RADIATIVE CORRECTIONS IN THE PHOTON SECTOR OF THE MYERS-POSPELOV MODEL

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PLAN OF THE TALK

- **INTRODUCTION AND THE MYERS-POSPELOV MODEL.**
- **THE PHOTON SECTOR: QUANTIZATION AND THE PROPAGATOR.**
- **THE INTERPOLATING REGULARIZATION PRESCRIPTION.**
- **THE FERMION SELF ENERGY.**
- **LIV CORRECTIONS WITH NO FINE TUNNING.**
- **FINAL COMMENTS.**

FIRST MODELS INVOLVING QG INSPIRED CORRECTIONS TO ELECTRODYNAMICS

- **Loop Quantum Gravity inspired** [R. Gambini and J. Pullin, Phys. Rev. D59(1999)124021]

$$\nabla \cdot \vec{E} = 0, \quad \partial_t \vec{E} = -\nabla \times \vec{B} + 2\,\xi\,\ell_P\,\nabla^2\,\vec{B},$$

$$\nabla \cdot \vec{B} = 0, \quad \partial_t \vec{B} = +\nabla \times \vec{E} - 2\,\xi\,\ell_P\,\nabla^2\,\vec{E}$$

$$\Rightarrow \omega_\pm(\vec{k}) = |\vec{k}|\left(1 \mp 2\,\xi\,\ell_P\,|\vec{k}|\right).$$

[J. Alfaro, H.A. Morales and L.F. Urrutia, Phys. Rev. Letts. 84(2000)2318, Phys. Rev. D65(2002)103509, Phys. Rev. D66(2002)124006.]

- **String Theory inspired** [J. Ellis et al., Gen. Relativ. Gravit. 32(2000)127; J. Ellis et al., Astrophys. J. 535(2000)139.]

$$\nabla \cdot \vec{E} + \vec{U} \cdot \partial_t \vec{E} = 0, \quad \nabla \times \vec{E} + \partial_t \vec{B} = 0,$$

$$\nabla \cdot \vec{B} = 0, \quad \nabla \times \vec{B} - (1 - |\vec{U}|^2)\partial_t \vec{E} + \vec{U} \times \partial_t \vec{B} + (\vec{U} \cdot \nabla)\vec{E} = 0,$$

$$\Rightarrow \omega(|\vec{k}|) = |\vec{k}|\left(1 - |\vec{k}|\,\xi\,\ell_P\right), \quad |\vec{U}| = O\left(|\vec{k}|\,\xi\,\ell_P\right).$$

THE MYERS-POSPELOV MODEL (MPM)

[R. Myers and M. Pospelov, PRL 90(2003)211601].

- **Effective field-theory model describing LIV using dimension five operators together with a non-dynamical constant four vector n^{μ}.**

- **Has been extensively used to discuss LIV Synchrotron Radiation** [R. Montemayor and L. F. Urrutia, PLB606(2005)66; PRD 72(2005)045018; AIP Conf. Proc. # 758, 2005].

- **Has been recently generalized to include interactions composed from Standard Model Fields.** [P.A. Bolokov and M. Pospelov, arXiv: hep-ph/0703291].

- **Quantum corrections to the original model have been also studied** [P.M. Crichigno and H. Vucetich, PLB 651(2007)313, arXiv: hep-th/0607214].

- **The model incorporates higher order time derivatives in the scalar and fermion sectors.**

- The Lagrangian is

$$\mathcal{L}_{MP} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\xi}{M_P}\left(n^{\mu}F_{\mu\nu}\right)\left(n^{\alpha}\partial_{\alpha}\right)\left(n_{\rho}\bar{F}^{\rho\nu}\right)$$

$$+ \ \bar{\Psi}i\gamma^{\mu}\left(\partial_{\mu} - m\right)\Psi + \frac{1}{M_P}\bar{\Psi}\left(n_{\alpha}\gamma^{\alpha}\right)\left(\eta_1 + \eta_2\gamma_5\right)\left(n^{\mu}\partial_{\mu}\right)^2\Psi.$$

- Photon contribution (Choose $n^{\mu} = (1, \vec{0})$ in the sequel.)

$$\sim \frac{\xi}{M_P}\vec{E}\cdot\partial_0\vec{B} \sim \frac{\xi}{M_P}\epsilon_{ijk}\dot{A}_i\partial_j\dot{A}_k, \qquad g \equiv \frac{\xi}{M_P}$$

- Fermion contribution

$$\sim \bar{\Psi}(\ldots\ldots)\ddot{\Psi}$$

- For the moment consider $\xi \neq 0, \eta_1 = \eta_2 = 0$, that is to say only corrections arising from the photon sector.

