SPINFOAM COSMOLOGY

with the COSMOLOGICAL CONSTANT

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References: 1003.3483, 1011.4705, 1101.4049.

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2 LOOP QUANTUM GRAVITY & COSMOLOGY

$$\begin{tikzcd}[column sep=huge, row sep=large]
\text{gravity} \arrow[r, "\text{quantization}"] \arrow[d, "\text{symmetry reduction}"'] & \text{quantum gravity} \arrow[d] \\
\text{cosmology} \arrow[r] & \text{quantum cosmology}
\end{tikzcd}$$

- How cosmology can be obtained from the full quantum gravity theory?
- **RESULTS**
  - There is a simple way to add the cosmological constant to the dynamics of LQG.
  - There are approximations in the quantum theory that yield cosmology.
  - The theory recover general relativity in the semiclassical limit, also for non-trivial solutions.

3 PLAN OF THE TALK

1. Definition of the complete theory
   - with the cosmological constant

2. Approximations
   - graph
   - vertex
   - spin

3. Study of the semiclassical limit (de Sitter solution)
   - transition amplitude
   - Hamiltonian constraint

4 KINEMATICS

Hilbert space: $\tilde{\mathcal{H}} = \bigoplus_{\Gamma} \mathcal{H}_\Gamma$ where $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N]$

Abstract graphs: Γ is determined by N=#nodes, L=#links and their adjacency

Identifications: $\tilde{\mathcal{H}}/\sim$

- if Γ is a subgraph of Γ' then we must identify $\mathcal{H}_\Gamma$ with a subspace of $\mathcal{H}_{\Gamma'}$
- divide $\mathcal{H}_\Gamma$ by the action of the discrete group of the automorphisms of Γ

Operators: U_f are diagonal and E_f are the left-invariant vector fields

States that solve gauge constraint: $|\Gamma, j_\ell, v_n\rangle \in \tilde{\mathcal{H}} = \bigoplus_{\Gamma} \bigoplus_{j_\ell} \bigotimes_{n} \mathcal{H}_n$

5 COHERENT STATES

$$\psi_{H_\ell}(U_\ell) = \int_{SU(2)^N} dg_n \quad \prod_{l\in\Gamma} K_t(\, g_{s(\ell)}\, U_\ell\, g_{t(\ell)}^{-1}\; H_\ell^{-1}) \tag{1}$$

"group average" to get gauge invariant states

The heat kernel K_t peaks each U_ℓ on H_ℓ

$$K_t(U) = \sum_j (2j+1)\; e^{-2t\hbar\, j(j+1)}\; \mathrm{Tr}\,[D^j(U)] \tag{2}$$

$$H = D^{\frac{1}{2}}(R_{\vec{n}})\; e^{-i(\xi+i\eta)\frac{\sigma_3}{2}}\; D^{\frac{1}{2}}(R_{\vec{n}'}^{-1}) \qquad H_\ell \in SL(2,\mathbb{C}) \tag{3}$$

$$z = \xi + i\eta$$

- **Geometrical interpretation** for the $(\vec{n}, \vec{n}', \xi, \eta)$ labels:

  $\vec{n}, \vec{n}'$ are the 3d normals to the faces of the cellular decomposition;
  $\xi \leftrightarrow$ extrinsic curvature at the faces and $\eta \leftrightarrow$ area of the face.

- Superposition of spinnetwork states, but peaked on a given geometry.

