Lifting General Relativity to Observer Space

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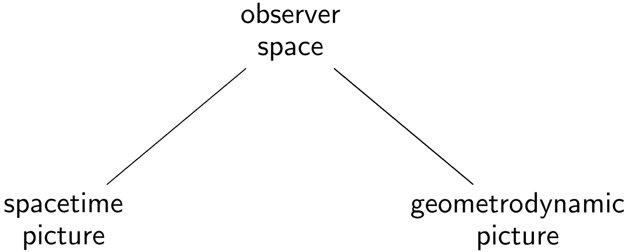

observer space
spacetime picture
geometrodynamic picture

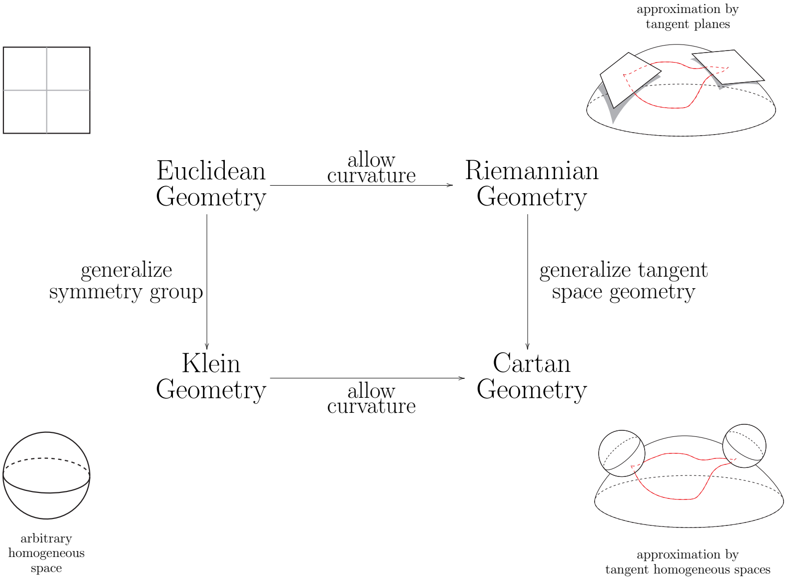
What it is:

observer space = space of unit future-timelike tangent vectors in spacetime

Why study it?

- Observers as logically prior to space or spacetime
- Link between covariant and canonical gravity
- Lorentz-violating theories
- Observer dependent geometry (Finsler, relative locality...)

How to study it? ...



(Adapted from diagram by R.W. Sharpe.)

Cartan geometry is 'geometry via symmetry breaking'.

Cartan geometry modeled on a homogeneous space G/H is described by a **Cartan connection**—a *pair* of fields:

A $\sim$ connection on a principal G bundle
(locally $\mathfrak{g}$-valued 1-form)

z $\sim$ symmetry-breaking field
(locally a function $z\colon M \to G/H$)

(satisfying a nondegeneracy property...)

Homogeneous model of spacetime is G/H with:

$$\boxed{G = \begin{cases} \mathrm{SO}(4,1) \\ \mathrm{ISO}(3,1) \\ \mathrm{SO}(3,2) \end{cases} \qquad H = \mathrm{SO}(3,1)} \quad \begin{matrix} \Lambda > 0 \\ \Lambda = 0 \\ \Lambda < 0 \end{matrix}$$

Break symmetry! As reps of $\mathrm{SO}(3,1)$:

$$\begin{array}{cccccc} & \mathfrak{g} & \cong & \mathfrak{so}(3,1)_z & \oplus & \mathbb{R}^{3,1}_z \\ \Longrightarrow & A & = & \omega & + & e \\ & & & \text{spin conn.} & & \text{coframe} \end{array}$$

$\mathbb{R}^{3,1}_z$ identified with tangent space of G/H

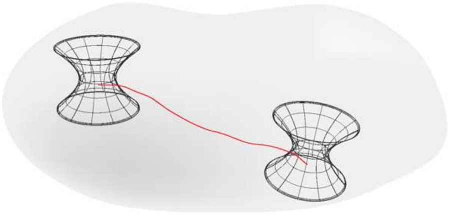
'Nondegeneracy condition' in CG means e nondegenerate.

Spacetime Cartan Geometry

MacDowell–Mansouri, Stelle–West (w. $\Lambda > 0$):

$$S[\underline{A, z}] = \int \varepsilon_{abcde} F^{ab} \wedge F^{cd} z^e$$

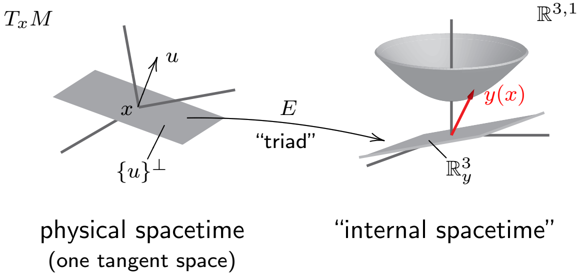
Cartan connection — curvature



"Rolling de Sitter space along physical spacetime" (gr-qc/0611154)

Can we think of Ashtekar variables as Cartan geometry?!

S. Gielen and D. Wise, 1111.7195



$\rightsquigarrow$ spacetime and internal splittings of fields.

