Eddington action and a unified source for dark matter and dark energy

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As often happens, one starts trying to solve problem "A" and end up writing a paper on problem "Z".

This talk will be an example of a scientific random walk.

We start with the energy,

$$E = M + \frac{l}{8G}. \qquad (1)$$

of a 2+1 black hole

$$ds^2 = -\left(-8GM + \frac{r^2}{l^2}\right) dt^2 + \frac{dr^2}{-8GM + \frac{r^2}{l^2}} + r^2 d\phi^2 \qquad (2)$$

The zero mass black hole has non-zero energy! There is a lot of physics in this relation.

- The *CFT* is quantized on a cylinder: The CFT central charge is $c = \frac{3l}{2G}$ (Brown-Henneaux) and $\frac{c}{12} = \frac{l}{8G}$ is precisely the Casimir energy.
- The state $M = 0$ is not the true background: it does not have the maximum number of symmetries.
- The state $M = -\frac{l}{8G}$ (with $E = 0$) is exactly anti-de Sitter space with the maximum number of isometries, and supersymmetries.

In five dimensions (relevant for the string/$\mathcal{N}=4$ *SYM* correspondence) the energy for a Schwarzschild-AdS black hole

$$ds^2 = \left(1 - \frac{8MG}{3\pi r^2} + \frac{r^2}{l^2}\right) dt^2 + \frac{dr^2}{1 - \frac{8MG}{3\pi r^2} + \frac{r^2}{l^2}} + r^2 d\Omega \tag{3}$$

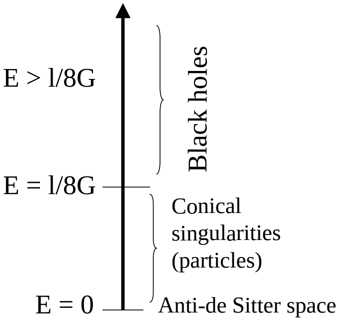
is

$$E = M + \frac{3\pi\, l^2}{32G}. \tag{4}$$

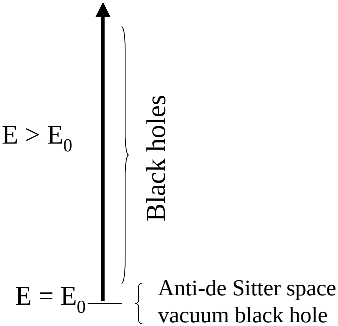
Again, for $M=0$, no black hole, there is a non-zero energy:

- The CFT is quantized on a sphere and $\frac{3\pi l^2}{32G}$ nicely matches the Casimir energy for $(N^2-1)\times$(1 vector + 4 spinors + 6 scalars) (Horowitz,Balasubramanian-Kraus).
- But.... what is the state with $E=0$? In this case, $M=0$ coincides with anti-de Sitter space having the maximum number of isometries and supersymmetries.

Three dimensions



Five dimensions



In five dimensions, the vacuum black hole and anti-de Sitter space are squashed together as the same state

In five dimensions, what state could have $E = 0$ and more symmetries than the maximally symmetric anti-de Sitter space?

The only state which comes to one's mind (at least some minds...) is

$$g_{\mu\nu}(x) = 0 \tag{5}$$

having the full set of diffeomorphism as "isometries".

We anticipate that this state will **not** *solve the above question because the very notion of energy requires boundary conditions, not satisfied by* $g_{\mu\nu}(x) = 0$*.*

General relativity with $g_{\mu\nu} = 0$ as an allowed state is not a new story...(Witten 1988, Horowitz 1990, Giddings 1991...)

What does it mean to set $g_{\mu\nu}(x) = 0$ *within general relativity?*

An analogy: without knowing quantum field theory, one can derive how much energy is necessary to create a particle:

The ground state matters. Spectrum of a massive particle

$E_N = \frac{p^2}{2m}$		m $E_N = 0$	p $E_N \neq 0$
$E = \sqrt{m^2c^4 + p^2c^2}$	$E = 0$	$E = mc^2$	p $E \neq 0$

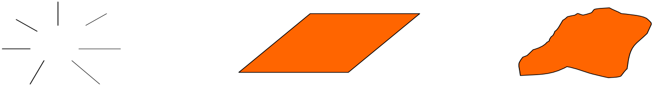
- mc^2 is the energy to *create* the particle.
- When coupled to gravity, the shift mc^2 is not an option. It *must* be included

We shall attempt a similar analysis for the Einstein tensor, and the creation of a metric.

