Observational signatures in LQC?

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International Loop Quantum Gravity Seminar, March 29 2011

Talk based on: I.A., A. Ashtekar, W. Nelson: IN PROGRESS!

1. Inflation and the generation of cosmic inhomogeneities

2. Loop Quantum Cosmology and Inflation

3. Concrete proposal: Observational signatures of LQC?

I. INFLATION AND THE GENERATION OF COSMIC INHOMOGENEITIES

- **Motivation (A.Guth '81): an exponential expansion of the early universe can solve the problems of the Big Bang model related with initial conditions (flatness, horizon problem)**

- **Criticized by prominent relativists. Then, why Inflation is so "popular": cosmic inhomogeneities**

> **Inflation provides a compelling argument, rooted on profound ideas on QFT in CST, for the origin of the cosmic inhomogeneities (CMB and galaxy distribution) as amplification of <u>vacuum</u> fluctuations**

I.1 Inflationary background

<u>Assumption</u>:$\exists \ \phi(\vec{x}, t)$, the inflaton, with a suitable $V(\phi)$, and there was a phase in the early universe dominated by $V(\phi)$

$$\phi(\vec{x}, t) = \phi_0(t) + \delta\phi(\vec{x}, t) \qquad \phi_0 \gg \delta\phi(\vec{x}, t) \quad \longrightarrow \quad \text{perturbation theory}$$

At zeroth order in perturb.

$$\begin{cases} \rho = 1/2\dot{\phi}_0^2 + V(\phi_0) \\ \\ p = 1/2\dot{\phi}_0^2 - V(\phi_0) \end{cases} \xrightarrow{\dot{\phi}_0 \ll V(\phi_0)} \begin{cases} T_{\mu\nu} = -V(\phi_0)\, g_{\mu\nu} \\ \\ \Lambda_{eff} = 8\pi G\, V(\phi_0) \end{cases} \longrightarrow a(t) \sim e^{Ht}$$

Slow-roll parameters: $\qquad \epsilon \equiv -\dfrac{\dot{H}}{H^2} \ll 1 \qquad\qquad \eta \equiv \dfrac{\ddot{H}}{2\dot{H}H} \ll 1$

I. 2 Cosmic inhomogeneities

$$\phi(\vec{x}, t) = \phi_0(t) + \delta\phi(\vec{x}, t)$$

- $\phi_0(t)$ **classical source producing the inflationary background**

- $\delta\phi(\vec{x}, t)$ **quantum filed propagating in the background produced by** $\phi_0(t)$

1) **The inflationary background expansion is able to amplify vacuum fluctuations of the field** $\delta\phi(\vec{x}, t)$ **: scalar perturbations ⟶ density perturbations**

2) **Inflation also amplifies vacuum fluctuations of metric tensor perturbations (gravity waves): primordial tensor perturbations**

These primordial perturbations are the "seeds" of the cosmic inhomogeneities that we observe today (CMB and galaxy distribution)

Scalar perturbations

$$\mathcal{R}(\vec{x}, t) = \Psi(\vec{x}, t) + \frac{H}{\dot{\phi}_0} \delta\phi(\vec{x}, t)$$ comoving curvature perturbations (gauge invariant)

Homogeneity : $\hat{\mathcal{R}}(\vec{x}, t) = \int d^3k \; \hat{\mathcal{R}}_{\vec{k}}(t) \; e^{i\vec{k}\vec{x}}$ where: $\hat{\mathcal{R}}_{\vec{k}}(t) = A_{\vec{k}} \mathcal{R}_k(t) + A^{\dagger}_{-\vec{k}} \mathcal{R}_k(t)^*$

● **Spacetime symmetries** (de Sitter): $\mathcal{R}_k(\tau) = \sqrt{\dfrac{-\tau\pi}{4(2\pi)^3 z(\tau)}} H^{(1)}_{\mu}(-k\tau)$ (1)

where: $z(\tau) = a(\tau)\dot{\phi}_0/H$ and $\mu = 3/2 + 2\epsilon + \eta$

● **Vacuum state:** $A_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k}$ **Bunch-Davies vacuum (adiabatic)**

NOTE: for observational purposes we only need to make assumptions about the range of modes that we are able to observe in the present universe $(k_0, 200k_0)$