Start with Holst action; split all fields internally and externally:

$$S = \int \hat{u} \wedge \Big[E^a \wedge E^b \wedge \pounds_u(\mathbf{A}_{ab}) + \cdots$$

"co-observer"
dual to u
($\sim dt$)

triad

spatial SO(3) connection
($\sim$ Ashtekar–Barbero)

proper time derivative
for observer u

$$S = \int \hat{u} \wedge \Big[E^a \wedge E^b \wedge \pounds_u(\mathbf{A}_{ab}) + \cdots$$

Fix $\hat{u}$, let $u = u(\hat{u}, y, E)$

Whenever $\ker \hat{u}$ is *integrable*, we get:

- Hamiltonian form clearly embedded in spacetime variables
- System of evolving spatial Cartan geometries.
  (Cartan connection built from $\mathbf{A}$ and E)
  "Cartan geometrodynamics"

But also:

- Manifestly Lorentz covariant
- Refoliation symmetry as special case of Lorentz symmetry

Observer Space

M a time-oriented Lorentzian 4-manifold.

O its **observer space**, i.e. unit future tangent bundle $O \to M$.

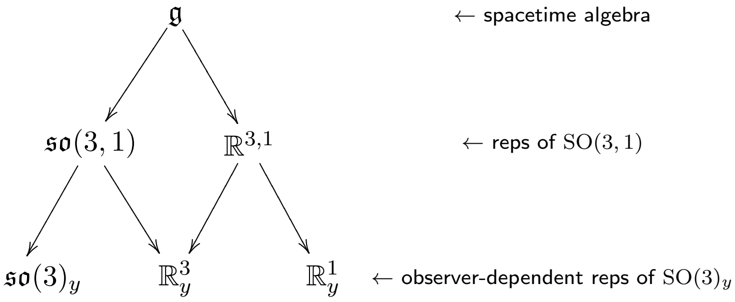
- Lorentzian 7-manifold
- Canonical "time" direction
- Contact structure
- Spatial and boost distributions

Three groups play important roles:

$$\boxed{G = \begin{cases} \mathrm{SO}(4,1) \\ \mathrm{ISO}(3,1) \\ \mathrm{SO}(3,2) \end{cases} \quad H = \mathrm{SO}(3,1) \quad K = \mathrm{SO}(3)} \quad \begin{matrix} \Lambda > 0 \\ \Lambda = 0 \\ \Lambda < 0 \end{matrix}$$

G/H = homogenous spacetime
H/K = velocity space (hyperbolic)
G/K = observer space

So, to do Cartan geometry on observer space, we do both levels of symmetry breaking we've already discussed:



As reps of SO(3):

$$\mathfrak{g} \cong \mathfrak{so}(3) \oplus (\mathbb{R}^3 \oplus \mathbb{R}^3 \oplus \mathbb{R})$$

Geometrically, these pieces are:

- tiny rotations around the observer
- tiny boosts taking us to another observer
- tiny spatial translations from the perspective of the observer
- tiny time translations from the perspective of the observer

So: a Cartan connection A splits into:

- an SO(3) connection
- a "heptad" or "siebenbein" with three canonical parts

Observer space Cartan geometry

Definition: An *observer space geometry* is a Cartan geometry modeled on G/K for one of the models just given.
That is...

- Principal G bundle with connection A
- A reduction of the G bundle to a principal K bundle P

(such that the nondegeneracy condition holds)

This definition doesn't rely on spacetime. Can we still talk about spacetime?

Given an observer space geometry
(with G-connection A, principal K bundle P)

Theorem:

1. If $F[A]$ vanishes on any "boost" vector, then the boost distribution is integrable
   ($\implies$ integrate out to get "spacetime")
2. If observer space is also "complete in boost directions", the boost distribution comes from a locally free H-action on P;
3. If the H action is free and proper, then P/H is a manifold, with spacetime Cartan geometry modeled on G/H;

Vacuum GR on observer space:

If an observer space Cartan geometry (A, P) satisfies:

1. $F(v, w) = 0$ for all boost vectors v and all vectors w
2. The field equations $[e, \star F] = 0$ (with e the spacetime part of the siebenbein)

Then we get both spacetime as a quotient of observer space, and Einstein's equations on the reconstructed spacetime.

Cartan geometrodynamics:

Cartan geometrodynamics is essentially a trivialization of observer space Cartan geometry: geometrically, the 'internal observer' y is a section of the observer bundle

$$\text{observer space} \;\rightarrow\; \text{spacetime.}$$

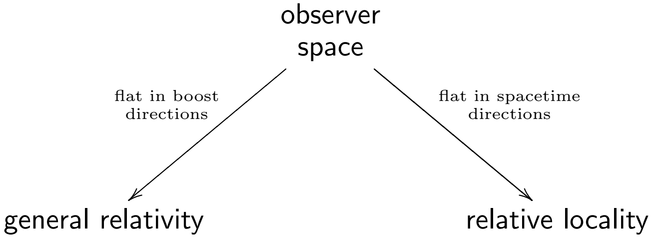
Pull fields down to get the 'geometrodynamic' description.

If the boost distribution is *not* integrable:

- each observer has local space, time, and boost directions in observer space
- boost directions give local notion of "coincidence"
- space/time directions give local notion of "spacetime"

Relation to 'relative locality' proposal? Very similar conclusions, but different starting point.

Morally speaking:



Outlook

Some things to work on:

- foundational issues: actions on observer space ...
- controlled way to relax "boost-flatness"; physical consequences?
- matter
- lightlike particles and boundary of observer space
- applications:
    - Relative locality
    - Lorentz-violating theories
    - Finsler
- quantum applications
    - spin foam / LQG?