Anisotropic Cosmological Constant and the Ellipsoidal Universe

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Summary

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Introduction and motivation

In [1] the possibility of an anisotropic dark energy modeled as close as possible to ΛCDM was explored.

Motivation:

- The CMB large angle anomalies (statistical isotropy violation) [2].
- Suggest other viable ways to model dark energy.

The significance and possible meaning of such anomalies is a disputed theme.

The ΛCDM fits very well in the WMAP data for intermediate multipoles, but perhaps the large angle anomalies are a sign of new physics.

[1] D.C. Rodrigues, "Anisotropic cosmological constant and the quadrupole anomaly", arXiv:0708.1168, to appear in PRD.

[2] e.g., de Oliveira-Costa A., Tegmark M., Zaldarriaga M., Hamilton A., "The Significance of the largest scale CMB fluctuations in WMAP" 2004, Phys. Rev., D69, 063516

Introduction and motivation

These anomalies led to the proposal of diverse anisotropic models, and renewed interested on the Bianchi cosmologies.

Others recent works on anisotropic dark energy[3].

We will consider a non-dynamical form of dark energy with anisotropic pressure.

The anisotropy magnitude will be estimated using the elliptical universe proposal to solve the quadrupole power anomaly [4].

[3] R. A. Battye and A. Moss, "Anisotropic perturbations due to dark energy," Phys. Rev. D **74**, 041301 (2006); J. Beltran Jimenez and A. L. Maroto, "Cosmology with moving dark energy and the CMB quadrupole," Phys. Rev. D 76, 023003 (2007); T. Koivisto and D. F. Mota, "Accelerating Cosmologies with an Anisotropic Equation of State," arXiv:0707.0279

[4] L. Campanelli, P. Cea and L. Tedesco, "Ellipsoidal Universe Can Solve The CMB Quadrupole Problem," Phys. Rev. Lett. **97**, 131302 (2006)

Summary

Bianchi Cosmologies

The standard cosmological model (ΛCDM) uses the (flat) Robertson-Walker metric $g = g_{\mu\nu} dx^\mu \otimes dx^\nu$, with $(g_{\mu\nu}) = \text{diag}\left(-N^2(t) \quad a^2(t) \quad a^2(t) \quad a^2(t)\right)$.

Bianchi classified a broader class of homogeneous cosmologies.

Let $g = \eta_{\mu\nu} e^\mu \otimes e^\nu$, with $e^\mu = e^\mu_a dx^a$ and $[e^\mu, e^\nu] = f^{\mu\nu}_\lambda e^\lambda$.

The simplest (Abelian) case has $f^{\mu\nu}_\lambda = 0$ and its line element reads

$$ds^2 = -N^2(t)dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2.$$

Bianchi I line element

It's the one closer to ΛCDM.

Bianchi I cosmology equations

The Einstein-Hilbert action

$$S[g, ...] = \int \left[\frac{k}{2} \, (R - 2\Lambda) + \mathcal{L}_M \right] \sqrt{-g} \; d^4x$$

with the metric $(g_{\mu\nu}) = \text{diag}\left(-1 \quad a^2(t) \quad b^2(t) \quad c^2(t)\right)$ leads to the Eqs.

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + H_b \, H_c = \Lambda - \frac{1}{k} T^1_1, \qquad H_a \, H_b + H_a \, H_c + H_b \, H_c = \Lambda - \frac{1}{k} T^0_0$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + H_a \, H_c = \Lambda - \frac{1}{k} T^2_2, \qquad H_a \equiv \frac{\dot{a}}{a}, H_b \equiv \frac{\dot{b}}{b}, H_c \equiv \frac{\dot{c}}{c}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + H_a \, H_b = \Lambda - \frac{1}{k} T^3_3, \qquad k \equiv \frac{1}{8\pi G}$$

On anisotropic pressure and E-M conservation

Let $(\mathcal{T}^{\mu}_{\nu})(t) = \text{diag}\begin{pmatrix}-\rho & p_x & p_y & p_z\end{pmatrix}$. Therefore, $\nabla_\mu \mathcal{T}^{\mu}_{i} = 0$ and

$$-\nabla_\mu \mathcal{T}^{\mu}_{0} = \dot{\rho} + (H_a + H_b + H_c)\rho + H_a \; p_x + H_b \; p_y + H_c \; p_z .$$

The cosmological constant case: $\dot{\rho} = \frac{d}{dt}(k\Lambda) = 0$, $p_x = p_y = p_z = -\rho$.