The spectrum of general relativity:

Flat space: $g_{\mu\nu} = \eta_{\mu\nu}$ Curved spaces: $g_{\mu\nu}(x)$

Could we consider the ground state as the state with no metric?



$g_{\mu\nu} = 0$ $g_{\mu\nu} = \eta_{\mu\nu}$ $g_{\mu\nu}(x)$

Flat space has a non-zero energy-momentum, with respect to the ground state.
Energy-momentum to create the metric: does this concept make sense?.

In this talk:

- we shall discuss some ingredients of a would-be (**???**) theory of gravity where $g_{\mu\nu}(x) = 0$ is sociable acceptable
- we shall postulate an action for the dynamics of the extra degrees of freedom involved
- we shall argue that these extra degree of freedom may have something to do with dark matter and dark energy.

A background connection

Does it make sense to extrapolate general relativity to $g_{\mu\nu} = 0$? Well, in a first order formulation (Witten, Horowitz,...) with e^a and w^{ab} as independent variables, Einstein equations read:

$$\begin{aligned} \epsilon_{abcd} R^{ab} \wedge e^c &= 0 \qquad (6) \\ \epsilon_{abcd} T^a \wedge e^b &= 0 \qquad (7) \end{aligned}$$

and they make perfect sense (and are satisfied!) at

$$e^a = 0 \qquad \Leftrightarrow \qquad g_{\mu\nu} = 0 \qquad (8)$$

Now, this state does not fix the connection which becomes arbitrary

$$w^{ab}(x) : \qquad \text{arbitrary} \qquad (9)$$

Is the "background" connection an artifact of the first order formalism?

Suppose we first solve for the connection and write either,

$$w^{ab}{}_{\mu}(e) = \frac{1}{2}(e^{\rho a} e^{b}_{\rho,\mu} - e^{\rho b} e^{a}_{\rho,\mu}) + \cdots \tag{10}$$

or, in a purely metric formulation,

$$\Gamma^{\mu}{}_{\nu\rho}(g) = \frac{1}{2} g^{\mu\alpha}(g_{\alpha\nu,\rho} + g_{\alpha\rho,\nu} - g_{\nu\rho,\alpha}) \tag{11}$$

We see that, as $e^{a}_{\mu} \longrightarrow 0$ or $g_{\mu\nu} \longrightarrow 0$, both w^{ab} and $\Gamma^{\mu}{}_{nu\rho}$ become

$$\frac{0}{0} \tag{12}$$

and thus completely arbitrary, but **finite**. We conclude:

> *Either in first order, second order, tetrad or metric formulations, if general general relativity make sense at all at $g_{\mu\nu}(x) = 0$, it will require the addition of a theory for the "background" connection.*

Eddington Theory

At $g_{\mu\nu} = 0$ there is an arbitrary connection. From now on we call it

$$A^{\mu}{}_{\nu\rho}(x). \tag{13}$$

We now postulate that this field make sense and we should write equations of motion for it. There are no too many options for an action. We start by defining its 2-form curvature

$$K = dA + A \wedge A \tag{14}$$

and its associated Ricci tensor and its determinant

$$K_{\mu\nu} = K^{\rho}{}_{\mu\rho\nu}, \qquad K \equiv |\det K_{(\mu\nu)}| \tag{15}$$

None of these quantities require a metric. An action for this field was introduced by Eddington

$$I[A] = l^2 \int \sqrt{K} \tag{16}$$

which is a sort of Born-Infeld theory for the back ground connection A.

It was shown by Eddington (Einstein,Schroedinger...) that the dynamics of this action is equivalent to a metric theory (with a cosmological constant). Let

$$\sqrt{q} q^{\mu\nu} \equiv \frac{\delta I[A]}{\delta K_{\mu\nu}} \tag{17}$$

then, in terms of $q_{\mu\nu}$, Eddington equations of motion ready simply

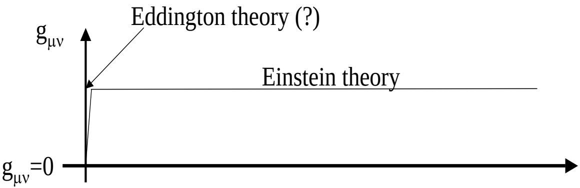
$$K_{\mu\nu} = \frac{1}{l^2} q_{\mu\nu} \tag{18}$$

with

$$A^{\mu}{}_{\nu\rho} = \frac{1}{2} q^{\mu\alpha}(q_{\alpha\nu,\rho} + q_{\alpha\rho,\nu} - q_{\nu\rho,\alpha}). \tag{19}$$

