# Loop quantum gravity from the quantum theory of impulsive gravitational waves

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# Motivation

- In LQG twistors were discovered by embedding the LQG phase space into a larger phase space with canonical Darboux coordinates.
- The construction is intricate and rests heavily upon the discretisation. It takes the LQG lattice phase $(T^*SL(2,\mathbb{C}))^{\#\text{links}}/SL(2,\mathbb{C})^{\#\text{nodes}}$ space for granted.
- Thus, the question arose: What has the LQG twistor to do with GR in the continuum? In this talk, I will give a definite answer: The LQG twistor is the natural gravitational boundary variable on a null surface.
- This is not just a technical result. I believe it has wide implications for the further development of our theory.

*L Freidel and S Speziale, From twistors to twisted geometries, Phys. Rev. D 82 (2010), arXiv:1006.0199.
*E Bianchi, Loop Quantum Gravity a la Aharonov-Bohm , Gen. Rel. Grav. 46 (2014), arXiv:0907.4388.
*ww, One-dimensional action for simplicial gravity in three dimensions, Phys. Rev. D 90 (2014), arXiv:1402.6708.
*S Speziale and ww, Twistorial structure of loop-gravity transition amplitudes, Phys. Rev. D 86 (2012), arXiv:1207.6348.
*S Speziale and M Zhang, Null twisted geometries, Phys. Rev. D 89 (2014) , arXiv:1311.3279.
*ww, Twistorial phase space for complex Ashtekar variables, Class. Quant. Grav. 29 (2012), arXiv:1107.5002.

# Outline of the talk

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# Spinors as gravitational boundary variables on a null surface

# Basic idea: Three-surface spinors as boundary DOF

***Boundary action crucial for discretisation:*** *To discretise gravity, we split the manifold into domains, with no local degrees of freedom inside. The only contribution to the action can, therefore, only come from the internal boundaries.*

***Spinors as natural gravitational boundary variables:*** *General relativity (with tetrads) is a background invariant gauge theory for the Lorentz group. At a boundary, a gauge connection couples naturally to its boundary charges. The boundary charge for an $SL(2,\mathbb{C})$ gauge connection is spin — thus, spinors as the fundamental boundary variables. At a null surface, this becomes even more natural, because the intrinsic three-dimensional null geometry is fully specified by a spinor $\ell^A$ and a spinor-valued two-form $\boldsymbol{\kappa}_A$.*

# A key observation: Self-dual area two-form on a null surface

- Consider the self-dual component $\Sigma_{AB}$ of the Plebański two-form $\Sigma_{\alpha\beta} = e_\alpha \wedge e_\beta$.

$$\begin{pmatrix} \Sigma^{A}{}_{B} & \emptyset \\ \emptyset & -\bar{\Sigma}_{\bar{A}}{}^{\bar{B}} \end{pmatrix} = -\frac{1}{8}[\gamma_\alpha, \gamma_\beta] e^\alpha \wedge e^\beta .$$

- Let $\varphi : \mathscr{N} \hookrightarrow \mathscr{M}$ be the canonical embedding of a null surface $\mathscr{N}$ into spacetime $\mathscr{M}$.
- It always exists then a spinor $\ell^A : \mathscr{N} \rightarrow \mathbb{C}^2$ and a spinor-valued two-form $\boldsymbol{\kappa}^A \in \Omega^2(\mathscr{N} : \mathbb{C}^2)$ on $\mathscr{N}$ such that

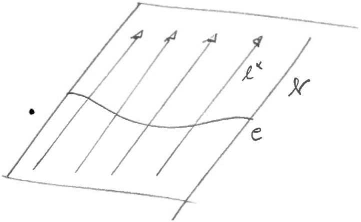
$$\varphi^* \Sigma_{AB} = \boldsymbol{\kappa}_{(A} \ell_{B)} .$$

Notation:

- $A, B, C, \dots = 0, 1$ transform under the fundamental representation of $SL(2, \mathbb{C})$.
- $\bar{A}, \bar{B}, \bar{C}, \dots$ transform under the complex conjugate representation.
- $\ell_A = \epsilon_{BA} \ell^B$, $\ell^A = \epsilon^{AB} \ell_B$.

