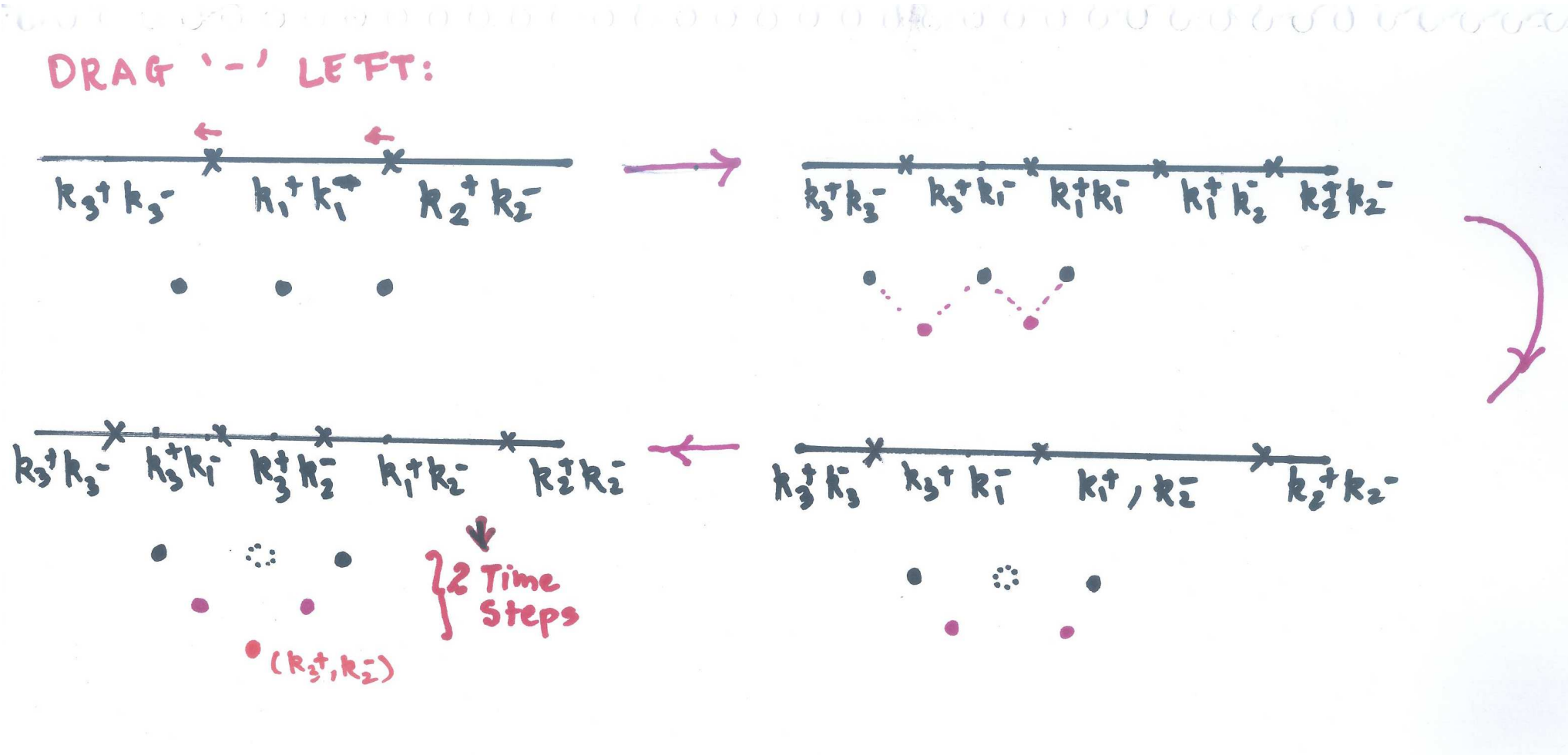
- $\hat{C}_{ham,\delta}$ acts nontrivially only in infinitesmal vicinity of vertices dragging `+' one way and `-' the other. Including matter labels, charge net describes data on a lattice version of Cauchy slice. Repeated action of $\hat{C}_{ham,\delta}$ can only generate single time step to past and future and then evolution gets stuck. Only immediate null related lattice point is generated with matter data. No large scale propagation



- This happens due to ultralocal action of $\hat{U}_{ham,\delta}$....