Spectrum of fluctuations (in momentum space)

1) $\langle 0|\hat{\mathcal{R}}_{\vec{k}}(t)|0\rangle = 0$

Power Spectrum

2) $\langle 0|\hat{\mathcal{R}}_{\vec{k}}(t)\hat{\mathcal{R}}_{\vec{k}'}(t)|0\rangle = \delta(\vec{k}+\vec{k}')|\mathcal{R}_k(t)|^2 \equiv \delta(\vec{k}+\vec{k}')4\pi k^3 \Delta_{\mathcal{R}}^2(k,t)$

Let us call t_k the "Hubble exit time" for the mode k defined as: $\lambda(t_k) = R_H \equiv \dfrac{1}{H}$

Then, at $t \gg t_k$

$$\Delta_{\mathcal{R}}^2(k) = \frac{1}{\pi m_{Pl}^2 \epsilon(t_k)} H(t_k)^2$$

Power Spectrum

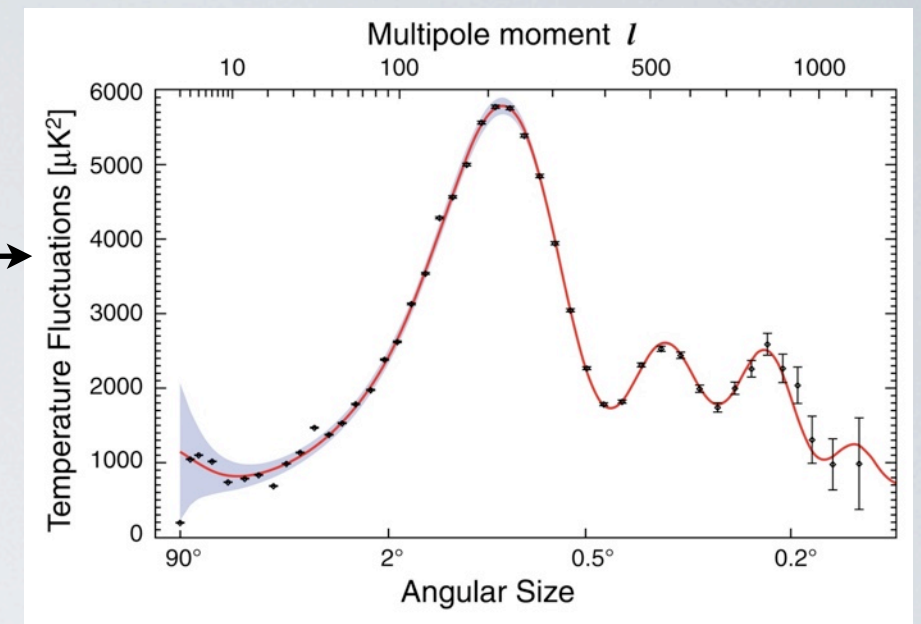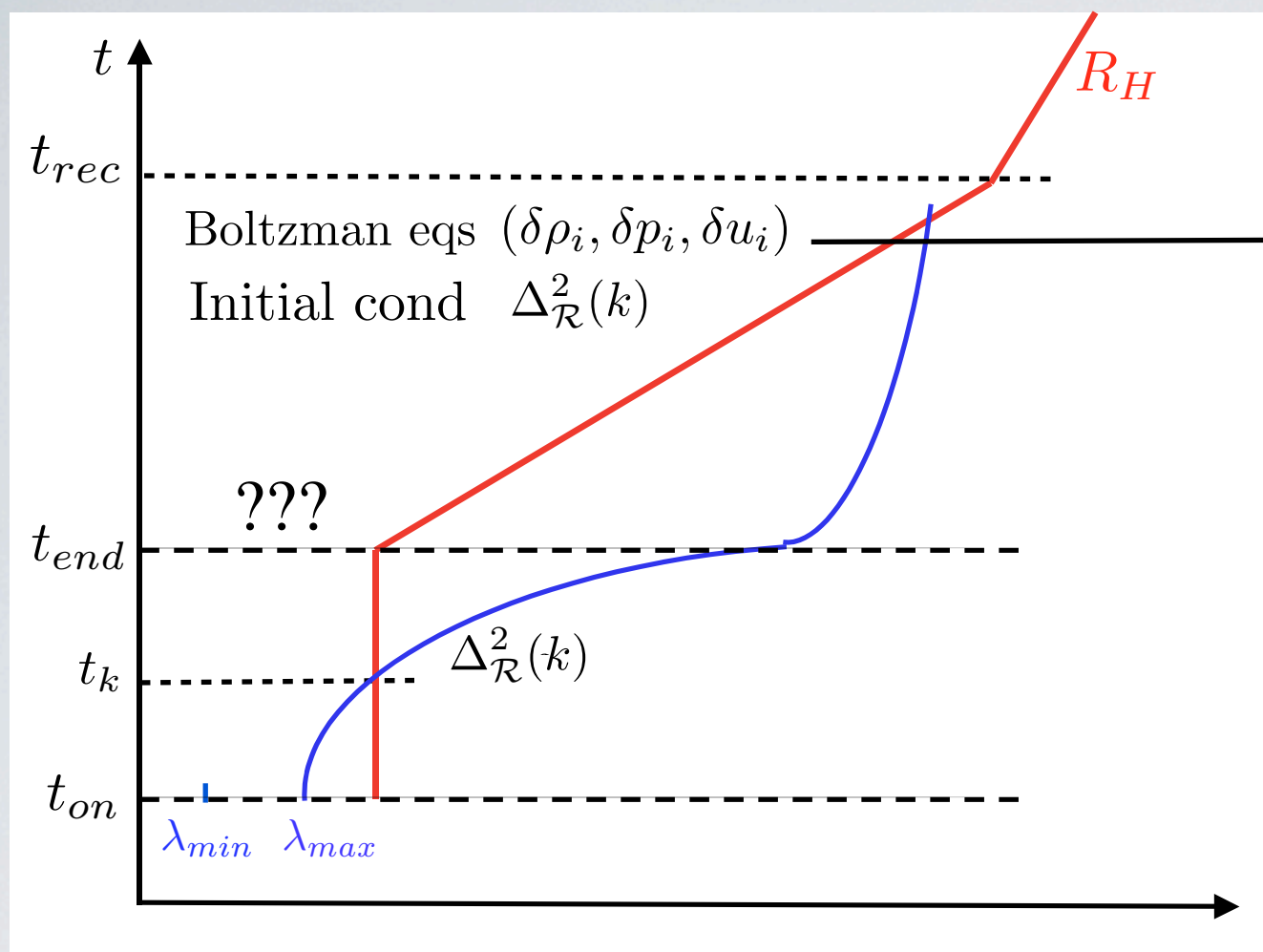$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2(k)}{d\ln k} = -4\epsilon - 2\eta$$

