Lifting General Relativity to Observer Space

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International Loop Quantum Gravity Seminar

2 October 2012
Three pictures of gravity

observer
space

spacetime picture

geometrodynamic picture
What it is:

\[
\text{observer space} \quad = \quad \text{space of unit future-timelike tangent vectors in spacetime}
\]

Why study it?

- Observers as logically prior to space or spacetime
- Link between covariant and canonical gravity
- Lorentz-violating theories
- Observer dependent geometry (Finsler, relative locality...)

*How to study it*? ...
Cartan geometry

 Euclidean Geometry \[\text{allow curvature}\] \rightarrow \text{Riemannian Geometry}

 generalize symmetry group

 Klein Geometry \[\text{allow curvature}\] \rightarrow \text{Cartan Geometry}

 arbitrary homogeneous space

 approximation by tangent planes

 approximation by tangent homogeneous spaces

(Adapted from diagram by R.W. Sharpe.)
Cartan geometry is ‘geometry via symmetry breaking’.

Cartan geometry modeled on a homogeneous space $G/H$ is described by a **Cartan connection**—a pair of fields:

- $A \sim$ connection on a principal $G$ bundle
  - (locally $g$-valued 1-form)
- $z \sim$ symmetry-breaking field
  - (locally a function $z: M \to G/H$)