5 DYNAMICS WITH COSMOLOGICAL CONSTANT

term that yields the
cosmological constant

↓

Transition amplitude:

boundary state $\psi \in \mathcal{H}$

(4)
$$Z_{\mathcal{C}} = \sum_{j_f, \mathrm{v}_e} \prod_f (2j+1) \prod_e e^{i\lambda \mathrm{v}_e} \prod_v A_v(j_f, \mathrm{v}_e)$$

Vertex amplitude: $A_v(j_f, \mathrm{v}_e) \longrightarrow W_v(H_\ell) = \langle A | \psi_{H_\ell} \rangle$ (5)

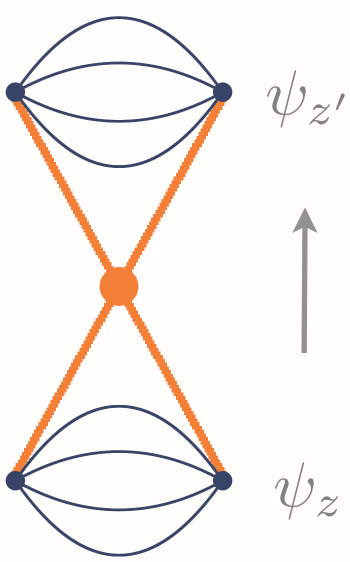
(6)
$$W_v(H_\ell) = \int_{SO(4)^N} dG_n \prod_\ell P_t(H_\ell\,,\, G_{s(\ell)} G^{-1}_{t(\ell)})$$

where

(7)
$$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)}\ \mathrm{Tr}\left[D^{(j)}(H) Y^\dagger D^{(j^+, j^-)}(G) Y \right]$$

7 INGREDIENTS TO DO COSMOLOGY

- **graph truncation** ↔ number of d.o.f we want to describe
  - *example:* 2 tetrahedra glued along all their faces = triangulated 3-sphere
- **geometry** ↔ coherent states can peaked on a given geometry
  - we choose an homogeneous and isotropic geometry
    - $z_\ell \to z$ where $Re(z) \sim \dot{a}$ and $\sqrt{Im(z)} \sim a$
    - transition amplitude from an initial to a final state (boundary states are fixed)



- **vertex expansion**
  - we consider the 1st order ↔ single vertex
- **semiclassicality** ↔ coherent states + large distance
  - large distance ⇒ large spin j (the graph truncation is well defined)

8 EVALUATION OF THE AMPLITUDE

(7) $$\langle W|\psi_{H_{(z,z')}}\rangle = W(z,z') = W(z)\,\overline{W(z')}$$

(8) $$P_t(H,G)=\sum_j (2j+1)e^{-2t\hbar j(j+1)}\ \mathrm{Tr}\left[D^{(j)}(H)Y^\dagger D^{(j^+,j^-)}(G)Y\right]$$

(9) $$H = D^{\frac{1}{2}}(R_{\vec{n}})\ e^{-iz\frac{\sigma_3}{2}}\ D^{\frac{1}{2}}(R^{-1}_{\vec{n}'}) \qquad\qquad H_\ell \in SL(2,\mathbb{C})$$

(10) $$D^{(j)}(H_\ell) = D^{(j)}(n_\ell)\ D^{(j)}(e^{-iz\frac{\sigma_3}{2}})\ D^{(j)}(n_\ell^{-1})$$

(11) $$\eta \gg 1 \qquad\qquad D^{(j)}(e^{-iz\frac{\sigma_3}{2}}) \approx e^{-izj}\ P$$

projection on the highest magnetic number

(12) $$P_t(H,G)=\sum_j (2j+1)e^{-2t\hbar j(j+1)}e^{-izj}\ \mathrm{Tr}\left[P\ Y^\dagger D^{(j^+,j^-)}(G)Y\right]$$

(13) $$W(z) = \sum_j (2j+1)\,\frac{N_o}{j^3}\ e^{-2t\hbar j(j+1)-izj-i\lambda \mathrm{v}_o j^{\frac{3}{2}}}$$

intertwiner $\mathrm{v}_e \sim \mathrm{v}_o\, j^{3/2}$

9 EVALUATION OF THE AMPLITUDE

$$W(z) = \sum_j (2j+1)\,\frac{N_o}{j^3}\; e^{-2t\hbar j(j+1) - izj - i\lambda \mathrm{v}_o j^{\frac{3}{2}}} \quad \sim \text{ gaussian sum} \tag{14}$$

$$j \sim j_o + \delta j \tag{15}$$

- max(real part of the exponent) gives where the gaussian is peaked;

