

Lorentz and CP symmetry in higher derivative theories and quantum gravity

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Outline

■ Introduction

- ◆ Loop quantum-gravity-phenomenology
- ◆ The construction of the U(1) effective gauge theory

- Non-duality extension of the modified electrodynamics
- Higher order Myers and Pospelov model
- Perturbative method for higher order Lagrangians
- Higher order extension of the Chern-Pontryagin theory
- Final remarks

Loop quantum gravity phenomenology

Loop quantum gravity phenomenology

What has been obtained?

- The standard Field Theories are modified by
 - ◆ Lorentz and Parity violating terms, suppressed by Planck scales
 - ◆ Deformed dispersion relations
 - ◆ The inclusion of a minimal length

[R.Gambini, J.Pullin, PRD 59(1999),124021; PRL 84 (2000), J. Alfaro,H. Tecotl, L. Urrutia; H. Sahlmann, T. Thiemann, CQG 23(2006)]

- Nevertheless, modified quantum field theories are fine tuned
 - ◆ Radiative corrections are of the order of the free parameters of the Standard Model [J. Collins, A. Perez, D. Sudarsky, L. Urrutia and H. Vucetich, PRL 93(2004), 191301]
- No agreement of which is the correct semiclassical approximation

Loop quantum gravity phenomenology

The construction of such effective theories can be summarized in the following steps

1) From the ADM split of the action one obtains a Hamiltonian constraint (for $U(1)$ gauge fields),

$$S_M \rightarrow H_M = \int d^3x N(x) \frac{q_{ab}}{\sqrt{q}} [E^a(x)E^b(x) + B^a(x)B^b(x)]$$

2) The Hamiltonian is promoted to operator acting on the kinematic Hilbert space of loop quantum gravity [T. Thiemann, CQG 15(1998), 839-873; CQG 15(1998), 1281-1314]

$$\hat{H}_M = \kappa \sum_{vertex, v} \epsilon^{JKL} \epsilon^{RNP} \hat{C}_{iL} \hat{C}_{iP} \left(\hat{\Phi}_{JK}(E) \hat{\Phi}_{RN}(E) + e^{\hat{\Phi}_{JK}(B)} e^{\hat{\Phi}_{RN}(B)} \right)$$

[J. Alfaro, H. Tecotl and L. Urrutia PRD 65(2002), 103509]

Loop quantum gravity phenomenology

3) Make expansion of the fields around vertices and use states that respects invariance under space rotations to compute expectation values. Here invariant tensors as ϵ^{abc} appears

$$\mathcal{H}_M^{eff} = \frac{1}{2}(\vec{E}^2 + \vec{B}^2) + g(\vec{E} \cdot \nabla \times \vec{E} + \vec{B} \cdot \nabla \times \vec{B})$$

→ At this level one has dynamics defined by

$$\{E^a(x), A_b(y)\} = \delta_a^b \delta(x - y)$$

→ The theory is invariant under the interchange of E and B

A generalization is possible considering odd permutation of indices in the resulting rotational invariant tensor

Non duality Extension

Non duality
Extension

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Schematically, the Hamiltonian can be expanded to each power in the segment s of the triangulation inside the expectation value of a state $|\psi\rangle$

$$\langle \psi | \left(\sum_{v,n} \hat{H}_{grav}^{(n)}(\hat{C}) \times \hat{H}_{matter}^{(n)}(\hat{\Phi}_E(v), \hat{\Phi}_B(v)) \right) | \psi \rangle$$

and in the box regions

$$= \sum_{x \in Box, n} \langle H_{grav}^{(n)}(\hat{C}_v) \rangle \times H_{matter}^{(n)}(\Phi_E(x(v)), \Phi_B(x(v)))$$

considering the rescaling $s \sim \ell_P$, $\sqrt{V} \sim \ell_P^{3/2}$ and for the gravitational connection $A \sim \frac{1}{\mathcal{L}}$, one arrives to

$$H^{eff} = \int d^3x \sum_{n=0} R^{(n)} \times H^{(n)}(E(x), B(x), \ell_P, \mathcal{L})$$