Spectral index

NEARLY SCALE-INVARIANT SPECTRUM

Observations (7 years WMAP): Nearly scale-invariant spectrum

$$\Delta_{\mathcal{R}}^2(k_p) = (2.43 \pm 0.091)10^{-9} \qquad n_s - 1 = 0.032 \pm 0.012$$

Main assumptions:

1) $\exists \; \phi(\vec{x}, t)$ **and a phase of** $V(\phi)$ **domination**

2) **At the onset of inflation all the modes of observational interest were well inside the Hubble radius and above the Planck length**

3) **Scalar perturbations were in the vacuum state at the onset of inflation**

Assumptions ad-hoc, but what one gets is far more than what one puts in. There must be a germ of truth

2. I Inflationary background

LQC: A coherent picture of the early universe has emerged in the last few years:

1) **Cosmological singularities have been shown to be resolved in a variety of models** (Ashtekar, Bentivegna, Bojowald, Corichi, Garay, Mena-Marugan, Martin-Benito, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

2) **FLRW models have been studied most extensively, using analytical and numerical methods** (Ashtekar, Corichi, Pawlowski, Singh).

3) **The Big Bang singularity is replaced by a quantum Bounce.**

4) **The Bounce is followed by a robust phase of superinflation** $(\dot{H} > 0)$

5) **Interestingly, the full quantum dynamics, including the Bounce, is well-approximated by effective equations** (Ashtekar, Bojowald, Corichi, Pawlowski, Singh, Taveras)

$$H^2 = \frac{8\pi G}{3} \rho \left[1 - \frac{\rho}{\rho_c} \right]$$

Is inflation natural in LQC?