# Geometric meaning of the boundary spinors $\boldsymbol{\kappa}_A$ and $\ell^A$

We now have spinors $\boldsymbol{\kappa}_A$ and $\ell^A$ on $\mathscr{N}$. What is their geometric meaning?




- The spin $(\frac{1}{2},\frac{1}{2})$ component $\ell^\alpha \simeq \mathrm{i}\ell^A\bar{\ell}^{\bar{A}}$ returns the null generators of $\mathscr{N}$.
- The spin $(1,0)$ component $\boldsymbol{\kappa}_{(A}\ell_{B)}$ returns the pull-back of the self-dual two-form $\varphi^*\Sigma_{AB}$ to $\mathscr{N}$.

- The Lorentz invariant spin $(0,0)$ component $\boldsymbol{\varepsilon} = -\mathrm{i}\boldsymbol{\kappa}_A\ell^A$ defines the *oriented* area of any two-dimensional cross section $\mathscr{C}$ of $\mathscr{N}$

$$\pm\,\mathrm{Area}[\mathscr{C}] = \int_{\mathscr{C}} \boldsymbol{\varepsilon} = -\mathrm{i}\int_{\mathscr{C}} \boldsymbol{\kappa}_A\ell^A.$$

- The reverse is also true: Given the hypersurface twistor $(\bar{\boldsymbol{\kappa}}_{\bar{A}}, \ell^A) = Z$, we can reconstruct the intrinsic signature $(0{+}{+})$ metric $q_{ab}$ on $\mathscr{N}$.

# New boundery term for the self-dual action

- The self-dual action is

$$S_{\mathscr{M}}[A,e] = \frac{\mathrm{i}}{8\pi G}\int_{\mathscr{M}} \Sigma_{AB}\wedge F^{AB}.$$

- The boundary conditions are that the pull-back $\varphi^*\delta e^\alpha$ of the variation of the tetrad vanishes, but $\varphi^*\delta A^A{}_B$ is arbitrary. This yields

$$\delta S_{\mathscr{M}}[A,e] = \mathrm{EOM} + \frac{\mathrm{i}}{8\pi G}\int_{\partial\mathscr{M}} \Sigma_{AB}\wedge \delta A^{AB}.$$

- For the action to be functionally differentiable, we introduce a boundary term, whose variation cancels the reminder from the bulk.
- If the boundary is null, i.e. if $\varphi^*\Sigma = \boldsymbol{\kappa}_{(A}\ell_{B)}$, we can work with the following $SL(2,\mathbb{C})$-invariant boundary action

$$S_{\partial\mathscr{M}}[A|\boldsymbol{\kappa},\ell] = \frac{\mathrm{i}}{8\pi G}\int_{\partial\mathscr{M}} \boldsymbol{\kappa}_A\wedge D\ell^A.$$

- The boundary condition $\varphi^*\Sigma_{AB} = \boldsymbol{\kappa}_{(A}\ell_{B)}$ is then derived as an equation of motion from the bulk+boundary variation of the self-dual action.

Introducing the Barbero – Immirzi parameter $\beta > 0$ amounts to twisting the self-dual and anti-self-dual sectors and add them up together.