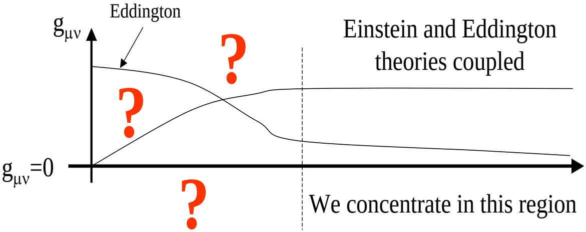
Thus, the solutions to the equations of motion can be parameterized in terms of a symmetric rank-2 tensor $q_{\mu\nu}$ with constant curvature.

But the fundamental field in Eddington theory is a connection!



The above interpretation does not look reasonable.

Assume a smooth transition:



- We then consider Einstein + Eddington's actions and couple them:

$$I[g, A] = \frac{1}{16\pi G}\int(\sqrt{g}R + l^2\sqrt{K} + \alpha\sqrt{K}(K^{-1})^{\mu\nu}g_{\mu\nu})$$

- Eddington action is a remnant of the creation process

The coupling is not too arbitrary because we would like $G_{\mu\nu} = -\frac{\delta I_{int}}{g^{\mu\nu}}$ not to be inconsistent at $g_{\mu\nu} = 0$. With the above interaction, and only this one (!), the equations are

$$G_{\mu\nu}(g) = -\alpha\sqrt{\frac{K}{g}}g_{\mu\alpha}K^{\alpha\beta}g_{\beta\nu} \tag{20}$$

becoming both $\frac{0}{0} = \frac{0}{0}$ (in four dimensions).

Summarizing, we have a bi-metric theory of gravity coupled by the equations (in an approximation for q large)

$$G_{\mu\nu}(g) = -\alpha\sqrt{\frac{q}{g}}g_{\mu\alpha}q^{\alpha\beta}g_{\beta\nu} \quad (21)$$

$$K_{\mu\nu}(q) = \frac{3}{l^2}q_{\mu\nu} \quad (22)$$

A condition on α. Since the metric $q_{\mu\nu}$ has constant curvature, it is natural to wonder if (for some l')there exists solutions with

$$R_{\mu\nu} = \frac{3}{l'^2}g_{\mu\nu} \quad (23)$$

Replacing in (21) we derive,

$$q_{\mu\nu} = \frac{\alpha l^2}{l'^2}g_{\mu\nu}$$

and replacing back in (22) we find $R_{\mu\nu} = \frac{3\alpha}{l'^2}g_{\mu\nu}$. Consistency with (23) requires

$$\alpha = 1. \quad (24)$$

Applications

- Cosmology
- Galactic velocity profiles

Consider the cosmological ansatz for $g_{\mu\nu}$ and $q_{\mu\nu}$

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \quad (25)$$

$$q_{\mu\nu}dx^\mu dx^\nu = -\frac{\dot{q}^2}{q^2}dt^2 + q(t)^2(dx^2 + dy^2 + dz^2) \quad (26)$$

We shall compare the (current) standard Friedman evolution

$$\frac{\dot{a}^2}{a^2} = \frac{0.24 + 0.03}{a^3} + 0.73 \quad (27)$$

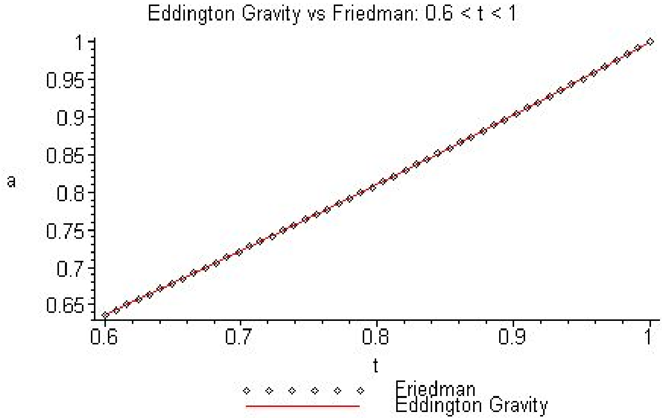
with that predicted by the Eddington+Einstein model