Let us <u>assume</u> $\exists\ \phi(\vec{x}, t)$ and $V(\phi)$

Let us allow ALL possible initial condition for ϕ_0 at the bounce, subject to the Hamiltonian constraint

QUESTION: are most of the trajectories such that slow-roll inflation compatible with observations (within the error bars) happens??

A. Ashtekar & D. Sloan:

- **For a broad family of $V(\phi)$, almost all dynamical trajectories undergo a period of slow-roll inflation compatible with WMAP observations (within the error bars)**

- **The probability of having inflation compatible with observations is greater than 0.999997 (for a quadratic potential)**

Then, is inflation inevitable in LQC??? NO.
It is inevitable only under the assumption that there is a phase in which matter density is dominated by a scalar field in a suitable potential.

2. LOOP QUANTUM COSMOLOGY AND INFLATION

- **At first seems bad news: there are not significant deviation with respect the inflationary predictions for the background**

- **In fact is good news: it opens a window to study perturbations on the inflationary background ➡ contact with observations**

2. 2 Perturbations in inflation

Lot of work, examples of ideas recently pushed:

1) Barrau, Cailleteau, Grain, Gorecki, Mielczarek : **holonomy corrections in tensor perturbations**

2) Bojowald, Calcagni, Tsujikawa: **inverse volume corrections in scalar and tensor perturbations.**

➡ **LQC effects can be incorporated into the study of perturbations <u>during</u> inflation, and corrections to the inflationary predictions can be obtained!**

CMB observations can be use to constrain the LQC corrections!

But, $\rho_I \approx 10^{-11}\rho_{Pl}$. This constitutes a huge handicap to make LQC corrections relevant <u>during</u> inflation.
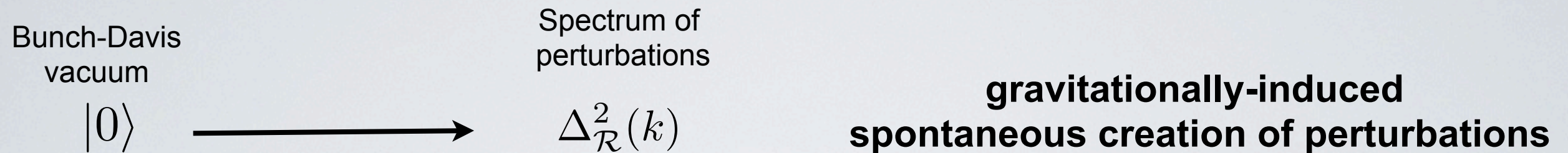
However, <u>before</u> inflation quantum gravity corrections can be important. In fact they <u>dominate</u> near the bounce.

IDEA: Can the pre-inflationary history of the universe, *<u>when quantum gravity corrections are of order one</u>*, modify the *<u>initial conditions</u>* for perturbations?

- Remember: Bunch-Davis vacuum for perturbations is assumed at the onset of inflation. Why the vacuum at this intermediate time???

- Old argument: the exponential expansion of the inflationary universe will DILUTE any possible quanta present in the initial state, and will drive any arbitrary state to the vacuum. Seems like bad news...

But, is this really true? The answer is NO.

Bunch-Davis
vacuum

Spectrum of
perturbations

$|0\rangle$ $\longrightarrow$ $\Delta_{\mathcal{R}}^{2}(k)$

**gravitationally-induced
spontaneous creation of perturbations**

L. Parker, PhD Thesis, Harvard University, 1966

Generically, spontaneous creation of quanta is accompanied by the corresponding *stimulated* (or induced) counterpart

I.A. and L. Parker (PRD 2011)**: if there are perturbation quanta present at the onset of inflation** ➡️ ***gravitationally-induced stimulated creation of quanta***

- **The stimulated creation process is enhanced by the same amplification factor as is responsible for the spontaneous creation from the vacuum**

- **This amplification factor grows enormously during inflation and compensates for the dilution of the particles initially present, keeping their average number density constant during inflation**

Non-vacuum state at the onset of inflation ➡️ **observable consequences**

Initial state

$$\rho = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \cdots p(n_1(\vec{k}_1), n_2(\vec{k}_2), \ldots) \, |n_1(\vec{k}_1), n_2(\vec{k}_2), \ldots\rangle \, \langle n_1(\vec{k}_1), n_2(\vec{k}_2), \ldots| \qquad (1)$$

with

$$\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \cdots p(n_1(\vec{k}_1), n_2(\vec{k}_2), \ldots) = 1 \qquad (2)$$

