Effective theories, continuum limit and background independence in LQG

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Outline

1. Motivation and statement of the problem
   - Target theories
   - “Background Independence”
   - Open Problems
2. Research Plan
   - Structures Developed
   - Objective
   - Today’s Question
3. RG for Loop Quantization
4. Study of Background Independence

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Target theories: Loop quantized theories (in particular LQG)

- $\star\star\star$ Gravity as a constrained gauge theory
- Other gauge theories (coupled to gravity or not)
- Scalar fields (coupled to gravity or not)
- Model theories (lower dimensional gravity, Husain-Kuchar model, 2dYM, “QM”, etc)

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“Background Independence”

- Assumed background: a 3-manifold Σ $(C^\infty, C^\omega, P\omega, PL)$ (without or with a fixed metric)
- Representation of “Diff_Σ”
  * But “Diff_Σ” acts discretely on $\mathcal{H}_\Sigma$!
- In gravity diffeos are gauge
  $\mathcal{H}_\Sigma \supset Cyl_\Sigma \xrightarrow{\eta_{\mathrm{Diff}}} \mathcal{H}_{\mathrm{Diff}} \subset Cyl^\star_\Sigma$
  where η_{Diff} is a group averaging
  * Needs “renormalization”

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Open Problems

- Dynamics (for gravity, etc)
  * Point splitting regularization does not work on $\mathcal{H}_\Sigma$
- Semiclassical limit, macroscopic limit, Newtonian limit, ...
  Recovery of / Comparison with / Deviations from standard low energy physics

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Structures Developed

An implementation Wilson's RG tailored to loop quantization

- A notion of “measuring” **scales**
- **Effective Theories** at given scales
  * Including regularization to that scale
- **Coarse Graining** (decimation operation)
- Construction of **Observables in the Continuum** (in particular dynamics) by a renormalization procedure

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Overall Objective

- Apply to known QFTs defined over metric backgrounds to test the method
  2d Ising (lattice filed theory with an introduction),
  "polymer QM", *2d YM*
- Apply to field theories without a metric background (ultimately to gravity coupled to matter)
  * Reconsider simple models developed by LQ techniques

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Today's Question

Do the added structures interfere with background independence?

– In analogy with –

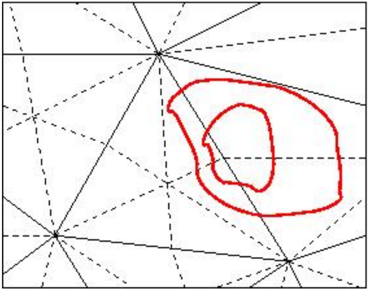
Do the lattices of lattice field theory interfere with the rotational symmetry of Euclidean field theory?

Measuring Scales

$$Cyl_\Sigma \xrightarrow{\text{Scale n: available measurements}} Cyl_n$$

$$\psi_j \text{ and } \psi_k \longmapsto \psi_{[j]_n}$$

for n=1, $(\Sigma, \mathrm{Sd}^1(\Delta))$



Effective Theories and Regularization

$$\overline{\mathcal{A}/\mathcal{G}}_{\Sigma,\star} = \mathrm{Hom}(\mathcal{P}_{\Sigma,\star}, SU(2))$$

$$\mathcal{P}_{\Sigma} \xrightarrow{\;[\,\cdot\,]_n\;} \mathcal{P}_n$$

$$\ell \text{ (mod. retracing)} \longmapsto \{\alpha_1, \ldots, \alpha_M\} \text{ (mod. retr.)}$$

$$\overline{\mathcal{A}/\mathcal{G}}_{\Sigma} \longleftarrow \mathcal{A}/\mathcal{G}_n$$

Loops in the same $[\,\cdot\,]_n$ share holonomies.

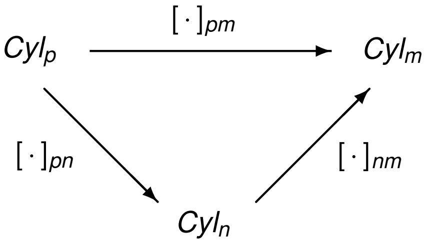
Effective Theories and Regularization

$$\begin{array}{ccc} Cyl_{\Sigma} & \xrightarrow{[\,\cdot\,]_n^{\star\star} \sim [\,\cdot\,]_n} & Cyl_n \\ \psi_j & \longmapsto & [\psi_j]_n = \psi_{[j]_n} \end{array}$$

It **regularizes observables** from the continuum to scale *n*.

Effective Theories and Regularization

If $m \leq n$, the map $[\,\cdot\,]_{nm}$ regularizes from scale n to scale m.

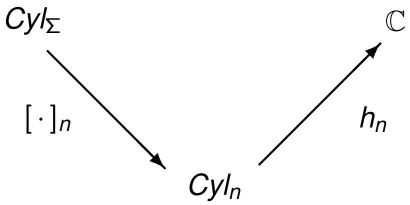
- 

commutes.

- $[\psi_j]_n = [\psi_k]_n$ for all n iff $\psi_j = \psi_k$.

Approximation of functionals in the continuum

E.g. expectation value of a Hamiltonian **functional in the continuum approximated** using $h_n(\psi) =< \psi|H_n|\psi >_n$.

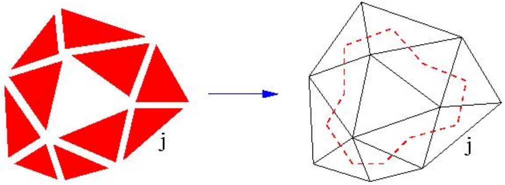


* Recall that point splitting regularization of operators on $\mathcal{H}_\Sigma$ (or Cyl_Σ) does not work.