$$j_o = \frac{Im(z)}{4t\hbar} \tag{16}$$

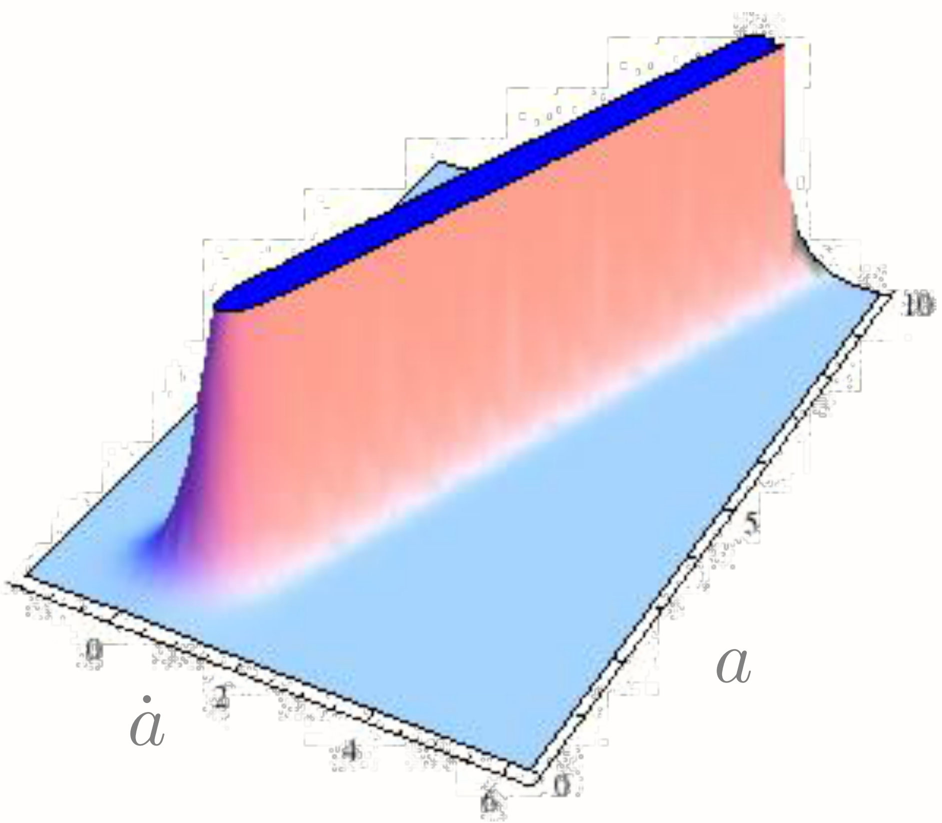
- imaginary part of the exponent=2kπ gives where the gaussian is not suppressed.