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= \frac{q^4}{\dot{q}}\frac{1}{a^3} + \frac{0.03}{a^3} \\ \left(\frac{q^4}{\dot{q}}\right)^{\cdot} &= 3\dot{q}\,\dot{a}\,a \end{aligned} \quad (28)$$

The only free parameter is $q_0 \equiv q(t)|_{t0}$. (t_0=today)

- We fix q_0 by demanding a best fit in the range $0.6 < t < 1$.
- Then we extrapolate the equations to larger values of t

$$q_0 = 0.912...$$


Eddington Gravity vs Friedman: 0.6 < t < 1
a
1
0.95
0.9
0.85
0.8
0.75
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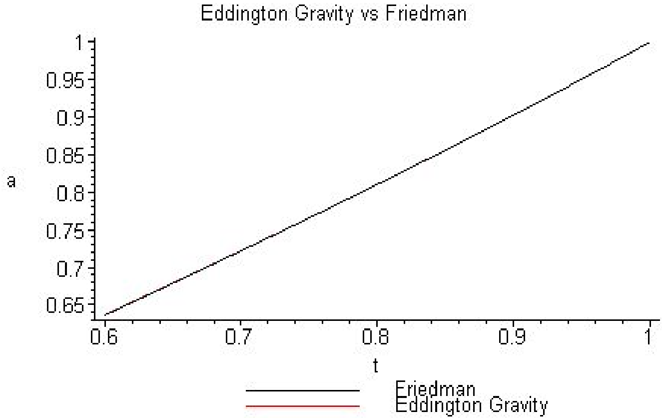
t
Friedman
Eddington Gravity

$q_0 = 0.912$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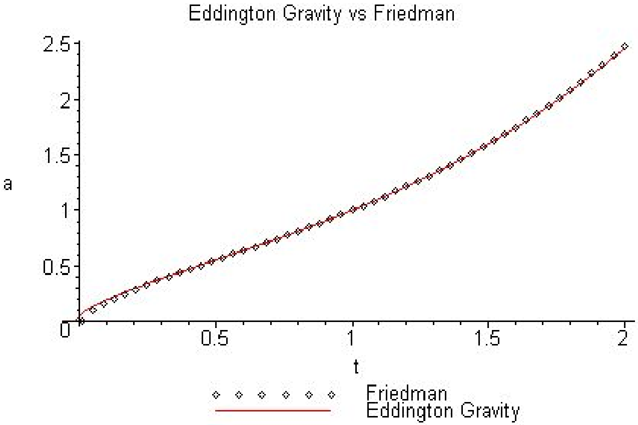
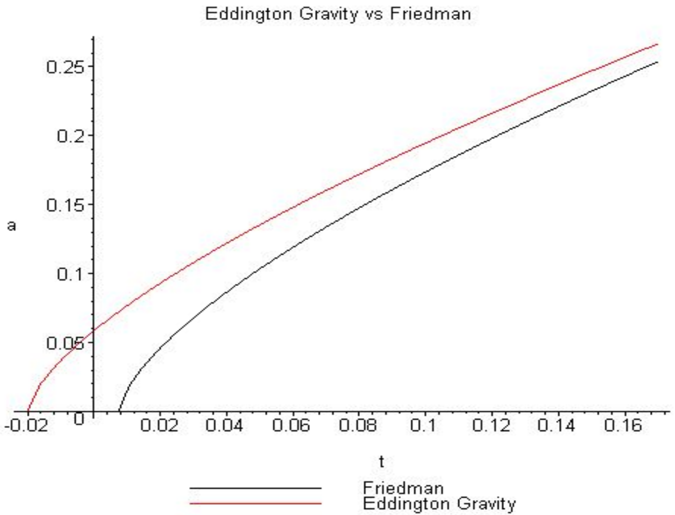
The accelerating Universe



The age of the Universe:



The agreement can also be established analytically via a power series:

- For t small

$$a(t) = a_0 t^{2/3} \left(1 + \frac{243}{56} \frac{Q_0^4 \; t^{4/3}}{a_0(4a_0^3 + 3C_m)} + \mathcal{O}(t^{7/3}\right) \qquad (29)$$