- Bulk action

$$S_{\mathscr{M}}[A, e] = \frac{\mathrm{i}}{8\pi\beta G} \int_{\mathscr{M}} \Big[ (\beta + \mathrm{i})\, \Sigma_{AB} \wedge F^{AB} \Big] + \mathrm{cc.}$$

- Boundary action

$$S_{\partial\mathscr{M}}[A|\boldsymbol{\pi}, \ell] = \int_{\partial\mathscr{M}} \boldsymbol{\pi}_A \wedge D\ell^A + \mathrm{cc.}$$

- Where we have defined the momentum spinor

$$\boldsymbol{\pi}_A := \frac{\mathrm{i}}{8\pi\beta G}(\beta + \mathrm{i})\boldsymbol{\kappa}_A.$$

# Reality conditions and $U(1)_{\mathbb{C}}$ gauge invariance

- The boundary action is

$$S_{\partial\mathscr{M}}[A|\boldsymbol{\pi},\ell] = \int_{\partial\mathscr{M}} \boldsymbol{\pi}_A \wedge D\ell^A + \text{cc.}$$

- The canonical area element is

$$\boldsymbol{\varepsilon} = -\frac{8\pi\beta G}{\beta+\mathrm{i}}\boldsymbol{\pi}_A\ell^A.$$

- For the area to be real-valued, we have to satisfy the reality conditions

$$\frac{\mathrm{i}}{\beta+\mathrm{i}}\boldsymbol{\pi}_A\ell^A + \text{cc.} = 0.$$

- N.b. So far, the action is defective: The spinors are unique modulo $U(1)_{\mathbb{C}}$ transformations and the action is not invariant under this symmetry

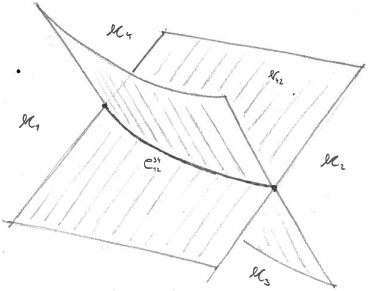
$$\boldsymbol{\pi}_A \longrightarrow \mathrm{e}^{-z/2}\boldsymbol{\pi}_A, \quad \ell^A \longrightarrow \mathrm{e}^{+z/2}\ell^A.$$

- We will see how this symmetry is restored when glueing together adjacent regions.

# Discretising/truncating gravity with topological null defects

# Topological setup

We introduce a cellular decomposition, such that the four-dimensional manifold $\mathscr{M}$ splits into four-dimensional cells $\mathscr{M}_1, \mathscr{M}_2, \dots$.




- The four-dimensional cells are bounded by three-dimensional interfaces $\mathscr{N}_{ij} = \mathscr{M}_i \cap \mathscr{M}_j \subset \partial\mathscr{M}_i$, $\mathscr{N}_{ij} = \mathscr{N}_{ji}^{-1}$.
- We will *impose* constraints requiring all $\mathscr{N}_{ij}$ to be null.
- The intersection of two such surfaces defines a two-dimensional corner $\mathscr{C}$.
- All corners $\mathscr{C}_{ij}^{mn}$ are adjacent to four such null surfaces.
- Edges and vertices do not appear in the construction.

We introduce a truncation, and demand that every four-cell is either flat or constantly curved inside (depending on the value of the cosmological constant).

- Hence we require
$$\forall \mathscr{M}_i : F_{AB} - \frac{\Lambda}{3}\Sigma_{AB} = 0.$$
- Adding the Barbero – Immirzi parameter amounts to working with the twisted momentum
$$\Pi_{AB} = \frac{\mathrm{i}}{16\pi\beta G}(\beta + \mathrm{i})\Sigma_{AB}.$$
- For the bulk action, we can thus choose

$$S_{\mathscr{M}_i}[A,\Pi] = \int_{\mathscr{M}_i} \left[\Pi_{AB} \wedge F^{AB} + \frac{8\pi \mathrm{i}\beta\Lambda G}{3}\frac{1}{\beta + \mathrm{i}}\Pi_{AB} \wedge \Pi^{AB}\right] + \mathrm{cc}.$$

# Definition of the action: Glueing conditions

- Each three-surface $\mathscr{N}$ bounds two bulk regions, say $\mathscr{M}$ and $\underset{\sim}{\mathscr{M}}$.
- All boundary variables appear twice, one from *below*, and the other from *above* the interface.
- We then have to impose conditions that determine the discontinuity at the interface. We require, as in Regge calculus, that the intrinsic three-geometry is the same from either side of the interface. All discontinuities are confined to the transversal directions.