Consider $\dot{\rho} = 0$ and $p_{x^i} = -\rho + C^{ij} H_{a_j}$, with $C^{ij} = C^{ij}(t)$.

Energy-momentum conservation only constrain the symmetric part of C^{ij}.

Brackets

Motivated by space-time noncommutativity, Ref. [1] considered the effects of $[a_i, a_j] \neq 0$ to the Wheeler-deWitt equation.

In the present case, $C^{ij} \propto B^{ij}$, where $B^{ij} = \{\pi^i, \pi^j\}_B$.

Momenta brackets deformations of field theories were explored in [2]; it may also be induced by spinor cosmology [3].

[1] H. Garcia-Compean, O. Obregon and C. Ramirez, "Noncommutative quantum cosmology," Phys. Rev. Lett. **88**, 161301 (2002).

[2] e.g., A. Das, J. Gamboa, J. Lopez-Sarrion and F. A. Schaposnik, "Gauge field theory in the infrared regime," Phys. Rev. D 72, 107702 (2005)

[3] M. Henneaux, "Bianchi Type I Cosmologies And Spinor Fields," Phys. Rev. D **21**, 857 (1980).

Two dynamical conditions on B^{ij} and the anisotropies/dark energy merging

The eqs. of motion are

$$\frac{\ddot b}{b} + \frac{\ddot c}{c} + H_b\, H_c = \Lambda - C^{1j} H_{a_j}, \qquad \frac{\ddot a}{a} + \frac{\ddot b}{b} + H_a\, H_b = \Lambda - C^{3j} H_{a_j},$$

$$\frac{\ddot a}{a} + \frac{\ddot c}{c} + H_a\, H_c = \Lambda - C^{2j} H_{a_j}, \qquad H_a\, H_b + H_a\, H_c + H_b\, H_c = \Lambda,$$

with $C^{12} = B^{12}/(\tilde k c)$, $C^{13} = B^{13}/(\tilde k b)$, $C^{23} = B^{23}/(\tilde k a)$.

Conditions on B^{ij}:

- The absolute value of a scale factor should not have dynamical impact.

  i.e., at any instant t_* one should be capable of selecting coordinates such that $a = b = c|_{t=t_*}$ and such rescaling should not modify the dynamics $\Rightarrow\ B^{ij} = B\epsilon^{ijk} a_k$.

- The vacuum dynamics should not depend on $\tilde k$, so $B \propto \tilde k$.

$[\tilde k] = m^{-3+2}$, $[C^{ij}] = m$, $[\Lambda] = m^2 \ \Rightarrow\ B = B_0 \sqrt{\Lambda}\ \tilde k$, with $[B_0] = 1$.

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + H_b\ H_c = \Lambda + B_0\ \sqrt{\Lambda}\ (H_c - H_b) - \frac{1}{k} T^1_1, \qquad H_a\ H_b + H_a\ H_c + H_b\ H_c = \Lambda - \frac{1}{k} T^0_0,$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + H_a\ H_c = \Lambda + B_0\ \sqrt{\Lambda}\ (H_a - H_c) - \frac{1}{k} T^2_2,$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + H_a\ H_b = \Lambda + B_0\ \sqrt{\Lambda}\,(H_b - H_a) - \frac{1}{k} T^3_3,$$

Some properties:

- Non-dynamical Dark energy with anisotropic pressures
- $\bar{\omega} \equiv \frac{\bar{p}}{\rho} = \frac{\int p(\theta,\phi)d\Omega}{4\pi\rho} = \frac{p_x+p_y+p_z}{3\rho} = -1$
- The phantom divide is crossed for some directions
- Cyclic symmetry is preserved, but permutation is not
- No isotropization is possible – Evades the cosmic no-hair theorem [4].

[4] R.M. Wald, "Asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant," Phys. Rev. D28 (1983) 2118

Summary

Ellipsoidal Last Scattering Surface (LSS)

The quadrupole power anomaly suggests the presence of some elliptical deformation of the LSS [1] – could be either of primary or secondary nature.

The contribution to the CMB temperature anisotropies due to an elliptical background reads

$$\frac{\Delta_B T}{\bar{T}}(\mathbf{n}) = -\frac{1}{2} e_{\text{LSS}}^2 \; n_A^2$$

$\mathbf{n_A}$ is the projection of $\mathbf{n}$ into the axis of symmetry.

Only the quadrupole ($l = 2$) is modified by the eccentricity e_{LSS}.

