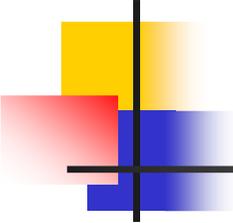


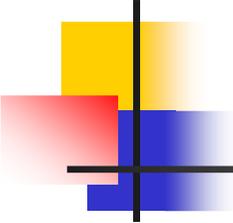
Questions & Answers about Perturbative Quantum Gravity

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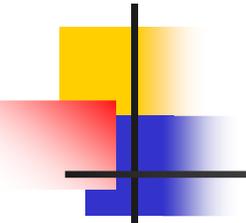
Seven Questions

1. Why is QGR so bad & CGR so good?
2. Why must we quantize gravity?
3. Why do QFT's have ∞ 's?
4. Why are those of QGR worse?
5. How bad is the problem?
6. What are the main approaches to it?
7. What would we do with QGR?



Sketch of General Relativity

1. Gravitational Field: $g_{\mu\nu}(t, \mathbf{x})$
 - $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
2. $g_{\mu\nu}$ affects fields thru Minimal Coupling
 - $-(\partial/\partial ct)^2 + \nabla^2 \rightarrow (-g)^{1/2} \partial_\mu [(-g)^{1/2} g^{\mu\nu} \partial_\nu]$
3. Other fields affect $g_{\mu\nu}$ thru Einstein Eqn
 - $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
 - Key Principle: **Energy gravitates**
4. Eqn also predicts gravitational radiation
 - Purely grav. DOF's not fixed by matter



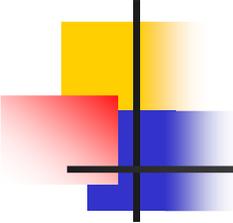
A Solution versus the GENERAL Solution

QM eqns same as in CM, eg $\ddot{x} + \omega^2 x = 0$

- $x(t) = x_0 \cos(\omega t) + \dot{x}_0/\omega \sin(\omega t)$ BUT
- $x_0=0$ & $\dot{x}_0=0$ OK in CM
- $x_0=0$ & $\dot{x}_0=0$ not OK in QM

Classic CGR tests have most IVD = 0

- $ds^2 = -[1-2GM/r]dt^2 + dr^2/[1-2GM/r] + r^2 d\Omega$
- Need $g_{\mu\nu}(t,x)$ for general IVD
- Lots of DOF's with 0-pt motion



Some Quantization Unavoidable

Dual role of force fields:

- Mediate interactions
- Harbor new quanta

■ Cf. EM

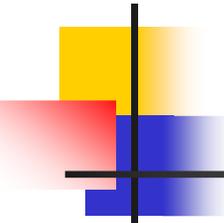
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B}$$

$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \frac{1}{\epsilon_0} \frac{\partial \vec{E}}{\partial t} = \vec{J} \qquad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{B}(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \int_0^t dt' \frac{\sin[ck(t-t')]}{ck\epsilon_0} i\vec{k} \times \tilde{\vec{J}}(t', \vec{k})$$

$$+ \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \left\{ \tilde{\vec{B}}_0(\vec{k}) \cos(ckt) - \frac{i}{ck} \vec{k} \times \tilde{\vec{E}}_0(\vec{k}) \sin(ckt) \right\}$$

- 1st term quantized because of matter, whether or not there are photons



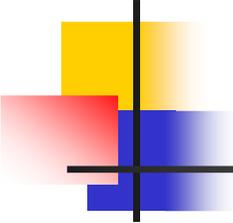
Matter quantized whether or not photons & gravitons are!

- Cf Hydrogen: $H = p^2/2m - e \Phi(q)$
 $\Phi(q) = e/(4\pi\epsilon_0 q)$ an operator from q
- Same for General Relativity
 $g_{\mu\nu} = (\text{functional of } T_{\mu\nu}) + (\text{gravitons})$
Fields in $T_{\mu\nu}$ are certainly quantum!
- Allowing quantum matter to interact gravitationally causes problems with or without gravitons

Asymptotic Series

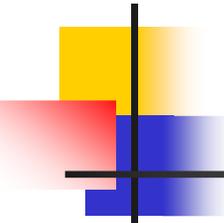
Are Your Friends

- Impossible to find general solution
→ MUST approximate
- Typical Asymptotic series
$$E_1(x) = \int_x^\infty dt/t e^{-t} \rightarrow e^{-x}/x \sum_{n=0}^{\infty} (-1/x)^n n!$$
 - Great for small $1/x$ at fixed n
 - But diverges for large n at fixed x
 - Hence use out to $n \sim x$ and no further
 - Not exact but often good enough