$$\varphi^* g_{ab} = \underset{\sim}{\varphi}^* g_{ab} \qquad (*)$$

- How to do it with spinors? Build all $SL(2,\mathbb{C})$ invariant bilinears, and match them across the interface. Matching conditions

$$\left.\begin{aligned} \boldsymbol{\pi}_A{}^a \ell^A &= \underset{\sim}{\boldsymbol{\pi}}_A{}^a \underset{\sim}{\ell}^A, \\ \boldsymbol{\pi}_A{}^a \boldsymbol{\pi}^{Ab} &= \underset{\sim}{\boldsymbol{\pi}}_A{}^a \underset{\sim}{\boldsymbol{\pi}}^{Ab}. \end{aligned}\right\} \quad (**)$$

- $(*)$ and $(**)$ are equivalent. $(**)$ implies there is an $SL(2,\mathbb{C})$ gauge transformation $h$ s.t.

$$\underset{\sim}{\ell}^A = h^A{}_B \ell^B, \quad \underset{\sim}{\boldsymbol{\pi}}^{Aa} = h^A{}_B \boldsymbol{\pi}^{Ba}.$$

Notation:

- $\boldsymbol{\pi}_A{}^a = \frac{1}{2}\tilde{\boldsymbol{\epsilon}}^{abc}\boldsymbol{\pi}_{Abc}$ is the dual density (a spinor-valued vector-density).

- For every interface, the boundary term contains variables from either side of the interface.
- We then introduce Lagrange multipliers $\omega$ and $\Psi_{ab}$ to match the variables across the interface.
- One further Lagrange multiplier $\lambda$ to impose the reality conditions.

Resulting action for e.g. a single interface

$$S_{\mathcal{N}}[\boldsymbol{\pi},\ell,\underset{\sim}{\boldsymbol{\pi}},\underset{\sim}{\ell}|A,\underset{\sim}{A}|\lambda,\omega,\Psi] = \int_{\mathcal{N}}\Big[\boldsymbol{\pi}_A\wedge(D-\omega)\ell^A - \underset{\sim}{\boldsymbol{\pi}}_A\wedge(\underset{\sim}{D}-\omega)\underset{\sim}{\ell}^A +$$
$$+\frac{\lambda}{2}\Big(\frac{\mathrm{i}}{\beta+\mathrm{i}}(\boldsymbol{\pi}_A\ell^A+\underset{\sim}{\boldsymbol{\pi}}_A\underset{\sim}{\ell}^A)+\mathrm{cc.}\Big)+\Psi_{ab}\Big(\boldsymbol{\pi}_A{}^a\boldsymbol{\pi}^{Ab}-\underset{\sim}{\boldsymbol{\pi}}_A{}^a\underset{\sim}{\boldsymbol{\pi}}^{Ab}\Big)\Big]+\mathrm{cc.}$$

# The different terms in the action

Let us discuss the boundary action more closely

$$S_{\mathcal{N}}[\boldsymbol{\pi},\ell,\utilde{\boldsymbol{\pi}},\utilde{\ell}|A,\utilde{A}|\lambda,\omega,\Psi]=\int_{\mathcal{N}}\Big[\boldsymbol{\pi}_A\wedge(D-\omega)\ell^A-\utilde{\boldsymbol{\pi}}_A\wedge(\utilde{D}-\omega)\utilde{\ell}^A+$$
$$+\frac{\lambda}{2}\Big(\frac{\mathrm{i}}{\beta+\mathrm{i}}(\boldsymbol{\pi}_A\ell^A+\utilde{\boldsymbol{\pi}}_A\utilde{\ell}^A)+\mathrm{cc.}\Big)+\Psi_{ab}\Big(\boldsymbol{\pi}_A{}^a\boldsymbol{\pi}^{Ab}-\utilde{\boldsymbol{\pi}}_A{}^a\utilde{\boldsymbol{\pi}}^{Ab}\Big)\Big]+\mathrm{cc.}$$