In order to generate $e_{\text{LSS}} = 6.4 \times 10^{-3}$ and solve the quadrupole anomaly

$$(\Delta T_2)^2_{\text{obs}} = 211 \mu K^2, \qquad (\Delta T_2)^2_{\Lambda\text{CDM}} = 1252 \mu K^2,$$

a cosmic homogeneous magnetic field (4.6×10^{-9} G) was considered [2].

[1] L. Campanelli, P. Cea and L. Tedesco, "Ellipsoidal Universe Can Solve The CMB Quadrupole Problem," Phys. Rev. Lett. **97**, 131302 (2006)

[2] L. Campanelli, P. Cea and L. Tedesco, "Cosmic Microwave Background Quadrupole and Ellipsoidal Universe," Phys.Rev.D76:063007,2007.

Implementing an axial symmetry

A system with $\{\pi^i, \pi^j\}_B = B\epsilon^{ijk} a_k$ cannot become axial symmetric with the universe expansion.

Let $b = c \;\Rightarrow\; \pi^b = \pi^c$, so inspired in the previous model we set $\{\pi^a, \pi^b\}_B = Bb$.

$\nabla_\mu T^\mu_\nu = 0$ holds.

Nevertheless, $\bar{\omega} = -1 + \frac{B_0}{3\sqrt{\Lambda}}(2H_b - 2H_a)$.

$$2\,\frac{\ddot{b}}{b} + H_b^2 = \Lambda - 2\,B_0\,\sqrt{\Lambda}\,H_b - \frac{1}{k}T^1_1, \qquad 2\,H_a\,H_b + H_b^2 = \Lambda - \frac{1}{k}T^0_0$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + H_a\,H_b = \Lambda + B_0\,\sqrt{\Lambda}\,H_a - \frac{1}{k}T^2_2,$$

Vacuum Exact Solutions

For the vacuum, the FLRW cosmology with Λ leads to $a_{\text{RW}} = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$.

The vacuum solutions for the present model are

$$a = a_0 \; e^{\sqrt{\frac{\tilde{\Lambda}}{3}} \; \left(1+2\tilde{B}_0\right) \; t}, \qquad b = b_0 \; e^{\sqrt{\frac{\tilde{\Lambda}}{3}} \; \left(1-\tilde{B}_0\right) \; t},$$

with $\tilde{\Lambda} = \Lambda(1 + \frac{B_0^2}{3})$, $\tilde{B}_0 = \frac{B_0}{\sqrt{3+B_0^2}}$.

So $\bar{\omega} = -1 + \frac{B_0}{3\sqrt{\Lambda}}(2H_b - 2H_a) = -\left(1 + \frac{2}{3}B_0^2\right)$.

Since $\omega_{\text{exp}} = -0.967^{+0.073}_{-0.072}$, we find $|B_0| \lesssim 0.2$.

These are supposed to describe the universe evolution for $z \lesssim 0.5$.

For higher redshifts one needs to insert matter ($z_{LSS} \approx 1000$).

Vacuum + Matter Exact Solutions

Let $\rho_M(t) = \rho_0/(ab^2)$ stand for usual matter + CDM energy density.

The standard solution reads $a_{\rm RW} = a_0 \sinh^{\frac{2}{3}} \left(\frac{3}{2}\sqrt{\frac{\Lambda}{3}} \right)$, while for the presented model it reads

$$a(t) = a_0 e^{\frac{2}{3} B_0 \sqrt{\Lambda}\,(t-t_*)} \sinh^{\frac{2}{3}} \left(\frac{3}{2} \sqrt{\frac{\tilde{\Lambda}}{3}}\, t \right) \qquad b(t) = a_0 e^{-\frac{1}{3} B_0 \sqrt{\Lambda}\,(t-t_*)} \sinh^{\frac{2}{3}} \left(\frac{3}{2} \sqrt{\frac{\tilde{\Lambda}}{3}}\, t \right)$$

with $a_0 = \left(\frac{\rho_0}{k\,\tilde{\Lambda}} \right)^{\frac{1}{3}}$, $\tilde{\Lambda} = \Lambda(1 + \frac{B_0^2}{3})$. t_* is an arbitrary constant. $a(t_*) = b(t_*)$.