- **Theoretical constraints: Renormalizability and backreaction from ρ :**

$$\left\{ \begin{array}{l} \langle N_{\vec{k}} \rangle \equiv \mathrm{Tr}[\rho N_{\vec{k}}] \;\; \text{has to fall of as } 1/k^{4+\delta} \text{ with } \delta > 0 \\[2em] \mathrm{Tr}[\rho N_{\vec{k}}] \;\; \text{has to be not much larger than } \mathcal{O}(1) \end{array} \right.$$

- **Predictions for the Power Spectrum:**

$$\Delta_{\mathcal{R}}^2(k)^{\rho} = \Delta_{\mathcal{R}}^2(k)^0 (1 + 2\,\mathrm{Tr}[\rho N_{\vec{k}}]) \qquad (3)$$

$$n_s^{\rho} - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2(k)^{\rho}}{d\ln k} = n_s^0 - 1 + \frac{d\ln(1 + 2\,\mathrm{Tr}[\rho N_{\vec{k}}])}{d\ln k} \qquad (4)$$

- **Observations** $\Longrightarrow$ $\left| \dfrac{d\ln(1 + 2\,\mathrm{Tr}[\rho N_{\vec{k}}])}{d\ln k} \right| \sim O(\epsilon)$

The power spectrum has limited potential in revealing information about the initial state

But we can go beyond the power spectrum:

$$\langle \hat{\mathcal{R}}_{\vec{k}_1}(t)\hat{\mathcal{R}}_{\vec{k}_2}(t)\hat{\mathcal{R}}_{\vec{k}_3}(t)\rangle = \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)B^{\rho}_{\mathcal{R}}(\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

Non-Gaussianities

$$B^{\rho}_{\mathcal{R}}(\vec{k}_1,\vec{k}_2,\vec{k}_3)$$ **Bispectrum**

Non-gaussianities have been proven to be a sharp tool to test many aspects of the inflationary dynamics: Single-field, Lagrangian, slow-roll, etc.

Squeezed configuration $k_1 \approx k_2 \ll k_3$ ➡ $B(\vec{k}_1,\vec{k}_2,\vec{k}_3) = f_{NL}\left[\frac{\Delta^2_{\mathcal{R}}(k_1)}{4\pi k_1^3}\right]\left[\frac{\Delta^2_{\mathcal{R}}(k_3)}{4\pi k_3^3}\right]$

The presence of particles at the onset of inflation produces an enhancement of the bispectrum in the squeeze configuration

$$f^{\rho}_{NL} = \frac{5}{6}\epsilon\frac{k_1}{k_3}\frac{2\mathrm{Tr}[\rho N_{\vec{k}_1}N_{\vec{k}_2}] + \mathrm{Tr}[\rho N_{\vec{k}_1}] + \mathrm{Tr}[\rho N_{\vec{k}_2}]}{\mathrm{Tr}[\rho(2N_{\vec{k}_1}+1)]\mathrm{Tr}[\rho(2N_{\vec{k}_2}+1)]}$$

➡ $\frac{f^{\rho}_{NL}}{f^0_{NL}} \approx \mathcal{O}(100)$

$$f^0_{NL} = \frac{5}{12}(4\epsilon + 2\eta)$$

The forthcoming observations of the CMB and galaxy distribution are going to be sensitive to $f_{NL}(k_1,k_3)$ **(size, sign, k-dependence....)**

By observing the non-gaussianities in the CMB and galaxy distribution we can get detailed information about the state of scalar perturbations at the onset of inflation

1) **The quantum state describing scalar perturbations at the onset of inflation depends on the pre-inflationary history**

2) **LQC provides a complete picture of the pre-inflationary universe**

3) **From the GR perspective most natural to specify the initial state at the Big Bang, but not possible**