At first order

$$H_M^{(1)} = \int d^3x \mathcal{R}^{abc} \left[g_1 E^a(x) \nabla_b E^c(x) + g_2 B^a(x) \nabla_b B^c(x) \right]$$

note that $\mathcal{R}^{abc} = \epsilon^{abc}$, and therefore

$$\mathcal{H}_M^{eff} = \frac{1}{2} (\vec{E}^2 + \vec{B}^2) + (g_1 \vec{E} \cdot \nabla \times \vec{E} + g_2 \vec{B} \cdot \nabla \times \vec{B})$$

where $|g_1| = |g_2|$

→ The generalization consists in considering a relative minus sign between the electric and magnetic first correction

→ The Hamiltonian is no more invariant under interchange of E and B

Is there any other theory that give rise to these kind of terms???

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Myers and Pospelov Model

Higher order theories have been proposed to account for quantum gravity effects. They are given by the free field Lagrangians [R. Myers and M.

Pospelov, PRL 90(2003), 211601]

$$\mathcal{L}_{scalar} = \partial^\mu \Phi^* \partial_\mu \Phi - m^2 \Phi^* \Phi + g_s \Phi^* \partial_0^3 \Phi$$

$$\mathcal{L}_{photon} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + g_p E_i \partial_0 B_i$$

$$\mathcal{L}_{ferm} = \bar{\Psi} (i\gamma^\mu \partial_\mu - m) \Psi + g_f \bar{\Psi} \gamma^0 (\eta_1 + \eta_2 \gamma_5) \partial_0^2 \Psi$$

Consequences of higher order terms

- Phase space is increased
- Hamiltonian is not positive defined
- The evolution is not unitary
- Ghosts in the Feynman integral

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In order to deal with this kind of quantum field theories one can apply a perturbative scheme to reduce phase space and produce positive Hamiltonians

The method consist in

- 1) Iterating the equations of motion. This way one finds higher order time derivatives in terms of lower ones
- 2) The original higher symplectic form is approximated with first time derivatives
- 3) Redefinition of the fields in order to diagonalize the symplectic form
 - One obtains an approximate (effective) Hamiltonian positive defined
 - The expansion parameter remains analytic in all steps

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Higher order Chern-Pontryagin term

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We consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + g\frac{\partial\mathcal{P}}{\partial t}$$

where $\mathcal{P} = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$ is the Chern-Pontryagin term and g is a small constant of the order of Planck scales.

Using the Gauss-Ostrogradski equations for higher time derivative theories

$$\frac{\partial\mathcal{L}}{\partial A_\sigma} - \partial_\lambda\frac{\partial\mathcal{L}}{\partial(\partial_\lambda A_\sigma)} + \partial_\mu\partial_\lambda\frac{\partial\mathcal{L}}{\partial(\partial_\mu\partial_\lambda A_\sigma)} = 0$$

we arrive to the non-modified equations of motion

$$\partial_\lambda F^{\lambda\sigma} = 0$$

We perform the canonical analysis defining momenta's using the expressions

$$\pi^\alpha = \frac{\partial \mathcal{L}}{\partial \ddot{A}_\alpha}$$

$$p^\alpha = \frac{\partial \mathcal{L}}{\partial \dot{A}_\sigma} - \partial_r \frac{\partial \mathcal{L}}{\partial (\partial_r \dot{A}_\sigma)} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \ddot{A}_\sigma}$$

We arrive to

$$\pi^0 = 0$$

$$\pi^i = g \nabla \times A^i$$

$$p^0 = 0$$

$$p^i = F_{0i} + g \nabla \times \dot{A}^i$$

The symplectic structure is defined by the canonical Poisson brackets

$$\begin{aligned} \{A_0(x), p^0(y)\} &= \delta_{ij} & \{A_i(x), p^j(y)\} &= \delta_{ij} \\ \{\dot{A}_0(x), \pi^0(y)\} &= \delta_{ij} & \{\dot{A}_i(x), \pi^j(y)\} &= \delta_{ij} \end{aligned}$$

