Black Hole Collapse in Loop Quantum Gravity

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Kelly, Santacruz, WE, PRD **102** (2020) 106024, `arXiv:2006.09302`
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ILQGS Panel on Black Holes in LQG

Reminder: Motivation

Black holes are one of the few places quantum gravity effects are expected to be important.

There are two main problems in black hole physics that any successful theory of quantum gravity should be able to solve:

- **Singularity**
  Black hole space-times in general relativity are singular. Can quantum gravity resolve the singularity?
- **Information loss problem**
  Hawking radiation is thermal. If a black hole fully evaporates, an initial pure state seems to evolve to a thermal state. Can quantum gravity somehow restore unitarity?

Black Hole Collapse

Our goal is to understand quantum gravity effects in black holes, starting from the initial collapse.

There are (at least) two good reasons to study black hole collapse:

1. **How is the singularity avoided?**
   This is presumably a dynamical process, so we should study space-times where (classically) the singularity forms dynamically.
2. **The role of matter**
   Classically, vacuum is often thought sufficient since matter from the collapse will eventually hit the singularity and 'disappear'. What if there is no singularity?
   - → During collapse an inner horizon forms; this is missed in vacuum.

For these reasons, it is interesting to consider the Lemaître-Tolman-Bondi (LTB) space-time: spherically symmetric, with a dust field.

Previous Work in Loop Quantum Gravity

There has been a lot of work studying black holes in loop quantum gravity, by many groups. The literature is too extensive to cite all relevant work here, but we built on what has come before.

In our work, we combine 3 key aspects that had already been considered separately, but not together:

- A Hamiltonian treatment of the full space-time (not just the interior) [Campiglia, Gambini, Pullin, 2007],
- Use the same 'improved dynamics' quantization as loop quantum cosmology [Boehmer, Vandersloot, 2007; Chiou, Ni, Tang, 2012; Gambini, Olmedo, Pullin, 2020],
- Include matter with local degrees of freedom [Bojowald, Harada, Tibrewala, 2008; Campiglia, Gambini, Olmedo, Pullin, 2016; Benitez, Gambini, Lehner, Liebling, Pullin, 2020].

See also the more recent [Han, Liu, 2020; Giesel, Han, Li, Liu, Singh, 2022].

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$$\mathrm{d}s^2 = -1\,\mathrm{d}t^2 + 2N^x\,\mathrm{d}t\mathrm{d}x + f(x,t)\,\mathrm{d}x^2 + x^2\mathrm{d}\Omega^2;$$

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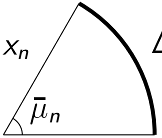
4. Do a loop quantization, using the improved dynamics:



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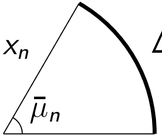

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5. Extract effective dynamics from the quantum theory, and take the continuum limit.

Effective Equations

The effective dynamics capture leading-order loop quantum gravity effects, and as usual are generated by a Hamiltonian (density) $\mathcal{H}^{\text{eff}}$.

I will focus on the 'marginally trapped' class of solutions with the metric

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There remains one degree of freedom b (the connection component in angular directions) that satisfies the non-linear equation of motion

$$\dot{b} + \frac{1}{2\ell_{\mathrm{Pl}}^2 x}\partial_x\left(x^3\sin^2\frac{\ell_{\mathrm{Pl}}b}{x}\right) = 0.$$

To find solutions to non-linear wave equations, it is typically necessary to allow weak solutions.

Weak Solutions

Weak solutions are not differentiable, so they cannot solve a differential equation—but they can solve an integral form of the equation of motion. For the conservation equation

$$\dot{u} + \partial_x[f(u)] = 0,$$

weak solutions $u(x, t)$ satisfy

$$\int_{x_1}^{x_2} \mathrm{d}x \; u \Big|_{t=t_1}^{t=t_2} \; + \; \int_{t_1}^{t_2} \mathrm{d}t \; f(u) \Big|_{x=x_1}^{x=x_2} \; = \; 0,$$

for all x_1, x_2, t_1, t_2.