- $SL(2,\mathbb{C})$ derivatives on either side
- Lagrange multiplier imposing the area-matching conditions. It restores $U(1)_{\mathbb{C}}$ gauge invariance of the boundary action.
- reality conditions
- shape matching conditions

Manifest gauge symmetry:

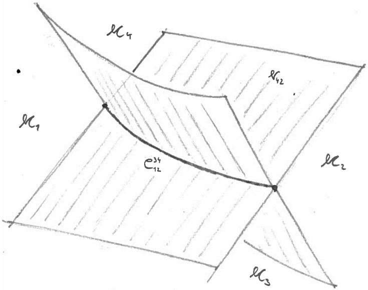
$$SL(2,\mathbb{C})\times SL(2,\mathbb{C})\times U(1)_{\mathbb{C}}$$

# Corner term

The action is the sum over bulk regions, three-dimensional interfaces and two-dimensional corners.

$$S = \sum_i S_{\mathscr{M}_i} + \sum_{\langle ij \rangle} S_{\mathscr{N}_{ij}} + \sum_{\langle ijmn \rangle} S_{\mathscr{C}_{ij}^{mn}}$$

$$S_{\mathscr{C}_{ij}^{mn}}[\alpha, \{\ell\}] = \int_{\mathscr{C}_{ij}^{mn}} \alpha \left[\ell_A^{im}\, \ell_{in}^A - \ell_A^{jm}\, \ell_{jn}^A + \ell_A^{mj} \ell_{mi}^A - \ell_A^{nj} \ell_{ni}^A\right]$$




- The action is functionally differentiable only if we add a term for the two-dimensional corners.
- Its $\ell^A$-variations yields boundary conditions at the corners. $\alpha$-variation vanishes *on-shell*.

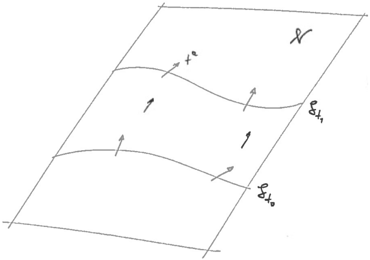
- The bulk action is topological.
- Only at the boundary does the Hamiltonian analysis become tricky.
- We integrate out the bulk momentum, and arrive at an entirely three-dimensional action: Two copies of the $SL(2,\mathbb{C})$ Chern – Simons action coupled to constrained boundary spinors.

$$
\begin{aligned}
S_{\mathscr{N}}[A,\underset{\sim}{A}|\boldsymbol{\pi},\ell,\underset{\sim}{\boldsymbol{\pi}},\underset{\sim}{\ell}|\lambda,\omega,\Psi] &= \frac{\gamma}{2}S_{\mathrm{CS},\mathscr{N}}[A]-\frac{\gamma}{2}S_{\mathrm{CS},\mathscr{N}}[\underset{\sim}{A}]+\\
&+\int_{\mathscr{N}}\Big[\boldsymbol{\pi}_A(D-\omega)\ell^A-\frac{\lambda}{2}\Big(\frac{\mathrm{i}}{\beta+\mathrm{i}}\boldsymbol{\pi}_A\ell^A+\mathrm{cc.}\Big)-\frac{1}{2}\Psi_{ab}\boldsymbol{\pi}_A{}^a\boldsymbol{\pi}^{Ab}\Big]+\\
&-\int_{\mathscr{N}}\Big[\underset{\sim}{\boldsymbol{\pi}}_A(\underset{\sim}{D}-\omega)\underset{\sim}{\ell}^A-\frac{\lambda}{2}\Big(\frac{\mathrm{i}}{\beta+\mathrm{i}}\underset{\sim}{\boldsymbol{\pi}}_A\underset{\sim}{\ell}^A+\mathrm{cc.}\Big)-\frac{1}{2}\Psi_{ab}\underset{\sim}{\boldsymbol{\pi}}_A{}^a\underset{\sim}{\boldsymbol{\pi}}^{Ab}\Big]+\\
&+\mathrm{cc.}
\end{aligned}
$$