4) **In LQC more natural to specify the initial state at the Bounce**
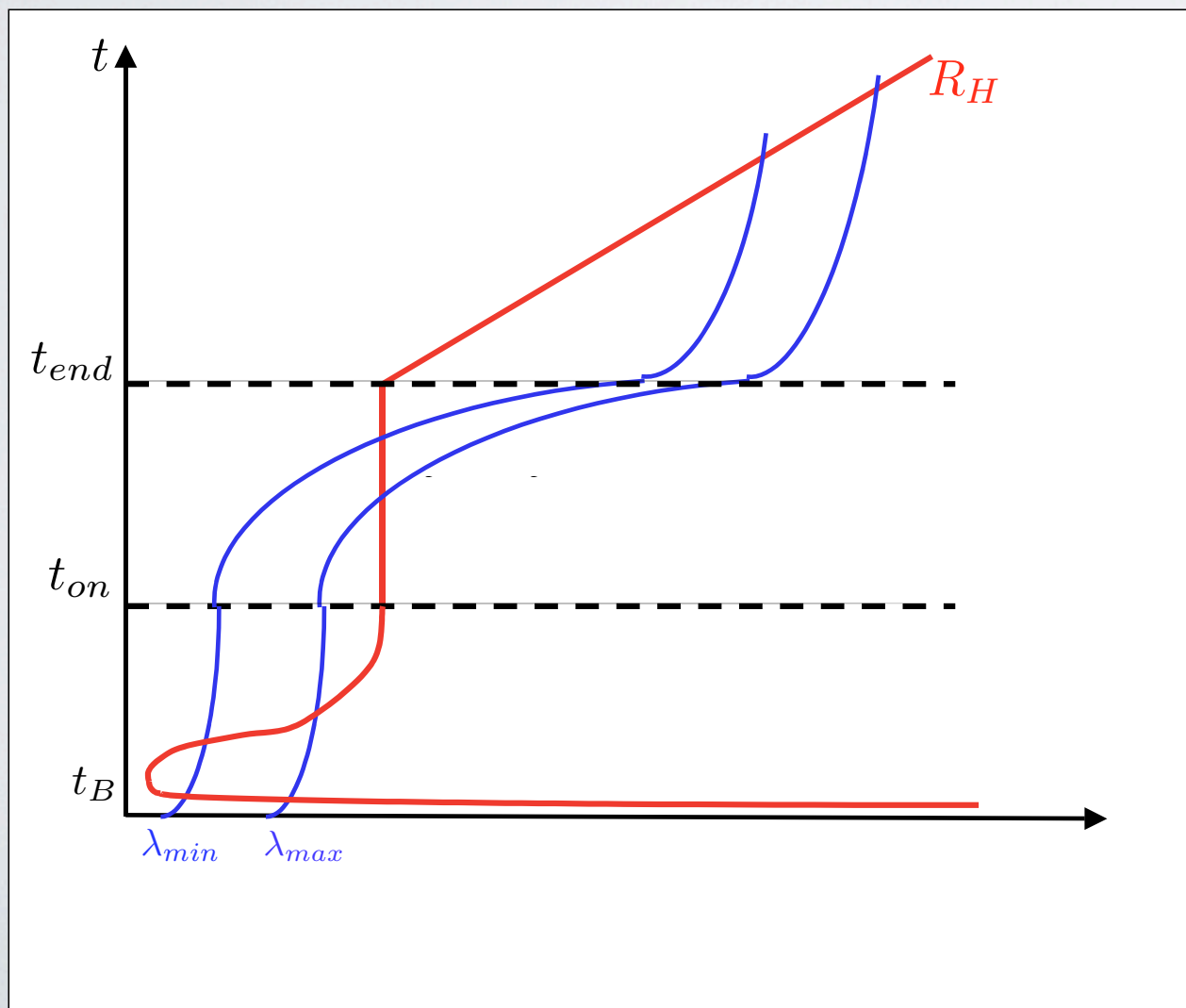
FRAMEWORK:

- Strictly one should do QFT in QST. The framework exists (Ashtekar, Kaminski, Lewandowski) and there is a program to use it in the inflationary context.