When the weak solution is discontinuous, the discontinuity is called a shock wave.

Weak Solutions in General Relativity

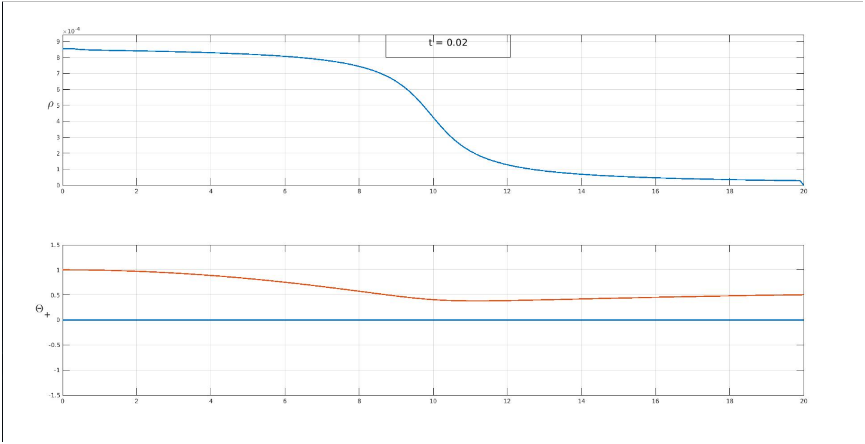
Examples of weak solutions in general relativity are thin shell solutions obtained using Israel's junction conditions [Israel, 1966], and the Dray-'t Hooft shock wave [Dray, 't Hooft, 1985].

It has also been argued that weak solutions should be considered for the LTB space-time in general relativity [Nolan, 2003; Lasky, Lun, Burston, 2006].

We will allow for weak solutions in the LQC effective dynamics for LTB space-times.

- Analytical methods are useful for simple configurations.
- Otherwise, numerics are typically necessary—we use the standard Godunov algorithm.

Video: Star-like Collapse



Numerical Results

- There is no singularity, it is replaced by a bounce.
- A shock wave forms, at the latest at the bounce time.
- The bounce is stable: there is no mass inflation [Poisson, Israel, 1989] and no instability to infalling matter like white holes in general relativity [Eardley, 1974].
- The lifetime seems robust: we find $T = \dfrac{8\pi M^2}{3m_{\rm Pl}}$ in various analytic and numeric solutions.

Implications for Information Loss

There is no singularity, and no event horizon: it should be possible to recover information once the apparent horizons are gone.

1. LQG corrections at the horizon are negligible:
   $\Rightarrow$ Hawking radiation will occur as usual.
2. The predicted black hole lifetime $T \sim M^2/m_{\rm Pl}$ is much less than the Page time $\sim M^3/m_{\rm Pl}^2$.
   $\Rightarrow$ The entropy of Hawking radiation $S_{HR} \ll A_{BH}/4\ell_{\rm Pl}^2$ always.
3. The Hawking radiation will presumably be entangled with matter/gravitational fields in the shock wave. These degrees of freedom will be accessible to outside observers once the apparent horizon vanishes when the shock passes beyond $x = 2GM$.

More work is required, but it seems likely that there is no information loss problem here.

Summary

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- There are apparent horizons but no event horizon,
- A shock wave forms, and slowly moves outwards,
- The lifetime of the black hole is $\sim M^2/m_{\rm Pl}$,
- Information loss seems to be avoided.

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Next steps:

- Explore other types of initial conditions [work in progress],
- Avoid fixing gauges [see Giesel, Han, Li, Liu, Singh, 2022],
- Couple other matter fields [see Benitez, Gambini, Lehner, Liebling, Pullin, 2020],
- Include Hawking radiation,
  $\Rightarrow$ **Can we explicitly show how to recover information from the black hole and purify Hawking radiation?**
- Extend to rotating black holes. . .

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Thank you for your attention!