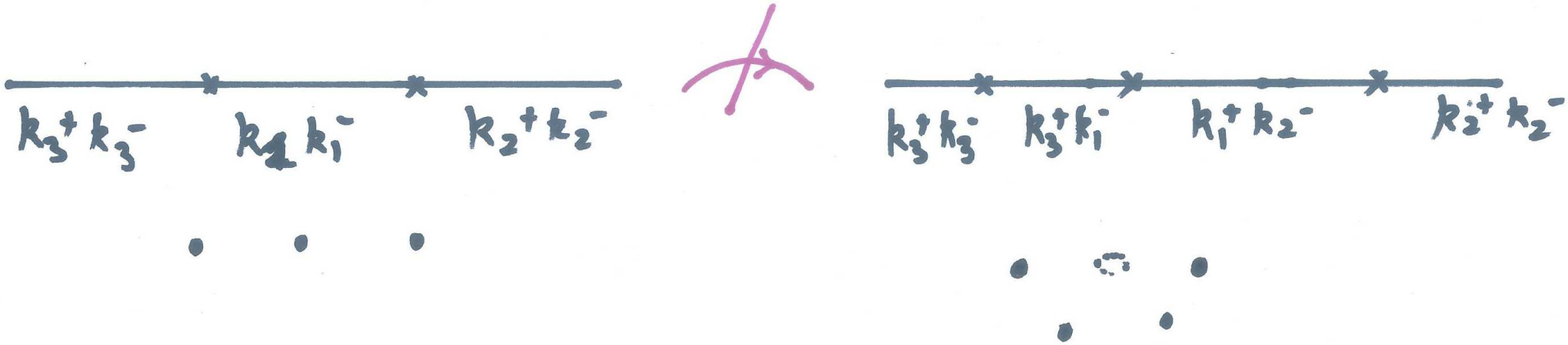
Ultralocality problem

- Recall: Finite gge transf $U_{\pm}(\Phi_{\pm})$ can drag `+', `-' edges past each other and get beyond single time step as follows:



- Problem: Above, necessary intermediate step is disappearance of (k_1^+, k_1^-) edge i.e. k_1^- is dragged past k_1^+.

Due to ultralocal action of $\hat{U}_{ham,\delta}$ we always have some (k_1^+, k_1^-) edge, cant drag k_1^- past k_1^+.



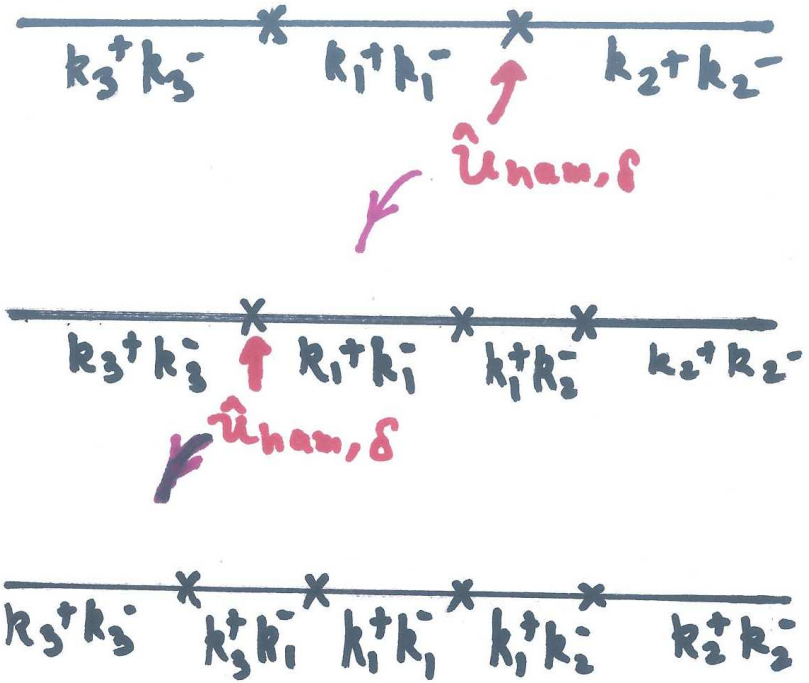
Propagation: Change of Perspective

- Rather than asking if repeated actions of $\hat{U}_{ham,\delta}$ (or $\hat{C}_{ham,\delta}$) give propagation, we should ask if joint kernel of diffeo constraint and $\hat{C}_{ham}$ describes propagation.
- Element Ψ of kernel is (distribtnl) sum of chargenet (bra) states. Ψ describes propagation if following statement holds: If the bra corresponding to some discrete Cauchy slice with matter data is in sum then the bra corresponding to any finite evolution of this slice and data must also be in the sum.
- By finite evolution we mean action of any finite gge transf $\Phi_{\pm}$ so this statement is equiv to showing elements of kernel same as physical states obtained by grp avging wrto finite transf generated by $H_{\pm}$.
- The key to showing statement holds for joint kernel of diffeo constr and Ham constraint $\hat{C}_{ham}$ is to show that if bra with (k_1^+, k_1^-) edge is in Ψ so is bra without this edge (previous slide).

