Questions & Answers about Perturbative Quantum Gravity

Richard Woodard
University of Florida

Seven Questions

1. Why is QGR so bad & CGR so good?
2. Why must we quantize gravity?
3. Why do QFT's have ∞'s?
4. Why are those of QGR worse?
5. How bad is the problem?
6. What are the main approaches to it?
7. What would we do with QGR?

Sketch of General Relativity

1. Gravitational Field: $g_{\mu\nu}(t,x)$
   - $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
2. $g_{\mu\nu}$ affects fields thru Minimal Coupling
   - $-(\partial/\partial ct)^2 + \nabla^2 \to (-g)^{½}\, \partial_\mu[(-g)^{½} g^{\mu\nu}\partial_\nu]$
3. Other fields affect $g_{\mu\nu}$ thru Einstein Eqn
   - $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$
   - Key Principle: Energy gravitates
4. Eqn also predicts gravitational radiation
   - Purely grav. DOF's not fixed by matter

A Solution versus the GENERAL Solution

QM eqns same as in CM, eg $\ddot{x} + \omega^2 x = 0$

- $x(t) = x_0 \cos(\omega t) + \dot{x}_0/\omega \sin(\omega t)$ BUT
- $x_0=0$ & $\dot{x}_0=0$ OK in CM
- $x_0=0$ & $\dot{x}_0=0$ not OK in QM

Classic CGR tests have most IVD = 0

- $ds^2= -[1-2GM/r]dt^2 + dr^2/[1-2GM/r] + r^2 d\Omega$
- Need $g_{\mu\nu}(t,x)$ for general IVD
- Lots of DOF's with 0-pt motion

Some Quantization Unavoidable

Dual role of force fields:

- Mediate interactions
- Harbor new quanta

- Cf. EM

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla}\cdot\vec{B}$$

$$\frac{1}{\mu_0}\vec{\nabla}\times\vec{B} - \frac{1}{\epsilon_0}\frac{\partial\vec{E}}{\partial t} = \vec{J} \qquad \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$$

$$\vec{B}(t,\vec{x}) = \int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\vec{x}}\int_0^t dt'\,\frac{\sin[ck(t-t')]}{ck\epsilon_0}\,i\vec{k}\times\tilde{\vec{J}}(t',\vec{k})$$
$$+\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\vec{x}}\left\{\tilde{\vec{B}}_0(\vec{k})\cos(ckt) - \frac{i}{ck}\vec{k}\times\tilde{\vec{E}}_0(\vec{k})\sin(ckt)\right\}$$

- 1st term quantized because of matter, whether or not there are photons

Matter quantized whether or not photons & gravitons are!

- Cf Hydrogen: $H = p^2/2m - e\,\Phi(q)$

  $\Phi(q) = e/(4\pi\epsilon_0 q)$ an operator from q
- Same for General Relativity

  $g_{\mu\nu}$ = (functional of $T_{\mu\nu}$) + (gravitons)

  Fields in $T_{\mu\nu}$ are certainly quantum!
- Allowing quantum matter to interact gravitationally causes problems with or without gravitons

Asymptotic Series Are Your Friends

- Impossible to find general solution

  ➔MUST approximate
- Typical Asymptotic series

  $E_1(x) = \int_x dt/t\ e^{-t} \rightarrow e^{-x}/x\ \sum_{n=0}(-1/x)^n\ n!$
  - Great for small 1/x at fixed n
  - But diverges for large n at fixed x
  - Hence use out to n ~ x and no further
  - Not exact but often good enough

Should Be Great for QGR

- QED: $\alpha = e^2/[4\pi\varepsilon_0\hbar c] \sim 1/137$

  Results = $(0^{th}$ order$)[1 + a_1\alpha + a_2\alpha^2 + \ldots]$
  - Begins diverging at L ~ 430
  - Best experiments sensitive to L ~ 4
- QGR: $\kappa = GE^2/\hbar c^5 \sim (E/10^{19}\text{Gev})^2$

  Results = $(0^{th}$ order$)[1 + b_1\kappa + b_2\kappa^2 + \ldots]$
  - Same factorials & E = 1 TeV ➔ $\kappa \sim 10^{-32}$
  - But the coefficients b_n diverge!

Physics behind the ∞'s: Recall the QM Harmonic Osc.