We can identify the set of primary constraints

$$\begin{aligned} \chi_0 &= p^0 - \partial_k \pi^k \approx 0 & \chi_1 &= \pi^0 \approx 0 \\ \varphi_2^i &= \pi^i - g \nabla \times A^i \approx 0 \\ \varphi_3^i &= p^i - F_{0i} - g \nabla \times \dot{A}^i \approx 0 \end{aligned}$$

Let us consider the canonical Hamiltonian

$H_c = \pi^i \ddot{A}_i + p^i \dot{A}_i - L$, which gives

$$H_c = p^i \dot{A}_i + \frac{1}{2} (F_{0i} F^{0i} + (\nabla \times A)^2) + g \epsilon^{ijk} \dot{A}_i \partial_j \dot{A}_k$$

Adjoining the primary constraints with the Lagrange multipliers σ we define the total Hamiltonian

$$H_T = p^i \dot{A}_i + \frac{1}{2}(F_{0i}F^{0i} + (\nabla \times A)^2) + g\epsilon^{ijk} \dot{A}_i \partial_j \dot{A}_k + \sigma_0 \chi_0 + \sigma_1 \chi_1 + \sigma_2^i \varphi_2^i + \sigma_3^i \varphi_3^i$$

Now we require the set of constraints to have vanishing Poisson bracket with H_T . The evolution of $\{\chi_1, H_T\} = 0$ and χ_0 produces the secondary constraint

$$\{\chi_0, H_T\} = \partial_k p^k \approx 0$$

which we define as $\chi_2 = \partial_k p^k \approx 0$, it can be checked that $\{\chi_2, H_T\} = 0$

and by the evolution of

$$\begin{aligned}\{\varphi_2^k, H_T\} &= -\varphi_3^k + \sigma_3^k \\ \{\varphi_3^k, H_T\} &= \nabla \times \nabla \times A^k - \sigma_2^k\end{aligned}$$

the Lagrange multipliers are $\sigma_3^k = \varphi_3^k$ and $\sigma_2^k = \nabla \times \nabla \times A^k$ Therefore the total Hamiltonian is

$$\begin{aligned}H_T &= p^i \dot{A}_i + \frac{1}{2}(F_{0i}F^{0i} + (\nabla \times A)^2) + g\epsilon^{ijk} \dot{A}_i \partial_j \dot{A}_k \\ &\quad + \sigma_0 \chi_0 + \sigma_1 \chi_1 + \sigma_2^i \varphi_2^i + \sigma_3^i \varphi_3^i + \sigma_4 \chi_4\end{aligned}$$

We identify the second class constraints

$$\{\varphi_2^i, \varphi_3^j\} = \delta_{ij}$$

And the first class

$$\{\chi_0, \chi_1, \chi_2\}$$

→ The degrees of freedom remain the same as the usual Maxwell theory

→ Second class constraints modify the symplectic structure

Finally, the symplectic structure can be written as

$$\Omega_{pert} = dp^i(x) \wedge (dA_i + g \nabla \times dA_i) + \mathcal{O}(g^2)$$

Using the redefinition

$$\bar{A}_i = A_i + g \nabla \times A_i$$

this transformation is unique modulus canonical transformations. The Hamiltonian in the new variables is

$$H_{pert} = \frac{1}{2}(p^2 + \bar{B}^2) + g(p \cdot \nabla \times p - \bar{B} \cdot \nabla \times \bar{B}) + \mathcal{O}(g^2)$$

with

$$\{p^i(x), \bar{A}_j(y)\} = \delta_j^i \delta(x - y)$$

→ We note the same corrected terms as in the loop quantum gravity modifications

→ However, the components of the electromagnetic tensor are known in this approach

Summary

- We have constructed the Non-dual extension of modified electrodynamics
- We have shown that a perturbative scheme produces the same corrected terms
- The perturbative reduction gives good Hamiltonians for higher order Lorentz invariant theories, analytic in the expansion parameter
- Effective Hamiltonians from loop quantum gravity may preserve Lorentz symmetry if one accepts that the redefined gauge fields are components of the electromagnetic tensor F
- Similar arguments apply when adding fermions