(satisfying a nondegeneracy property...)

```plaintext
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Homogeneous model of spacetime is $G/H$ with:

$$G = \begin{cases} 
\text{SO}(4, 1) & \Lambda > 0 \\
\text{ISO}(3, 1) & \Lambda = 0 \\
\text{SO}(3, 2) & \Lambda < 0 
\end{cases}$$

Break symmetry! As reps of $\text{SO}(3, 1)$:

$$g \cong \mathfrak{so}(3, 1)_z \oplus \mathbb{R}^{3,1}_z$$

$$\implies A = \omega + e$$

$\mathbb{R}^{3,1}_z$ identified with tangent space of $G/H$

‘Nondegeneracy condition’ in CG means $e$ nondegenerate.
Spacetime Cartan Geometry

MacDowell–Mansouri, Stelle–West (w. $\Lambda > 0$):

$$S[A, z] = \int \varepsilon_{abcde} F^{ab} \wedge F^{cd} z^e$$

"Rolling de Sitter space along physical spacetime" (gr-qc/06111154)
Can we think of Ashtekar variables as Cartan geometry?!

S. Gielen and D. Wise, 1111.7195

physical spacetime (one tangent space)  “internal spacetime”

→ spacetime and internal splittings of fields.
Start with Holst action; split all fields internally and externally:

$$S = \int \hat{u} \wedge \left[ E^a \wedge E^b \wedge \mathcal{L}_u(A_{ab}) + \cdots \right]$$

- “co-observer” dual to $u$ ($\sim dt$)
- triad
- spatial SO(3) connection ($\sim$ Ashtekar–Barbero)
- proper time derivative for observer $u$
Cartan geometrodynamics

\[ S = \int \hat{u} \wedge \left[ E^a \wedge E^b \wedge \mathcal{L}_u(A_{ab}) + \cdots \right] \]

Fix \( \hat{u} \), let \( u = u(\hat{u}, y, E) \)

Whenever \( \ker \hat{u} \) is \textit{integrable}, we get:

- Hamiltonian form clearly embedded in spacetime variables
- System of evolving spatial Cartan geometries.
  (Cartan connection built from \( A \) and \( E \))
  “Cartan geometrodynamics”

But also:

- Manifestly Lorentz covariant
- Refoliation symmetry as special case of Lorentz symmetry
Observer Space
Observer space of a spacetime

$M$ a time-oriented Lorentzian 4-manifold.

$O$ its **observer space**, i.e. unit future tangent bundle $O \rightarrow M$.

- Lorentzian 7-manifold
- Canonical “time” direction
- Contact structure
- Spatial and boost distributions
Three groups play important roles:

\[
G = \begin{cases} 
\text{SO}(4, 1) \\
\text{ISO}(3, 1) \\
\text{SO}(3, 2) 
\end{cases} \quad H = \text{SO}(3, 1) \quad K = \text{SO}(3) \\
\Lambda > 0 \quad \Lambda = 0 \quad \Lambda < 0
\]

\[G/H = \text{homogenous spacetime}\]
\[H/K = \text{velocity space (hyperbolic)}\]
\[G/K = \text{observer space}\]
So, to do Cartan geometry on observer space, we do both levels of symmetry breaking we’ve already discussed:

\[ \mathfrak{g} \]

\[ \mathfrak{so}(3, 1) \quad \mathbb{R}^{3,1} \]

\[ \mathfrak{so}(3)_y \quad \mathbb{R}^3_y \quad \mathbb{R}^1_y \]

\[ \leftarrow \text{spacetime algebra} \]

\[ \leftarrow \text{reps of SO}(3, 1) \]

\[ \leftarrow \text{observer-dependent reps of SO}(3)_y \]
As reps of $\text{SO}(3)$:

$$\mathfrak{g} \cong \mathfrak{so}(3) \oplus (\mathbb{R}^3 \oplus \mathbb{R}^3 \oplus \mathbb{R})$$

Geometrically, these pieces are:

- tiny rotations around the observer
- tiny boosts taking us to another observer
- tiny spatial translations from the perspective of the observer
- tiny time translations from the perspective of the observer

So: a Cartan connection $A$ splits into:

- an $\text{SO}(3)$ connection
- a “heptad” or “siebenbein” with three canonical parts
**Definition:** An *observer space geometry* is a Cartan geometry modeled on $G/K$ for one of the models just given. That is...

- Principal $G$ bundle with connection $A$
- A reduction of the $G$ bundle to a principal $K$ bundle $P$
(such that the nondegeneracy condition holds)

This definition doesn’t rely on spacetime. Can we still talk about spacetime?
Given an observer space geometry
(with $G$-connection $A$, principal $K$ bundle $P$)

**Theorem:**

1. If $F[A]$ vanishes on any “boost” vector, then the boost distribution is integrable
   ($\implies$ integrate out to get “spacetime”)
2. If observer space is also “complete in boost directions”, the boost distribution comes from a locally free $H$-action on $P$;
3. If the $H$ action is free and proper, then $P/H$ is a manifold, with spacetime Cartan geometry modeled on $G/H$;
**Vacuum GR on observer space:**

If an observer space Cartan geometry \((A, P)\) satisfies:

1. \(F(v, w) = 0\) for all boost vectors \(v\) and all vectors \(w\)
2. The field equations \([e, \star F] = 0\) (with \(e\) the spacetime part of the siebenbein)

Then we get both spacetime as a quotient of observer space, and Einstein’s equations on the reconstructed spacetime.

**Cartan geometrodynamics:**

Cartan geometrodynamics is essentially a trivialization of observer space Cartan geometry: geometrically, the ‘internal observer’ \(y\) is a section of the observer bundle

\[
\text{observer space} \rightarrow \text{spacetime.}
\]

Pull fields down to get the ‘geometrodynamic’ description.
Relative spacetime

If the boost distribution is *not* integrable:

- each observer has local space, time, and boost directions in observer space
- boost directions give local notion of “coincidence”
- space/time directions give local notion of “spacetime”

Relation to ‘relative locality’ proposal? Very similar conclusions, but different starting point.
Morally speaking:

Relative spacetime

observer space

- flat in boost directions
- flat in spacetime directions

general relativity

relative locality
Outlook

Some things to work on:

- foundational issues: actions on observer space . . .
- controlled way to relax “boost-flatness”; physical consequences?
- matter
- lightlike particles and boundary of observer space
- applications:
  - Relative locality
  - Lorentz-violating theories
  - Finsler
- quantum applications
  - spin foam / LQG?