Notation:

- $S_{\mathrm{CS},\mathscr{N}}[A]=\int_{\mathscr{N}}\mathrm{Tr}\left[A\wedge\mathrm{d}A+\tfrac{2}{3}A\wedge A\wedge A\right]$
- Complex-valued level: $\gamma=3(\beta+\mathrm{i})/(8\pi\mathrm{i}\,\beta\Lambda G)$

## Hamiltonian analysis: Symplectic structure, constraints




- We perform a $2+1$ split of the boundary action.
- $t = \mathrm{const.}$ slices $\mathcal{S}_t = \{t\} \times \mathcal{S}$: $\mathcal{N} \simeq (0,1) \times \mathcal{S}$.
- Choose a transversal vector field $t^a\partial_a t = 1$, not necessarily null.

- Canonical variables obtained from pull-back $\varphi_t : \mathcal{S}_t \hookrightarrow \mathcal{N}$ through

$$\boldsymbol{\pi}_A = \varphi_t^* \boldsymbol{\pi}_A, \quad A^A{}_{Ba} = [\varphi_t^* A^A{}_B]_a,$$
$$\underset{\sim}{\boldsymbol{\pi}}_A = \varphi_t^* \underset{\sim}{\boldsymbol{\pi}}_A, \quad \underset{\sim}{A}{}^A{}_{Ba} = [\varphi_t^* \underset{\sim}{A}{}^A{}_B]_a.$$

- Canonical symplectic structure (inverse signs for *"tilde"* variables)

$$\{\, \boldsymbol{\pi}_A(p), \ell_B(q)\} = +\, \epsilon_{AB}\delta^{(2)}(p,q),$$
$$\{A^{AB}{}_a(p), A_{CDb}(q)\} = +\, \frac{1}{\gamma} \underset{\sim}{\epsilon}_{ab}\delta_C^{(A}\delta_D^{B)}\delta^{(2)}(p,q).$$

## Hamiltonian analysis: Constraints

- Two-copies of $SL(2,\mathbb{C})$ Gauss constraints — *all first-class.*

$$G_{AB}[\Lambda^{AB}] = \int_{\mathcal{S}} \Lambda^{AB}\left[\frac{\gamma}{2}\hat{\epsilon}^{ab}F_{ABab} + \boldsymbol{\pi}_A\ell_B\right] \stackrel{!}{=} 0$$

$$\underset{\sim}{G}_{AB}[\underset{\sim}{\Lambda}^{AB}] = \int_{\mathcal{S}} \underset{\sim}{\Lambda}^{AB}\left[\frac{\gamma}{2}\hat{\epsilon}^{ab}\underset{\sim}{F}_{ABab} + \underset{\sim}{\boldsymbol{\pi}}_A\underset{\sim}{\ell}_B\right] \stackrel{!}{=} 0$$

- Two-dimensional diffeomorphism constraint — *all first-class.*

$$H_a[N^a] = \int_{\mathcal{S}} N^a\left[\boldsymbol{\pi}_A D_a\ell^A - \underset{\sim}{\boldsymbol{\pi}}_A\underset{\sim}{D}_a\underset{\sim}{\ell}^A + \mathrm{cc.}\right] \stackrel{!}{=} 0$$

- $U(1)_{\mathbb{C}}$ area-matching — *one complex first-class constraint.*

$$M[\varphi] = \int_{\mathcal{S}} \varphi\left(\boldsymbol{\pi}_A\ell^A - \underset{\sim}{\boldsymbol{\pi}}_A\underset{\sim}{\ell}^A\right) \stackrel{!}{=} 0$$

- shape matching, generating residual BF-shift symmetry — *three first-class constraints, one second-class.*

$$M_a[\boldsymbol{J}^a] = \int_{\mathcal{S}} \boldsymbol{J}^a\left(\ell_A D_a\ell^A - \underset{\sim}{\ell}_A\underset{\sim}{D}_a\underset{\sim}{\ell}^A\right) \stackrel{!}{=} 0$$