THE PHOTON SECTOR

- The minimal substitution in the fermion effective Lagrangian plus the free photon contribution of the MPM lead to

$$\begin{aligned} \mathcal{L}_{MP} &= \mathcal{L}_0(\psi) + J^\mu(\psi)A_\mu + \mathcal{L}_{int}(A^i,\psi) \\ &\quad + \frac{1}{2}\left(\dot{A}^i - \partial^i A^0\right)^2 - \frac{1}{4}F_{ij}F^{ij} + g\epsilon_{ijk}\dot{A}^i\partial_j\dot{A}^k . \end{aligned}$$

- The corresponding canonically conjugated momenta are

$$\Pi_0 = 0, \quad \Pi_i = \left(\delta_{ik} + 2g\epsilon_{ijk}\partial_j\right)\dot{A}^k + \partial_i A^0 .$$

- leading to the velocities

$$\begin{aligned} \dot{A}^i &= \frac{1}{(1+4g^2\nabla^2)}\left(\delta_{ij} - 2g\epsilon_{irj}\partial_r + 4g^2\partial_i\partial_j\right)\left(\Pi_j - \partial_j A^0\right) \\ \dot{A}^i &\equiv M_{ij}(\partial)(\Pi_j - \partial_j A^0) \end{aligned}$$

- The Hamiltonian is

$$\mathcal{H}_C = \frac{1}{2}\Pi_p M_{pr}(\partial)\Pi_r + \left(\partial_p\Pi_p + J^0\right)A^0 + \frac{1}{4}F_{ij}F^{ij} - J^i A^i$$

- The Gauss law $\partial_i \Pi_i + J_0 = 0$ is recovered. Separating the fields in transverse and longitudinal parts the gauge fixing can be done as

$$\Pi_0 = 0, \quad A_0 = -\frac{1}{\nabla^2} J_0,$$

$$\partial_k A^k = 0, \rightarrow A^i_L = 0, \quad \Pi^L_i = -\frac{1}{\nabla^2} \partial_i J^0.$$

- The independent transverse fields satisfy

$$\{A^i_T(x),\ \Pi^T_m(y)\} = \left(\delta_{im} - \frac{\partial_i \partial_m}{\nabla^2}\right) \delta^3(x-y).$$

- In terms of them the Hamiltonian density is

$$\mathcal{H}_C = \frac{1}{2} \Pi^T_p M_{pr}(\partial) \Pi^T_r - \frac{1}{2} J^0 \frac{1}{\nabla^2} J^0 + \frac{1}{4} F_{ij} F^{ij} - J^i_T A^i_T$$

- In order to reproduce the commutation relations for the field operators starting from the annhilation-creation operators of the normal mode expansion we need to have $\Pi = \dot{A}$ or, equivalently, the momentum contribution normalized to $\frac{1}{2}\Pi^2$. To this end we perform the following canonical transformation: $A^i \rightarrow \bar{A}^i, \Pi_i \rightarrow \bar{\Pi}_i$. Here we denote $\Delta = g^2 \nabla^2$.

$$A_T^i = \frac{\sqrt{1+\sqrt{(1+4\Delta)}}}{\sqrt{2}\sqrt{(1+4\Delta)}}\left[\delta_{iq} - \frac{2g}{\left(1+\sqrt{(1+4\Delta)}\right)}\epsilon_{imq}\partial_m\right]\bar{A}_T^q,$$

$$\Pi_r^T = \frac{\sqrt{1+\sqrt{(1+4\Delta)}}}{\sqrt{2}}\left[\delta_{rq} + \frac{2g}{\left(1+\sqrt{(1+4\Delta)}\right)}\epsilon_{rmq}\partial_m\right]\bar{\Pi}_q^T.$$