- Reasonable to use in the first step: study perturbations on the effective spacetime of LQC that replaces FLRW (smooth metric but coefficients containing important quantum corrections)

What about LQG corrections to the perturbation equations? Strategy:

- Choose a background solution (i.e. the value of ϕ_0 at the Bounce) such that the physical wave-length of ALL observationally relevant modes is always well above the Planck length

- For instance: for the reasonable value $\phi_0(t_B) = 0.95\, m_{Pl} \implies \lambda(t_B) \in [2000, 20]\ell_{Pl}$

  Reasonable to assume in the first step that LQC quantum corrections to the evolution of these modes will be subdominant

Initial conditions at the bounce:

- **PROBLEM: In QFT in CST it is known that for** $g^{ab}\nabla_a\nabla_b\hat{\phi} = 0$ **there is a problem with constructing a FOCK space at a time instant. "Natural" Fock rep at a given time is unitarily inequivalent to that at another instant: the vacuum evolves to a state with infinite number of particles at an arbitrarily small future time.**

- **Neat solution from the Iberian group** (Cortez, Mena-Marugan, Olmedo and Velinho)**:**

  1) **The quantum field theory problem is equivalent to the QFT of the field satisfying** $\eta^{ab}\partial_a\partial_b\hat{f} + m^2(t)\hat{f} = 0$ **in flat spacetime!**

  2) **Furthermore, for this** $\hat{f}$ **THERE IS a canonically defined Fock space rep. Vacuum at any time evolves to a well defined number of particles at any other later instant of time.**

  Proposal: use this quantization for quantum perturbations on the effective spacetime of LQC.

The million dollars question(s):

If we start with the vacuum state at the **bounce time** and evolve it:

1) **Will it give a state close to the Bunch-Davis vacuum at the onset of inflation? Or will it be observationally ruled out right away?**

Unclear because the density drops 11 order of magnitude and there is a lot of time for LQC dynamics to make a difference $\sim 10^4 - 10^5$ Planck seconds

2) **Will it be so close that we won't be able to distinguish it form the B-D vacuum?**

This is what cosmologists generally assume

Our claim is:

There is a window of background spacetimes (initial values of ϕ_0 at the bounce) compatible with observations (Observations can constrain what happen at the bounce!!)

In this window, the state is NOT the Bunch-Davis vacuum at the onset of inflation: there are particles

The average number of particles in the mode k can be computed using the technique of Bogoliubov transformations in CST

$$\hat{\mathcal{R}}(\vec{x}, t) = \int d^3k \, \hat{\mathcal{R}}_{\vec{k}}(t) \, e^{i\vec{k}\vec{x}} \qquad \textbf{where:} \qquad \hat{\mathcal{R}}_{\vec{k}}(t) = A_{\vec{k}}\mathcal{R}_k(t) + A^{\dagger}_{-\vec{k}}\mathcal{R}_k(t)^*$$

● Let us call $\mathcal{R}_k(t)$ the mode function corresponding to the notion of vacuum state at the bounce

$$A_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k} \qquad \blacktriangleright \qquad |0\rangle \qquad \text{vacuum at the bounce}$$