Coarse Graining (decimation map)

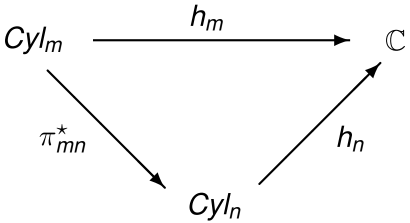
$$Cyl_n \xrightarrow{\pi_n^\star} Cyl_\Sigma$$

$$\psi_{[j]_n} = (\psi, [j]_n) \longmapsto \psi_{\mathrm{barEmb}}([j]_n)$$



- $\pi^\star$-triangle diagrams commmute
- $[\cdot]_{mn} \circ \pi^\star_{mn} = id_{Cyl_m}$ and $\pi^\star_{mn} \circ [\cdot]_{mn}$ projector in Cyl_n

Correcting Observables



- The coupling constants in each h_n are tuned for them to agree with one standard measurement at scale m_0.
- $\pi_{mn}(h_n)$ includes microscopic corrections to h_m.

Observables in the Continuum

At scale m we study the convergence of the microscopic corrections

$$\begin{aligned} h_m^{ren} : \quad Cyl_m &\to \mathbb{C} \\ \psi_{[j]_m} &\mapsto \lim_{n \to \infty} h_n(\pi_{mn}^{\star} \psi_{[j]_m}) \end{aligned}$$

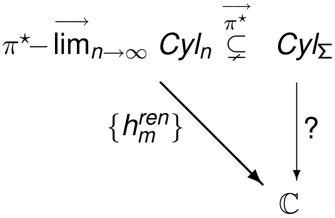
Convergence means ...

Observables in the Continuum

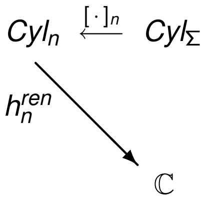
The collection $\{h_m^{ren}\}$ is π-compatible

$$h_m^{ren} = \pi_{mn}(h_n^{ren})$$

It can predict with complete accuracy at any scale.



Observables in the Continuum



- These maps may converge as $n \to \infty$.
- If they do they **extend** $\{h_m^{ren}\}$ to act on Cyl_Σ.

Solution of the diffeomorphism constraint in LQG

Diffeomorphism invariant states as linear functionals on Cyl_Σ

$$\begin{array}{ccc} Cyl_\Sigma \times Cyl_\Sigma & \xrightarrow{\eta_\Sigma} & \mathbb{C} \\ (\psi_j, \psi_k) & \longmapsto & \text{“}\sum_{f \in Diff_\Sigma} < \hat{f}\psi_j, \psi_k >_\Sigma\text{”} \end{array}$$

"Diffeos" at scale m (??!!)

Consider a "diffeo" $f : \Sigma \to \Sigma$ $\qquad (\hat{f}\psi_j = \psi_{f(j)})$

$$
\begin{array}{ccc}
Cyl_\Sigma & \xrightarrow{\quad \hat{f} \quad} & Cyl_\Sigma \\
\uparrow \pi^\star & & \downarrow [\,\cdot\,] \\
Cyl_n & \xrightarrow{\quad \hat{f}_n \quad} & Cyl_n \\
\uparrow \pi^\star & & \downarrow [\,\cdot\,] \\
Cyl_m & \xrightarrow{\quad \hat{f}_m \quad} & Cyl_m \\
\uparrow \pi^\star & & \downarrow [\,\cdot\,]
\end{array}
$$

- $f : \Sigma \to \Sigma$ leads to $\{\hat{f}_n\}$ compatible
- The sets $Diff_n = \{\hat{f}_n : f \in Diff_\Sigma\}$ are finite
- $f = g \iff \hat{f}_n = \hat{g}_n \; \forall \; n$

However . . .

- At any given scale n we do not have a representation of $Diff_\Sigma$ (or a related group).
- Even $\pi^\star\text{–}\overrightarrow{\lim}_{n\to\infty}\ Cyl_n \overset{\overrightarrow{\pi^\star}}{\subsetneq}\ Cyl_\Sigma$ is ill-suited in that respect.
- There is a representation in the huge space

  $$[\cdot]\text{–}\overleftarrow{\lim}_{n\to\infty}\ Cyl_n \overset{\overleftarrow{[\cdot]}}{\supsetneq}\ Cyl_\Sigma\ .$$
- The number of “coupling constants” in the sum over $Diff_n$ $\sum \beta([j]_n, [k]_n) < \hat{f}_n \psi_{[j]}, \psi_{[k]} >_n$ grows as $n \to \infty$.
- The “coupling constants” are constrained by the diffeo invariance requirement which becomes a complicated combinatorial relation.
- Since symmetry does not help when restricting to a single scale there are no useful truncations. Only difference when compared with the lattice treatment of EQFTs.

Summary

- With a single framework (Loop Quantization + an implementation of Wilson's RG) we have treated QFTs on metric and non metric backgrounds.
- When we can compare with standard treatments there is complete agreement:
  2d Ising (lattice filed theory with an introduction),
  "polymer QM", *2d YM*, "polymer QM", TQFTs,
  H-K model (with difficulty).
- The source of difficulty is the pour relation between the non-locality of knot classes of graphs and our notion of measuring scales.
  If a diffeo invariant QFT takes us to locally calculable knot invariants (as QG may do), our framework could be useful.

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