- $H = p^2/2m + \frac{1}{2} m\omega^2 q^2$

  → $q(t) = q_0 \cos(\omega t) + p_0/m\omega \sin(\omega t)$
- $\cos(\omega t) = \frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$

  → $q(t) = \frac{1}{2}(q_0 + ip_0/m\omega)e^{-i\omega t} + \frac{1}{2}(q_0 - ip_0/m\omega)\, e^{i\omega t}$

  $= [a\, e^{-i\omega t} + a^\dagger\, e^{i\omega t}]/(2\omega\}^{1/2}$
- Mode coordinate: $a = (\omega/2)^{1/2}\,(q_0 + ip_0/m\omega)$
- Mode function: $\varepsilon(t) = e^{-i\omega t}/(2\omega)^{1/2}$

  → $q(t) = a\,\varepsilon(t) + a^\dagger\,\varepsilon^*(t)$

Field Theories have ∞ Modes E.g., EM for $0 \leq x_i \leq L$

$E_1(t,x) = \Sigma_k [a_k \varepsilon_k(t,x) + a_k^\dagger \varepsilon_k^*(t,x)]$

where $k = \pi/L\,(n_1,n_2,n_3)$ & $\omega = \pi nc/L$

$\varepsilon_k=[\pi^3/2\omega L^3]^{1/2} \sin(k_1x_1)\cos(k_2x_2)\cos(k_3x_3)e^{-i\omega t}$

$a_k = \int^L dx_1 \int^L dx_2 \int^L dx_3\, \varepsilon_k^*(0,x)\, \{\omega\, E_1(0,x)$

$+ ic\, [\partial_2 B_3(0,x) - \partial_3 B_2(0,x)]\}$

Two modes for every (n1,n2,n3)

Conspiracy of Four Principles

1. Continuum Field Theory ➔ ∞ Modes
2. Q. Mechanics ➔ Can't have $q_0=p_0=0$
   - Each mode has $\frac{1}{2}\hbar\omega$ + interactions
   - Changes shift energies (Casimir & Lamb)
3. General Relativity ➔ Energy gravitates
4. Perturbation theory ➔ shifts <u>add</u>

"Too many modes interacting too strongly"

Renormalization 1: Classical EM in a medium

- Dumb way: $\epsilon_o \text{div}(E) = \rho_{free} + \rho_{bnd}$
- $\rho_{bnd} = \Sigma_{atm} q\, [\delta^3(x-x_{atm}) - \delta^3(x-x_{atm}-\Delta x)]$
  $\rightarrow -\Delta\epsilon\, \text{div}(E) + O(\Delta x)$
- Smart way:
  $[\epsilon_o + \Delta\epsilon]\text{div}(E) = \rho_{free} + O(\Delta x)$

Renormalization 2: $\varepsilon_0 \,\mathrm{div}(E) = \rho_{exc} + \rho_{0pt}$ in QED

e^+ w. $\epsilon(k)=[(\hbar ck)^2+m^2c^4]^{1/2}$ & e^- w. $\epsilon(p-k)$

live $\Delta t \sim \hbar/[\epsilon(k) + \epsilon(p-k)]$

They polarize $e\Delta x$, where

$d/dt\,[\epsilon/c^2\,\Delta x] = eE \rightarrow \Delta x \sim c^2 \Delta t^2\, eE/2\epsilon$

Sum over k to get total polarization

$$\vec{P}(\vec{k}) = \frac{8}{3}\int \frac{d^3k}{(2\pi)^3}\frac{e^2\hbar^2c^2\vec{E}(\vec{k})}{[\mathcal{E}(\vec{k})+\mathcal{E}(\vec{p}-\vec{k})]^3}$$

Renormalization: $\Delta\varepsilon(p) = \Delta\varepsilon(0)$ + finite

Renorm 3: $c^4/8\pi G\,[G_{\mu\nu}+\Lambda\, g_{\mu\nu}] = (T_{\mu\nu})_{exc} + (T_{\mu\nu})_{0pt}$

Cf. $-\varepsilon_0\nabla^2\Phi = \rho_{exc} + \rho_{0pt}$

$$\begin{aligned}\big(\rho(\vec{x})\big)_{0\mathrm{pt}} &= \int\frac{d^3p}{(2\pi)^3}\,e^{i\vec{p}\cdot\vec{x}}\int\frac{d^3k}{(2\pi)^3}\,\frac{\frac{8}{3}e^2\hbar^2c^2p^2\Phi(\vec{p})}{[\mathcal{E}(\vec{k})+\mathcal{E}(\vec{p}-\vec{k})]^3}\\ &= A\alpha\epsilon_0\ln(K^2)\Phi(\vec{x}) + \mathrm{Finite}\end{aligned}$$

$$\big(T_{\mu\nu}(\vec{x})\big)_{0\mathrm{pt}} = \int\frac{d^3p}{(2\pi)^3}\,e^{i\vec{p}\cdot\vec{x}}\int\frac{d^3k}{(2\pi)^3}\,\frac{\mathcal{E}^4h(\vec{p})}{[\mathcal{E}(\vec{k})+\mathcal{E}(\vec{p}-\vec{k})]^3}$$

$$= \hbar c\Big\{AK^4h(\vec{x}) + BK^2\nabla^2h(\vec{x}) + C\ln(K^2)\nabla^4h(\vec{x}) + \mathrm{Finite}\Big\}$$

$A\,K^4 \rightarrow \Lambda$, $B\,K^2 \rightarrow G$ but $C\ln(K^2)$ new

What's wrong with higher ∂'s?