We quantify the anisotropy at a given instant by

$$A(t) = \frac{\sigma}{H} = 3\frac{H_a - H_b}{H_a + 2H_b} = 3\ \tilde{B}_0\ \mathrm{Tanh} \left(\frac{3}{2} \sqrt{\frac{\tilde{\Lambda}}{3}}\ t \right)$$

$\lim_{t\to\infty} A(t) = 3\ \tilde{B}_0 \ \Rightarrow$ No isotropization, as expected.

Bound from the temperature anisotropies

The CMB temperature anisotropies put strong bounds on anisotropic cosmologies, $\Delta T(\theta,\phi)/\bar{T} \lesssim 10^{-5}$.

Hence, $\Delta_B T(\theta,\phi)/\bar{T} \lesssim 10^{-5}$, where Δ_B is the variation due to the background.

For the preferential direction, $\Delta_B T(0,\pi/2) = \bar{T} - a(t_{\rm dec})\, T_x$. $T_x =$ original temperature in that direction.

Since $|A(t_{\rm dec})| \ll |A(t_0)|$, $T_x \approx \bar{T}_e$, the mean original temperature.

$\bar{T} \approx a_{\rm RW}(t_{\rm dec})\bar{T}_e$

$\frac{\Delta_B T(0,\pi/2)}{\bar{T}} = 1 - \frac{a(t_{\rm dec})\ \bar{T}_e}{\bar{T}} = 1 - \frac{a(t_{\rm dec})}{a_{\rm RW}(t_{\rm dec})} \lesssim 10^{-5} \ \Rightarrow\ |B_0| \lesssim 10^{-5}$.

Eccentricity and the Ellipsoidal universe Proposal

The eccentricity reads

$$
\mathrm{e}(t, t_*) = \begin{cases} \sqrt{1 - \left(\frac{a}{b}\right)^2} = \sqrt{1 - e^{2B_0\sqrt{\Lambda}\,(t - t_*)}}, & \text{for } B_0(t - t_*) \leq 0 \\ \sqrt{1 - \left(\frac{b}{a}\right)^2} = \sqrt{1 - e^{2B_0\sqrt{\Lambda}\,(t_* - t)}}, & \text{for } B_0(t - t_*) \geq 0 \end{cases}
$$

In order to generate the proper eccentricity, following [3], we set $t_* = t_0$ and find $|B_0| \simeq 1.4 \times 10^{-5}$.

[3] L. Campanelli, P. Cea and L. Tedesco, Phys.Rev.Lett. 97 (2006) 131302, Phys.Rev.D76:063007,2007.

Plot

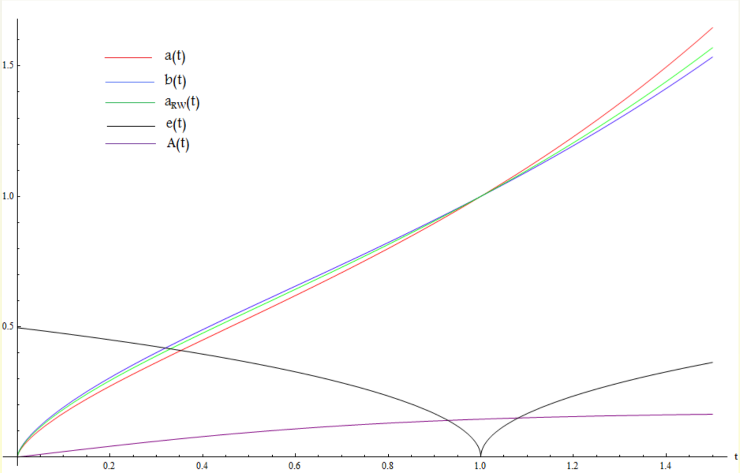

Figure: $\Lambda = 1.2 \times 10^{-35} s^{-2}$, $t_0 = 4.3 \times 10^{17} s$, $B_0 = 0.1$, $a(t_0) = b(t_0) = 1 \;\Rightarrow\; e(t_0) = 0$. The horizontal axis is the time in t_0 unities. a_{RW} acts as a mean scale factor ($a_{RW} = (ab^2)^{\frac{1}{3}}$.

Summary

Conclusions

General issues

- Ref. [1] introduced the elliptical universe proposal and favored the presence of a cosmic uniform magnetic field.
- In [2] dark energy was considered as the source of anisotropies and it was modeled as close as possible to ΛCDM.
- The effects to the CMB quadrupole power are the same, but the underlying physics and consequences for different z are not.
- The presented case has exact solutions.

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[2] D. C. Rodrigues, "Anisotropico cosmological constant and the quadrupole anomaly" arXiv:0708.1168, to appear PRD

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