Should Be Great for QGR

- QED: $\alpha = e^2/[4\pi\epsilon_0\hbar c] \sim 1/137$
 - Results = (0th order)[1 + a₁α + a₂α² + ...]
 - Begins diverging at L ~ 430
 - Best experiments sensitive to L ~ 4
- QGR: $\kappa = GE^2/\hbar c^5 \sim (E/10^{19}\text{Gev})^2$
 - Results = (0th order)[1 + b₁κ + b₂κ² + ...]
 - Same factorials & E = 1 TeV → κ ~ 10⁻³²
 - But the coefficients b_n diverge!



Physics behind the ∞ 's: Recall the QM Harmonic Osc.

- $H = p^2/2m + 1/2m\omega^2 q^2$
→ $q(t) = q_0 \cos(\omega t) + p_0/m\omega \sin(\omega t)$
- $\cos(\omega t) = 1/2 (e^{i\omega t} + e^{-i\omega t})$
→ $q(t) = 1/2(q_0 + ip_0/m\omega)e^{-i\omega t} + 1/2(q_0 - ip_0/m\omega) e^{i\omega t}$
 $= [a e^{-i\omega t} + a^\dagger e^{i\omega t}]/(2\omega)^{1/2}$
- **Mode coordinate:** $a = (\omega/2)^{1/2} (q_0 + ip_0/m\omega)$
- **Mode function:** $\varepsilon(t) = e^{-i\omega t}/(2\omega)^{1/2}$
→ $q(t) = a \varepsilon(t) + a^\dagger \varepsilon^*(t)$

Field Theories have ∞ Modes

E.g., EM for $0 \leq x_i \leq L$

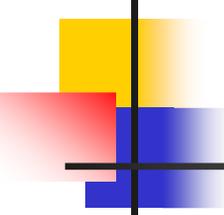
$$E_1(t, x) = \sum_k [a_k \varepsilon_k(t, x) + a_k^\dagger \varepsilon_k^*(t, x)]$$

where $k = \pi/L (n_1, n_2, n_3)$ & $\omega = \pi n c/L$

$$\varepsilon_k = [\pi^3/2\omega L^3]^{1/2} \sin(k_1 x_1) \cos(k_2 x_2) \cos(k_3 x_3) e^{-i\omega t}$$

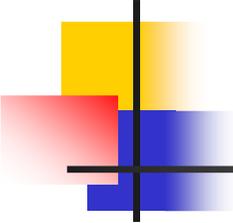
$$a_k = \int^L dx_1 \int^L dx_2 \int^L dx_3 \varepsilon_k^*(0, x) \{ \omega E_1(0, x) \\ + ic [\partial_2 B_3(0, x) - \partial_3 B_2(0, x)] \}$$

Two modes for every (n_1, n_2, n_3)



Conspiracy of Four Principles

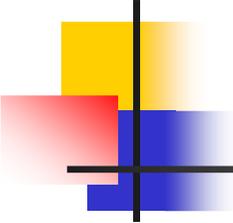
1. Continuum Field Theory $\rightarrow \infty$ Modes
 2. Q. Mechanics \rightarrow Can't have $q_0=p_0=0$
 - Each mode has $\frac{1}{2}\hbar\omega$ + interactions
 - Changes shift energies (Casimir & Lamb)
 3. General Relativity \rightarrow Energy gravitates
 4. Perturbation theory \rightarrow shifts add
- “Too many modes interacting too strongly”



Renormalization 1: Classical EM in a medium

- Dumb way: $\epsilon_0 \text{div}(\mathbf{E}) = \rho_{free} + \rho_{bnd}$
- $\rho_{bnd} = \sum_{atm} q [\delta^3(\mathbf{x}-\mathbf{x}_{atm}) - \delta^3(\mathbf{x}-\mathbf{x}_{atm}-\Delta\mathbf{x})]$
 $\rightarrow -\Delta\epsilon \text{div}(\mathbf{E}) + O(\Delta\mathbf{x})$
- Smart way:
 $[\epsilon_0 + \Delta\epsilon] \text{div}(\mathbf{E}) = \rho_{free} + O(\Delta\mathbf{x})$