- **The non-interacting sector of the Hamiltonian density results**

$$\mathcal{H}_C = \frac{1}{2}\bar{\Pi}_p^T\bar{\Pi}_p^T + \frac{1}{2}\bar{A}_T^r\left(\frac{1}{1+4g^2\nabla^2}\right)\left(-\nabla^2\right)\left[\delta_{rp} - 2g\epsilon_{rnp}\partial_n\right]\bar{A}_T^p.$$

- **The mode expansion of $\bar{A}_T^k$ is**

$$\bar{A}_T^i(x) = \int\frac{d^3\vec{k}}{\sqrt{(2\pi)^3}}\sum_{\lambda=\pm}\sqrt{\frac{1}{2\omega_\lambda(\vec{k})}}\left[a_\lambda(\vec{k})\,\varepsilon^i(\lambda,\vec{k})e^{-ik^\lambda\cdot x} + h.c.\right],$$

where: $\lambda = \pm 1$, $a_\lambda(\vec{k}), a_\lambda^\dagger(\vec{k})$ are standard annihilation, creation operators and $\varepsilon^i(\lambda,\vec{k})$ defines a circularly polarized basis.

- The corresponding modified frequencies are

$$\omega_\lambda = \frac{|\vec{k}|}{\sqrt{1 + 2\lambda g |\vec{k}|}}, \qquad \lambda = \pm 1.$$

- One can verify that the normal ordered version of the free part of the above photon Hamiltonian is indeed

$$\mathcal{H}_{0\gamma} = \int d^3\vec{k} \sum_{\lambda=\pm} \omega_\lambda(\vec{k})\, a^\dagger_\lambda(\vec{k})\, a_\lambda(\vec{k}).$$

- Incorporating the Coulomb contribution, in momentum space the photon propagator is

$$\begin{aligned}
\Delta_{\mu\nu}(k) &= \frac{1}{\left((k^2)^2 - 4g^2 \left|\vec{k}\right|^2 k_0^4\right)} \Bigg[\eta_{\mu\nu}\left(k^2 - 4g^2 \left|\vec{k}\right|^2 k_0^2\right) \\
&\quad + 4g^2 \left|\vec{k}\right|^2 k_0^2 \delta_{\mu 0}\delta_{\nu 0} + 2ig n^\rho \epsilon_{\rho\mu\nu\sigma} k^\sigma \left|\vec{k}\right|^2\Bigg], \\
\Delta_{\mu\nu}(k) &= \Delta^*_{\nu\mu}(k)
\end{aligned}$$

THE REGULARIZATION PRESCRIPTION

- **Our hypothesis is that the MPM is an effective field theory continuosly interpolating between QED and a LIV perturbation of it. This means that at the quantum level we should recover the standard QED results when allowing $g \to 0$.**

- **The higher (second) order derivative character of the MPM in the free fermionic sector has to be properly taken into account by setting up a perturbation scheme which preserves the number of fermionic degrees of freedom of standard electrodynamics and provides a zeroth order Hamiltonian bounded from below.**

- **The behavior of the MDR indicates that we have the natural upper bound $|\vec{k}| = 2g$, where at least on of the frequencies goes to infinity.**

- **The effective character of the model is provided by the scale**

$$M <<< 1/2g, \qquad gM <<< 1,$$

which is incorporated via a Lorentz covariant *regulator* $F(k^2/M^2)$.

THE FERMION SELF ENERGY

- Let us start from

$$\Sigma(p) = e^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \frac{\gamma(p-k)+m}{(p-k)^2 - m^2 + i\epsilon} \gamma^\nu \Delta_{\mu\nu}(k)$$

- The above structure can be parameterized as

$$\begin{aligned} \Sigma(p) \quad = \quad & \tau_0 + \ (\gamma_0 p^0)\ \tau_1 + \ (\gamma_i p^i)\ \tau_2 + \\ & (\gamma^i \gamma_0 \gamma^j)\epsilon_{0ijm} p^m p^0 \ \tau_{31} + (\gamma^i \gamma^j)\epsilon_{0ijm} p^m \ \tau_{33} \\ & + (\gamma^i \gamma_r \gamma^j)\epsilon_{0ijm} \left(\delta^{mr} \tau_{321} + p^m p^r \ \tau_{322}\right) \end{aligned}$$

where each of the functions $\tau_{ij\ldots}$ depends on $p_0^2, \vec{p}^{\,2}$. We expand them in the corresponding powers.