- reality conditions — *one second-class constraint*

$$S[N] = \int_{\mathcal{S}} N\left[\frac{\mathrm{i}}{\beta+\mathrm{i}}\left(\boldsymbol{\pi}_A\ell^A + \underset{\sim}{\boldsymbol{\pi}}_A\underset{\sim}{\ell}^A\right) + \mathrm{cc.}\right] \stackrel{!}{=} 0$$

# Hamiltonian analysis: Zero physical degrees of freedom

- The kinematical phase space has canonical coordinates $(\boldsymbol{\pi}_A, \ell^A, A^A{}_{Ba})$ and $(\underset{\sim}{\boldsymbol{\pi}}_A, \underset{\sim}{\ell}^A, \underset{\sim}{A}^A{}_{Ba})$.
- These are $2 \times (2 \times 2 + 2 \times 2 + 2 \times 6) = 40$ phase space dimensions per point.
- The constraints remove forty local directions from phase space as well.
- The boundary has no local degrees of freedom.

| constraints | Dirac classification | DOF removed |
|---|---|---|
| $G_{AB}[\Lambda^{AB}]$ | three $\mathbb{C}$-valued first class constraints | $2 \times 6 = 12$ |
| $\underset{\sim}{G}_{AB}[\underset{\sim}{\Lambda}^{AB}]$ | three $\mathbb{C}$-valued first class constraints | $2 \times 6 = 12$ |
| $H_a[N^a]$ | two first class constraints | $2 \times 2 = 4$ |
| $M[\varphi]$ | one $\mathbb{C}$-valued first class constraint | $2 \times 2 = 4$ |
| $M_a[\boldsymbol{J}^a]$ | one second class, three first class | $1 + 2 \times 3 = 7$ |
| $S[N]$ | one first class constraint | $1$ |
| | | $40$ |

# Spinors as gravitational boundary variables on a null surface

***take-home message:*** *The Hamiltonian formalism (constraints: area matching constraint, reality conditions,. . . and the symplectic structure) for the boundary spinors matches exactly what has been postulated earlier in LQG by hand. The LQG twistor variables are the canonical boundary variables of the gravitational field on a null surface.*


*L Freidel and S Speziale, From twistors to twisted geometries, Phys. Rev. D 82 (2010), `arXiv:1006.0199`.
*E Bianchi, Loop Quantum Gravity a la Aharonov-Bohm , Gen. Rel. Grav. 46 (2014), `arXiv:0907.4388`.
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*ww, Twistorial phase space for complex Ashtekar variables, Class. Quant. Grav. 29 (2012), `arXiv:1107.5002`.

# Special solutions: Plane fronted gravitational waves

- Consider the following ansatz for the tetrad in the vicinity of a defect

$$\left.\begin{array}{c} e^\alpha = \ell^\alpha \mathrm{d}u + \left[k^\alpha + \Theta(v)\left(f m^\alpha + \bar{f}\bar{m}^\alpha + f\bar{f}\ell^\alpha\right)\right]\mathrm{d}v + \\ + \bar{m}^\alpha \mathrm{d}z + m^\alpha \mathrm{d}\bar{z} \end{array}\right\} \quad (*)$$