Proof:

- We use language "$|c\rangle$ is in Ψ" to mean that $\langle c|$ is a summand in the sum representing distribution Ψ.
- Let Ψ be in kernel of diffeo, Ham constr. Then it follows that $|c\rangle$ is in Ψ if and only if
  - all sptl diffeo images of $|c\rangle$ are in Ψ.
  - $\hat{U}_{ham,\delta}|c\rangle$ are in Ψ for suff small δ
  (so that $\Psi(\hat{U}_{ham,\delta} - \mathbf{1})|c\rangle) = 0$ in $\delta \to 0$ limit)
  - Similarly $\hat{U}^{\dagger}_{ham,\delta}|c\rangle$ are in Ψ for suff small δ.
- We shall show that the desired chargenets (with and without the (k_1^+, k_1^-) edge) are in Ψ by relating them thru the action of $\hat{U}_{ham,\delta}, \hat{U}^{\dagger}_{ham,\delta}$.

Proof (Contd):


$k_3^+ k_3^-$
$k_1^+ k_1^-$
$k_2^+ k_2^-$
$\hat{u}_{nam,\delta}$
$k_3^+ k_3^-$
$k_1^+ k_1^-$
$k_1^+ k_2^-$
$k_2^+ k_2^-$
$\hat{u}_{nam,\delta}$
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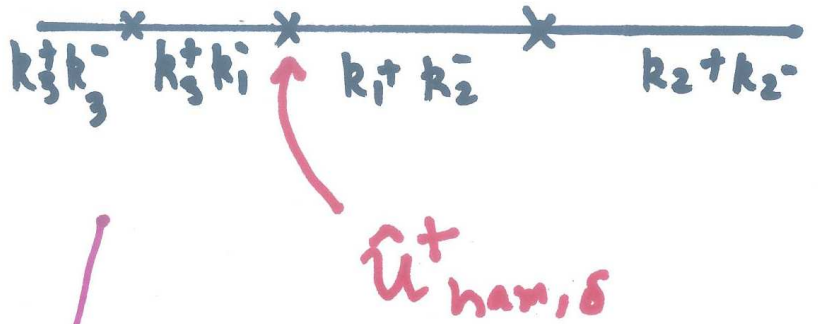
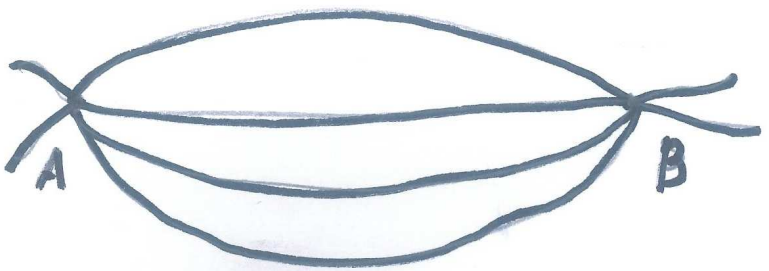
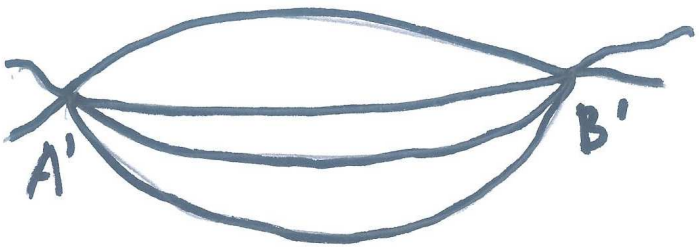

$k_3^+ k_3^-$
$k_3^+ k_1^-$
$k_1^+ k_2^-$
$k_2^+ k_2^-$
$\hat{u}^+_{nam,\delta}$

Conclusions

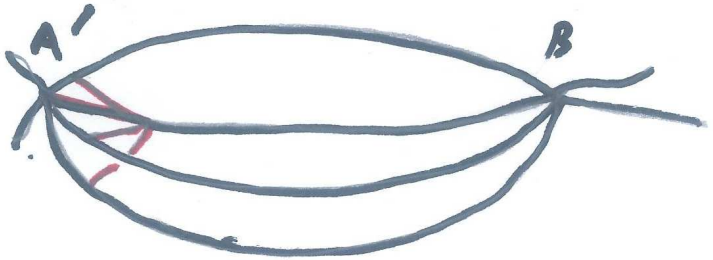
- Ultralocal action of Ham constraint does not preclude propagation.
- Propagation is to be seen as a pattern written on the entire set of kinematic bras which comprise a given physical state rather than as the repeated action of the Ham constraint on a fixed kinematic ket.
- Seen in this way propagation is a result of:
  - Particular "$\hat{U} - 1$" structure of the (finite triangulation) Ham constraint.
  - The imposition of the kinematic adjoint "$\hat{U}^\dagger - 1$" also as a Ham constraint.
- General structural lessons are robust and should be used in LQG to find phys correct Ham constraint.Seem to find v.interesting preliminry results for "$U(1)^3$" model.





$\downarrow \hat{c}_A$

$\downarrow \hat{c}^+_{B'}$



$\xrightarrow{\text{Drffeo}}$