- Recalling that $\Delta_{\mu\nu} \sim 1/k^2$ the LIV terms which might induce fine tuning are the constant pieces arising from $\tau_0, \tau_1, \tau_2, \tau_{33}$ and τ_{321}. These correspond to the zeroth and first order tems in the power expansion in the external momentum.

TERMS LINEAR IN THE GAMMA MATRICES

- They correspond to

$$\Sigma_L(p) = (\gamma_0 p^0)(\tau_1)_{p=0} + (\gamma_i p^i)(\tau_2)_{p=0} = (\gamma_0 p^0) B + (\gamma_i p^i) C$$
$$= (\gamma_0 p^0)(B - C) + (\gamma_\mu p^\mu) C$$

- We have

$$B-C = \frac{4e^2}{(2\pi)^4} \int d^4k \, \frac{k^2 \left(k_0^2 + \frac{1}{3}|\vec{k}|^2\right) + 4g^2 |\vec{k}|^2 k_0^2 \left(k^2 - m^2\right)}{(k^2 - m^2 + i\epsilon)^2 \left((k^2)^2 - 4g^2 |\vec{k}|^2 k_0^4\right)} \left(\frac{M^2}{M^2 - k^2}\right)$$

- The integrals can be exactly calculated by:

(1) Changing variables to $k_0 \to iq, \; k = |\vec{k}|$ and performing the angular integration, since the integrand depends only upon $q^2, \; k^2$. The integrations regions are $0 < k < \frac{1}{2g} \; , -\infty < q < +\infty$

(2) Going to polar coordinates in the (q, k) plane

$$q = r \cos\theta, \; k = r \sin\theta$$

- The required integrals are of the type

$$\int_0^{\frac{\pi}{2}} d\theta \left(- \sin^2\theta + \frac{4}{3} \sin^4\theta \right) \int_0^{\frac{1}{2g\sin\theta}} \frac{M^2 r^3 dr}{\left(r^2+m^2\right)^2 \left(1 - r^2 c^2\right)\left(M^2 + r^2\right)}$$

$$\int_0^{\frac{\pi}{2}} d\theta \sin^4\theta \cos^2\theta \int_0^{\frac{1}{2g\sin\theta}} \frac{M^2 r^3 dr}{\left(r^2+m^2\right)\left(1 - r^2 c^2\right)\left(r^2 + M^2\right)}$$

with $c = 2g\sin\theta\cos^2\theta$

- There is an apparent singularity in the upper limit of the r integration, near $\theta = 0$ arising from

$$\frac{1}{1 - c^2 r^2} \to \frac{1}{1 - \cos^4\theta} = \frac{1}{\left(1 + \cos^2\theta\right)\sin^2\theta}$$

which is in fact cancelled by the remaining $\sin^2\theta$ dependence.

- The final result is

$$\begin{aligned} B - C &= i\frac{4e^2}{\pi^3}\left[0.71(gM)^2 + 0.19(gM)^2\ln(gM)\right] \\ &\quad - i\frac{4e^2}{\pi^3}\left[0.03\,(gm)^2 + 0.19\,(gm)^2\ln(gm)\right] \end{aligned}$$

with $gm << gM << 1$.

- Recall $gM \sim \xi\frac{M}{M_P}$, where generically we have $\xi << 1$.

- The Lorentz covariant case is recovered by first taking $g = 0$ and subsequently $M \to \infty$.

- **THERE IS NO FINE TUNING !!!!!**

FINAL COMMENTS

- **Need to verify the additional possible dangerous terms which contain $\Sigma(0)$, $\partial\Sigma(0)/\partial p$, together with two and three gammas.**

- **Investigate whether one can smooth out the upper limit in the $|\vec{k}|$ integration via dimensional regularization, for example, in order to allow the momentum integral from $-\infty$ to $+\infty$.**

- **Incorporate a renormalization prescription in the model and understand its behavior regarding the characteristic energy scale M.**

- **Incorporate the fermions.**

- **Calculate physical effects which are non-zero in standard QED and verify that the regularizing prescription is adequate.**