- $(\ell^\alpha, k^\alpha, m^\alpha, \bar{m}^\alpha)$ is a normalized Newman – Penrose null tetrad $\ell^\alpha \simeq \mathrm{i}\ell^A\bar{\ell}^{\bar{A}}$, $k^\alpha \simeq \mathrm{i}k^A\bar{k}^{\bar{A}}$, $m^\alpha \simeq \mathrm{i}\ell^A\bar{k}^{\bar{A}}$, built from the spinors $(\ell^A, \ell^A) : k_A\ell^A = 1$, which are constant w.r.t. the fiducial derivative $\mathrm{d}k^A = \mathrm{d}\ell^A = 0$.
- $\Theta(v)$ is the Heaviside step function, metric discontinuous in transversal null direction
- Geometry fully specified by a function $f(u, v, z, \bar{z})$. For $\partial_u f = 0 = \partial_v f$, the ansatz $(*)$ solves EOM and glueing conditions derived from the discrete action for $\Lambda = 0$.
- Distributional Weyl curvature

$$\Psi_{ABCD} = \delta(v)\partial_z f(z, \bar{z})\ell_A\ell_B\ell_C\ell_D$$

- Energy momentum tensor

$$T_{ab} = \frac{1}{8\pi G}\delta(v)\left(\partial_{\bar{z}} f + \partial_z \bar{f}\right)\ell_a\ell_b$$

# Outlook and conclusion: A third way towards LQG

- Conjugate momentum $\boldsymbol{\pi}_A$ is a spinor-valued density on a cross-section $\mathscr{S}$ of $\mathscr{N}$.

$$\widehat{\boldsymbol{\pi}_A(x)} = \frac{\hbar}{\mathrm{i}}\frac{\delta}{\delta \ell^A(x)}, \qquad \widehat{\bar{\boldsymbol{\pi}}_{\bar{A}}(x)} = \frac{\hbar}{\mathrm{i}}\frac{\delta}{\delta \bar{\ell}^{\bar{A}}(x)}$$

- Cylindrical functions are wave functionals created from distributional point defects over spinorial version of the AL vacuum

$$\Psi_f[\ell] = f\big(\ell(x_1), \dots \ell(x_N)\big)$$

- Inner product

$$\langle \Psi_f, \Psi_{f'}\rangle = \int \mathrm{d}\mu(\ell_1, \dots, \ell_N)\overline{f(\ell_1, \dots, \ell_N)} f'(\ell_1, \dots, \ell_N)$$

- Amplitudes through path integral evaluation of boundary states, w.r.t. topological boundary action

$$\sum_{\langle ij\rangle} \int_{\mathscr{N}_{ij}} \left[\frac{\gamma}{2}\,\mathrm{Tr}\left(A\mathrm{d}A + \frac{2}{3}A^3\right) + \boldsymbol{\pi}_A D\ell^A\right] + \text{constraints} + \text{cc.}$$

*ww, Complex Ashtekar variables, the Kodama state and spinfoam gravity (2011), `arXiv:1105.2330`.
*H Haggard, M Han, W Kaminski, A Riello, Four-dimensional Quantum Gravity with a Cosmological Constant from Three-dimensional Holomorphic Blocks, Phys Lett. B 752 (2016), `arXiv:1509.00458`.

# LQG as a topological field theory with null defects

## All very similar to standard LQG

- We require that the quantum states $\Psi_f[\ell]$ carry a unitary representation of the Lorentz group and lie in the kernel of the reality conditions. This fixes the measure $d\mu(\ell_1, \dots)$ and recovers LQG quantisation of area.
- Oriented area operator (with positive (negative) discrete eigenvalues).

$$\widehat{\mathrm{Ar}[\mathcal{S}]} = -\frac{4\pi\beta G}{\beta + \mathrm{i}} \, :\!\int_{\mathcal{S}} \ell^A \frac{\delta}{\delta \ell^A}\!: + \mathrm{h.c.}$$

## But conceptually also very different from standard LQG

- The area defects are confined to three-dimensional null surfaces.
- No reduction to compact $SU(2)$ gauge group ever necessary.
- $SL(2,\mathbb{C})$ gauge symmetry manifest.
- Hence, LQG area spectrum is fully compatible with local Lorentz invariance and the universal speed of light.
- The four-dimensional geometry is distributional, curvature is confined to three-dimensional interfaces, which represent a system of colliding null surfaces.
- The classical discretised theory admits well-known distributional solutions of GR: Plane fronted gravitational waves, which represent e.g. the gravitational field of a massless point particle.