Renormalization 2:


$$\epsilon_0 \operatorname{div}(\mathbf{E}) = \rho_{\text{exc}} + \rho_{0\text{pt}} \text{ in QED}$$

$$e^+ \text{ w. } \mathcal{E}(k) = [(\hbar ck)^2 + m^2 c^4]^{1/2} \text{ \& } e^- \text{ w. } \mathcal{E}(p-k)$$
$$\text{live } \Delta t \sim \hbar / [\mathcal{E}(k) + \mathcal{E}(p-k)]$$

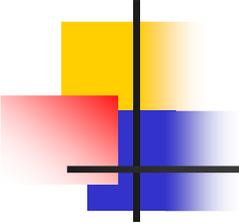
They polarize $e\Delta x$, where

$$d/dt [\mathcal{E}/c^2 \Delta x] = eE \rightarrow \Delta x \sim c^2 \Delta t^2 eE/2\mathcal{E}$$

Sum over k to get total polarization

$$\vec{P}(\vec{k}) = \frac{8}{3} \int \frac{d^3 k}{(2\pi)^3} \frac{e^2 \hbar^2 c^2 \vec{E}(\vec{k})}{[\mathcal{E}(\vec{k}) + \mathcal{E}(\vec{p} - \vec{k})]^3}$$

Renormalization: $\Delta\epsilon(p) = \Delta\epsilon(0) + \text{finite}$



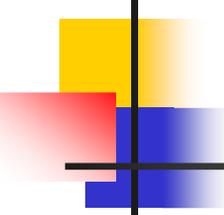
$$\text{Renorm 3: } c^4/8\pi G [G_{\mu\nu} + \Lambda g_{\mu\nu}] \\ = (T_{\mu\nu})_{\text{exc}} + (T_{\mu\nu})_{0\text{pt}}$$

$$\text{Cf. } -\epsilon_0 \nabla^2 \Phi = \rho_{\text{exc}} + \rho_{0\text{pt}}$$

$$\begin{aligned} (\rho(\vec{x}))_{0\text{pt}} &= \int \frac{d^3 p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \int \frac{d^3 k}{(2\pi)^3} \frac{\frac{8}{3} e^2 \hbar^2 c^2 p^2 \Phi(\vec{p})}{[\mathcal{E}(\vec{k}) + \mathcal{E}(\vec{p} - \vec{k})]^3} \\ &= A\alpha\epsilon_0 \ln(K^2) \Phi(\vec{x}) + \text{Finite} \end{aligned}$$

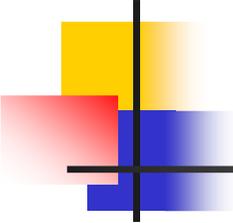
$$\begin{aligned} (T_{\mu\nu}(\vec{x}))_{0\text{pt}} &= \int \frac{d^3 p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \int \frac{d^3 k}{(2\pi)^3} \frac{\mathcal{E}^4 h(\vec{p})}{[\mathcal{E}(\vec{k}) + \mathcal{E}(\vec{p} - \vec{k})]^3} \\ &= \hbar c \left\{ AK^4 h(\vec{x}) + BK^2 \nabla^2 h(\vec{x}) + C \ln(K^2) \nabla^4 h(\vec{x}) + \text{Finite} \right\} \end{aligned}$$

$A K^4 \rightarrow \Lambda$, $B K^2 \rightarrow G$ but $C \ln(K^2)$ new



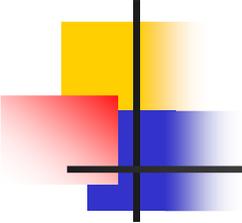
What's wrong with higher ∂ 's?

- Newton: $m\ddot{x} = F(x, \dot{x})$
 - $L = L(x, \dot{x}) \rightarrow Q = x$ and $P = \partial L / \partial \dot{x}$
 - $H(Q, P) = P\dot{x}(Q, P) - L(Q, \dot{x}(Q, P))$
- Ostrogradsky: $d^4x/dt^4 = f(x, \dot{x}, \ddot{x}, d^3x/dt^3)$
 - $L = L(x, \dot{x}, \ddot{x}) \rightarrow Q_1 = x, Q_2 = \dot{x}, P_2 = \partial L / \partial \ddot{x},$
 $P_1 = \partial L / \partial \dot{x} - d/dt \partial L / \partial \ddot{x}$
 - $H(Q_1, Q_2, P_1, P_2) = P_1 Q_2 + P_2 \ddot{x}(Q_1, Q_2, P_2) -$
 $L(Q_1, Q_2, \ddot{x}(Q_1, Q_2, P_2))$
- Why physics is based on 2nd order eqns!