● Let us call $\mathcal{R}_k^{BD}(t)$ the mode function corresponding to the Bunch-Davis vacuum during inflation
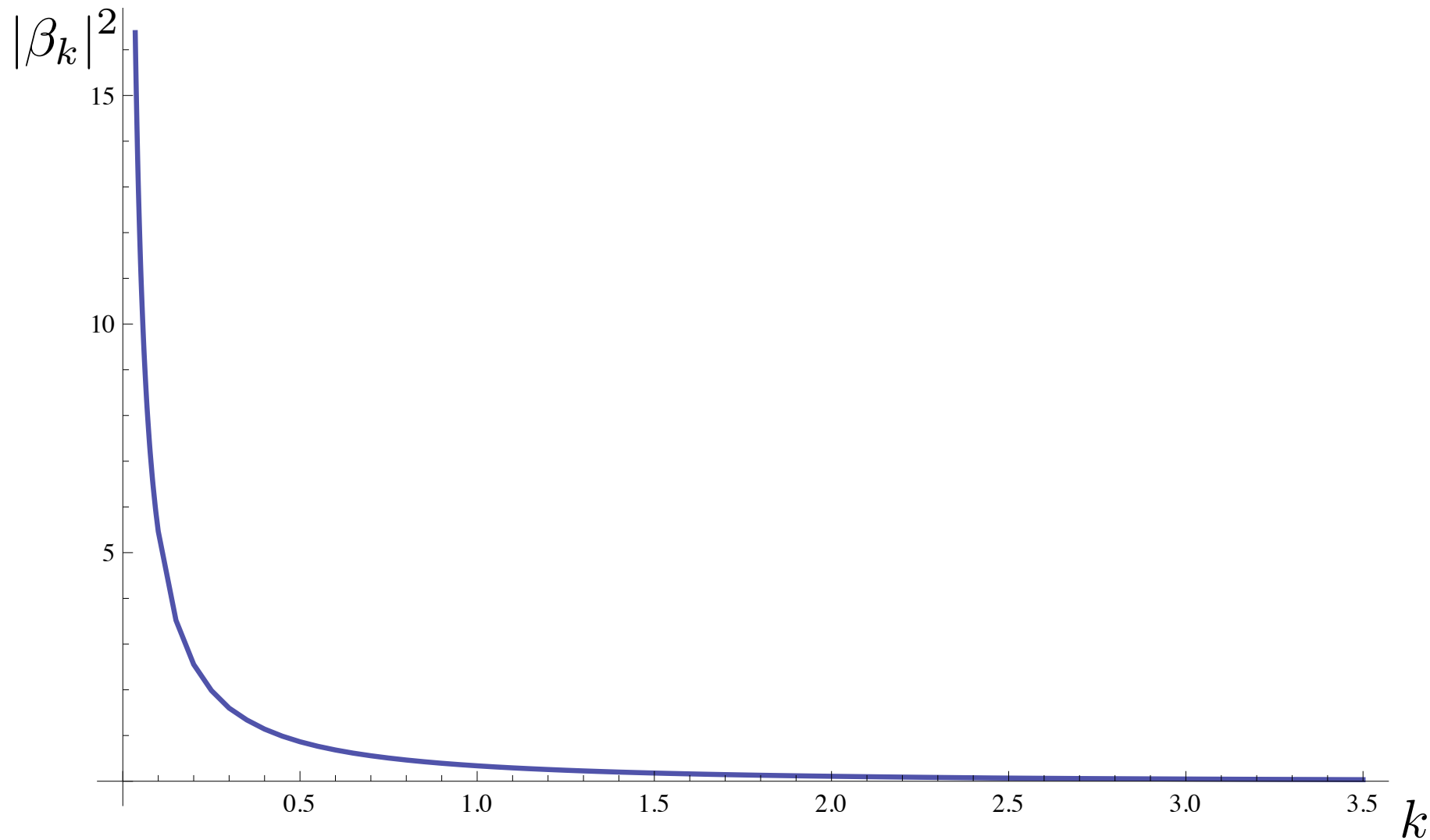
During inflation we can write $\quad \mathcal{R}_k(t) = \alpha_k \mathcal{R}_k^{BD}(t) + \beta_k (\mathcal{R}_k^{BD})^*(t)$

where: $\quad |\alpha_k|^2 - |\beta_k|^2 = 1 \qquad \alpha_k, \beta_k \quad$ Bogoliubov coefficients

▶ **Average number of particles created in the mode k :** $\quad \langle N_{\vec{k}} \rangle = |\beta_k|^2$

Preliminary computation

Satisfies all the constraints and is big enough to produce observable signatures!!

Inflationary scenario + QFT in CS:

1) **Inflation provides a compelling mechanism that satisfactorily accounts for the primordial cosmic perturbations responsible for the inhomogeneities in the present universe (CMB and galaxy distribution)**

2) **The stimulated creation of quanta during inflation makes the inflationary predictions sensitive to the quantum state of perturbations at the onset of inflation**

3) **By observing non-gaussianities in CMB and galaxy distribution in the squeezed configuration we can obtain detailed information about the initial state**

Quantum Cosmology:

4) **By assuming $\exists\ \phi(\vec{x}, t)$ and $V(\phi)$, a phase of slow-roll inflation compatible with observations naturally appears in LQC**

5) **The main features of the pre-inflationary history of the universe, when quantum gravity corrections dominate, can be imprinted in the state of perturbations at the onset of inflation**

6) **Observations of non-uniformities of the universe open a window to explore phenomenological aspects of QFT in CST and quantum gravity**

7) **We have presented a first attempt to quantitatively analyze those questions**

This trip across the largest possible range of physical scales, from the Planck scale to cosmological observations, is the most fascinating adventure that physics can face