- Newton: $m\ddot{x} = F(x,\dot{x})$
  - $L = L(x,\dot{x}) \Rightarrow Q = x$ and $P = \partial L/\partial \dot{x}$
  - $H(Q,P) = P\dot{x}(Q,P) - L(Q,\dot{x}(Q,P))$
- Ostrogradsky: $d^4x/dt^4 = f(x,\dot{x},\ddot{x},d^3x/dt^3)$
  - $L = L(x,\dot{x},\ddot{x}) \Rightarrow Q_1 = x,\ Q_2 = \dot{x},\ P_2 = \partial L/\partial \ddot{x},$
    $P_1 = \partial L/\partial \dot{x} - d/dt\ \partial L/\partial \ddot{x}$
  - $H(Q_1,Q_2,P_1,P_2) = P_1Q_2 + P_2\ddot{x}(Q_1,Q_2,P_2) -$
    $L(Q_1,Q_2,\ddot{x}(Q_1,Q_2,P_2))$
- Why physics is based on 2nd order eqns!

Perturbative QGR differs

- Other forces couple the same $\forall$ modes
- GR couples more strongly to large k modes
- This requires 4^{th} order counterterms which would make the universe blow up instantly

Divergent Opinions

1. Relativists (love General Relativity)
   - "Perturbation theory is wrong!"
   - Nonlinear grav. ints cancel the ∞'s
2. Particle Theorists (love Pert. Theory)
   - "General Relativity is wrong!"
   - Superstrings have ± 0-point energies and interact more weakly at large k

Repartee with Relativists

Fact: GR is weak & QGR unobs. small

Question: How can PT be wrong?

1. Correct series nonanalytic in G
   - Eg (0^{th}) $\{1 + GE^2/\hbar c^5 \ln(GE^2/\hbar c^5) + \ldots\}$
2. 0^{th} order may diverge for $G \rightarrow 0$

Charged shell of radius R → 0 (ADM 1960)

- Without GR: $mc^2 = m_0c^2 + q^2/8\pi\epsilon_0 R$

  "renormalize" with $m_0c^2 = m_{obs}c^2 - q^2/8\pi\epsilon_0 R$
- With GR: $mc^2=m_0c^2+q^2/8\pi\epsilon_0 R-Gm^2/2R$

$$m = \frac{Rc^2}{G}\left[-1 + \sqrt{1 + \frac{2Gm_0}{Rc^2} + \frac{Gq^2}{4\pi\epsilon_0 R^2c^2}}\right] \longrightarrow \sqrt{\frac{q^2}{4\pi\epsilon_0 G}}$$

- Perturbative Result:

  ➔ Oscillating series of ever-higher ∞'s

All Proposed Fixes Involve E < 0

1. Relativist's dream: gravity regulates
   ➔ Negative grav. int. energy cancels ∞'s nonperturbatively
2. Particle Theorist's dream: superstrings
   ➔ SUSY adds E > 0 fermions which contribute $-\frac{1}{2}\hbar\omega$
3. Pert. Gravity's wish: induce higher ∂'s
   ➔ E < 0 particles

The Other Problem
→ Experiments!

- Theoretical theory (Wizards dueling)
- Study weak ints using unique features
  - $\mu^- \to e^- \nu_e^\dagger \nu_\mu$ doesn't occur in QED
- Unique features of gravity
  - Negative interaction energy
  - M=0 gravitons without conformal invariance
    Photons can't distinguish $g_{\mu\nu}(x)$ from $\Omega^2(x)\, g_{\mu\nu}(x)$
- Cosmology is a natural venue
  - $ds^2 = -dt^2 + a^2(t)\, dx^2 = a^2\, [-d\eta^2 + dx^2]$
  - Gravity is long range and knows about a(t)

Seven Questions Answered

1. CGR works when almost all DOF's off, QGR hurt by 0-point motion of DOF's
2. Must quantize because part of $g_{\mu\nu}$ from quantum matter
   - This has been seen in CMB anisotropies!
3. QFT ∞'s from infinite modes
4. QGR is worse because it couples to energy instead of charge

Seven Questions Answered

5. GR + most matter fails a one loop, pure GR fails at two loops
6. Divergent Opinions
   - Relativists: "GR is right & PT is wrong!"
   - Particle Theorists: "PT is right & GR is wrong!"
7. Phenomenology of QGR exploits unique
   - Negative interaction energy
   - M = 0 gravitons w/o conformal invariance