- At the other side, one can check that

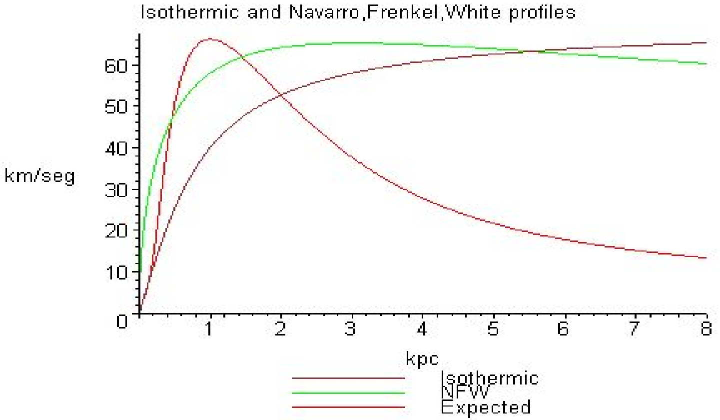
$$a(t) = e^{t/l} \qquad (30)$$

  is an exact solution. The Universe does accelerate like Λ.
- The numerical solution interpolates between matter and Λ

$$t^{2/3} \qquad \text{and} \qquad e^{t/l} \qquad (31)$$

Galactic velocity profiles.

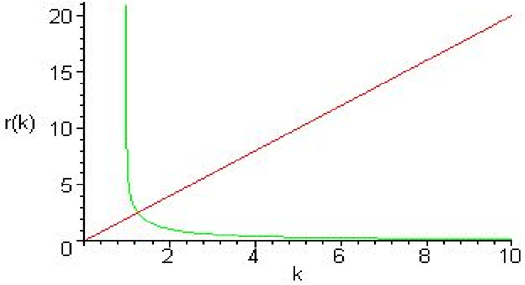
At small scales, the "Eddington fluid" behave like matter. It is thus natural to check the galactic problem where the circular motion is not the expected one, due to dark matter.



Let us analyze this problem in the Eddington-Einstein system.
We solve the same equations with the ansatz:

$$
\begin{aligned}
g_{\mu\nu}dx^{\mu}dx^{\nu} &= -f(r)^2dt^2 + \frac{dr^2}{h(r)^2} + r^2d\Omega \\
q_{\mu\nu}dx^{\mu}dx^{\nu} &= -c_1^2\left(1 - \frac{2w_0}{k} - \frac{k^2}{l^2}\right)dt^2 + \frac{dr^2k'^2}{1 - \frac{2w_0}{k} - \frac{k^2}{l^2}} + k^2d\Omega
\end{aligned}
$$

(c_1 is the velocity of light for the $q_{\mu\nu}$ metric.)



The linear branch:

If we choose the solution $k(r) = ar$ the metric becomes the usual Schwarzschild-dS solution:

$$ds^2 = -c^2 \left(1 - \frac{2m_0}{r} - \frac{r^2}{l'^2}\right) dt^2 + \frac{dr^2}{1 - \frac{2m_0}{r} - \frac{r^2}{l'^2}} + r^2 d\Omega \qquad (32)$$

with

$$\frac{1}{l'^2} = \frac{c_1^2}{c^2}\frac{1}{l^2} \qquad (33)$$

$$m_0 = \frac{c\, w_0}{c_1} \qquad (34)$$

This also shows that the "expected" physics (e.g. Solar system experiments) can be recovered.

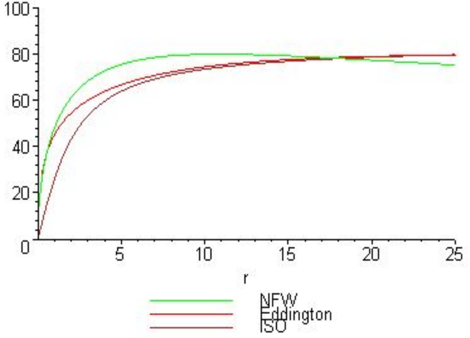
The cosmological constant is not fundamental and appears as an integration constant!

The logarithmic branch

If we choose instead the other solution (combined with the linear one appropriately)

$$r(k) = A\left(-k\left(1-\frac{w_0}{2k}\right)\left(\ln\left(1-\frac{w_0}{k}\right)+B\right)-1\right) \tag{35}$$

we find a flat velocity profile



w_0 must be non-zero to have a flat profile.

Conclusions

- Eddington theory is a well-defined field theory, it can be coupled to Einstein theory, and may provide a source for both dark matter and dark energy.
- Calculating the growth of primordial fluctuations will ultimately decide whether the Eddington "fluid" is, or is not, a good candidate. (Work in progress.)

- The interpretation by looking at $g_{\mu\nu} = 0$ may be feasible and would have interesting consequences.
- A proper "theory" for the intermediate regime is needed

THANK YOU