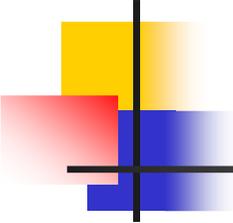
Perturbative QGR differs

- Other forces couple the same \forall modes
- GR couples more strongly to large k modes
- This requires 4th order counterterms which would make the universe blow up instantly



Divergent Opinions

1. Relativists (love General Relativity)
 - “Perturbation theory is wrong!”
 - Nonlinear grav. ints cancel the ∞ 's
2. Particle Theorists (love Pert. Theory)
 - “General Relativity is wrong!”
 - Superstrings have ± 0 -point energies and interact more weakly at large k



Repartee with Relativists

Fact: GR is weak & QGR unobs. small

Question: How can PT be wrong?

1. Correct series nonanalytic in G
 - Eg (0^{th}) $\{1 + GE^2/\hbar c^5 \ln(GE^2/\hbar c^5) + \dots\}$
2. 0^{th} order may diverge for $G \rightarrow 0$

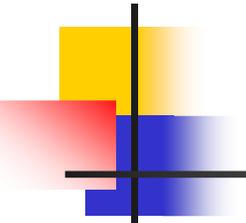
Charged shell of radius $R \rightarrow 0$ (ADM 1960)

- Without GR: $mc^2 = m_0c^2 + q^2/8\pi\epsilon_0 R$
“renormalize” with $m_0c^2 = m_{\text{obs}}c^2 - q^2/8\pi\epsilon_0 R$
- With GR: $mc^2 = m_0c^2 + q^2/8\pi\epsilon_0 R - Gm^2/2R$

$$m = \frac{Rc^2}{G} \left[-1 + \sqrt{1 + \frac{2Gm_0}{Rc^2} + \frac{Gq^2}{4\pi\epsilon_0 R^2 c^2}} \right] \rightarrow \sqrt{\frac{q^2}{4\pi\epsilon_0 G}}$$

- Perturbative Result:
→ Oscillating series of ever-higher ω 's

All Proposed Fixes Involve $E < 0$

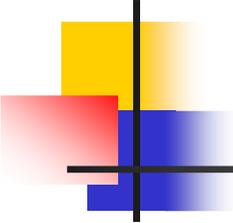


1. Relativist's dream: gravity regulates
 - Negative grav. int. energy cancels ∞ 's nonperturbatively
2. Particle Theorist's dream: superstrings
 - SUSY adds $E > 0$ fermions which contribute $-1/2\hbar\omega$
3. Pert. Gravity's wish: induce higher ∂ 's
 - $E < 0$ particles

The Other Problem

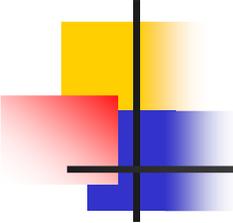
→ Experiments!

- Theoretical theory (Wizards dueling)
- Study weak ints using unique features
 - $\mu^- \rightarrow e^- \nu_e^\dagger \nu_\mu$ doesn't occur in QED
- Unique features of gravity
 - Negative interaction energy
 - $M=0$ gravitons without conformal invariance
Photons can't distinguish $g_{\mu\nu}(x)$ from $\Omega^2(x) g_{\mu\nu}(x)$
- Cosmology is a natural venue
 - $ds^2 = -dt^2 + a^2(t) dx^2 = a^2 [-d\eta^2 + dx^2]$
 - Gravity is long range and knows about $a(t)$



Seven Questions Answered

1. CGR works when almost all DOF's off, QGR hurt by 0-point motion of DOF's
2. Must quantize because part of $g_{\mu\nu}$ from quantum matter
 - This has been seen in CMB anisotropies!
3. QFT ∞ 's from infinite modes
4. QGR is worse because it couples to energy instead of charge



Seven Questions Answered

5. GR + most matter fails a one loop, pure GR fails at two loops

6. Divergent Opinions

- Relativists: "GR is right & PT is wrong!"
- Particle Theorists: "PT is right & GR is wrong!"

7. Phenomenology of QGR exploits unique

- Negative interaction energy
- $M = 0$ gravitons w/o conformal invariance