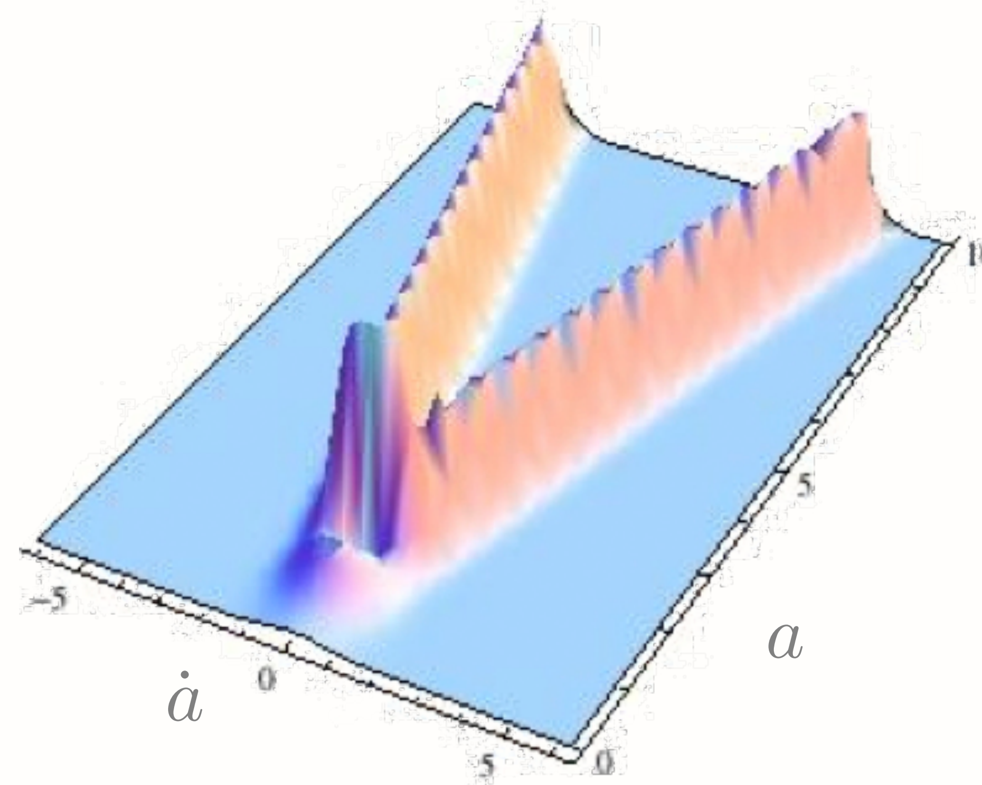
$$Re(z) + \lambda \mathrm{v}_o j^{\frac{1}{2}} = 0. \tag{17}$$

- $Re(z) \sim \dot{a}$ and $\sqrt{Im(z)} \sim a$

- $$\frac{Re(z)^2}{Im(z)} = \frac{\lambda^2 \mathrm{v}_o^2}{4t\hbar} \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \quad \text{with} \quad \Lambda = const\,\lambda^2 G^2 \hbar^2 \tag{18}$$

10 NUMERICAL EVALUATION





- $\dot{a} \propto a$
  - The numerical study confirms the validity of the approximations taken to perform the previous calculation. The extrinsic curvature is linear in the scale factor, as characteristic in de Sitter space.

- **The sign of the λ-term.**

Expanding or contracting solutions have different signs. The quantum gravity vertex (with or without cosmological constant) is the sum of two terms with opposite time orientation. Thus the final amplitude does not depend of the choice of the sign.

11 A HAMILTONIAN CONSTRAINT

Our amplitude happens to satisfy an equation

$$\hat{H}(z, \frac{d}{dz})W(z', z) = \left(\hat{z}^2 - \hat{\bar{z}}^2 - \cancel{3t}\hbar\right) W(z', z) = 0 \tag{19}$$

With the cosmological constant:

$$(z + \frac{3}{2}\lambda \mathrm{v}_o j_o^{\frac{1}{2}})^2 - \overline{(z + \frac{3}{2}\lambda \mathrm{v}_o j_o^{\frac{1}{2}})^2} = 0 \tag{20}$$

$$i4\, Im(z)\ (Re(z) + \frac{3}{2}\lambda \mathrm{v}_o j_o^{\frac{1}{2}}) = 0 \tag{21}$$

- $a^3 \sim 2\mathrm{v}_o j_o^{\frac{3}{2}}$

$$\frac{Re(z)^2}{Im(z)} = \frac{\lambda^2 \mathrm{v}_o^2}{4t\hbar} \longrightarrow \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \tag{22}$$

It is possible to translate the information from the transition amplitude to an effective Hamiltonian constraint.

12 SUMMARY & RESULTS

- A simple way to add the cosmological constant to the dynamics of LQG.

- It is possible to compute quantum transition amplitudes explicitly in suitable approximations:
  - graph expansion
  - vertex expansion
  - large volume expansion

- There are approximations in the quantum theory that yield cosmology.

- The transition amplitude computed appears to give the correct Friedmann dynamics with Λ in the classical limit.

- The theory recovers general relativity in the semiclassical limit, also for non-trivial solutions (de Sitter space).