Entropy of Hawking Radiation

Ahmed Almheiri (IAS) — ILQG Panel — 11/30/21

Central Dogma of Black Holes

- A black hole is a quantum system with

1. $\frac{\text{Area}_H}{4G_N}$ number of d.o.f.
2. Evolves unitary under time evolution

Is this assumption justified?

What we *know* about black holes

Black holes have *thermal entropy*

…assigned by the *gravity path integral*

- Canonical partition function; sum over the phase space

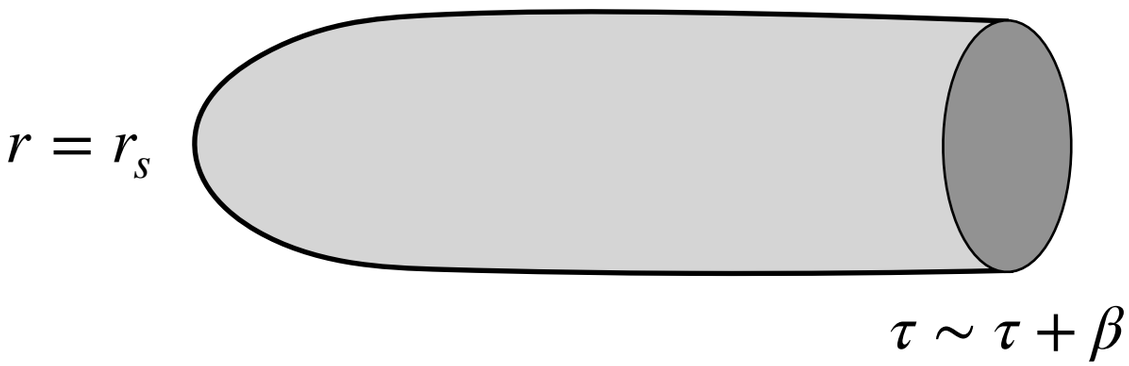
$$Z[\beta] = \mathrm{Tr}[e^{-\beta H}]$$

…assigned by the *gravity path integral*

- Canonical partition function; sum over the phase space

$$Z[\beta] = \mathrm{Tr}[e^{-\beta H}] = \int Dg_{\mu\nu} e^{-I[g]} \times Z_{\mathrm{m}}[g] \supset e^{-I[g_{cg}]} \times Z_{\mathrm{m}}[g_{\mathrm{cg}}]$$

Boundary condition: $M_{d+1} \sim S^1 \times R^d$



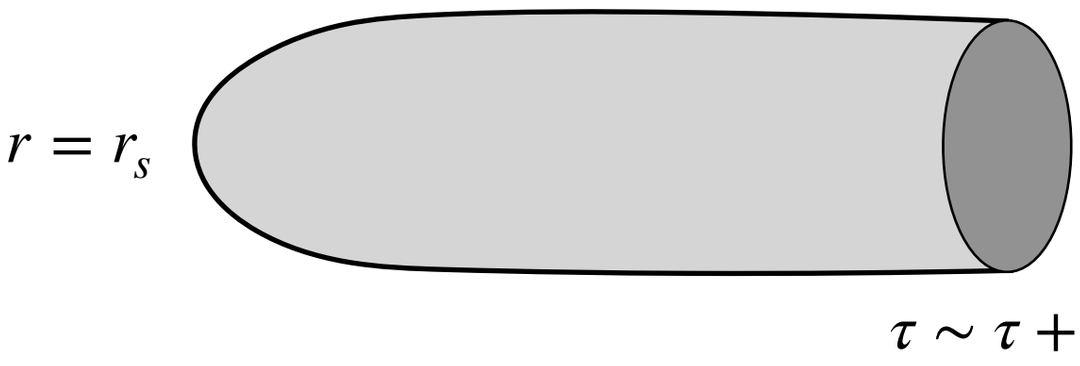
$$ds^2 = \left(1 - \frac{r_s}{r}\right) d\tau^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

…assigned by the *gravity path integral*

- Canonical partition function; sum over the phase space

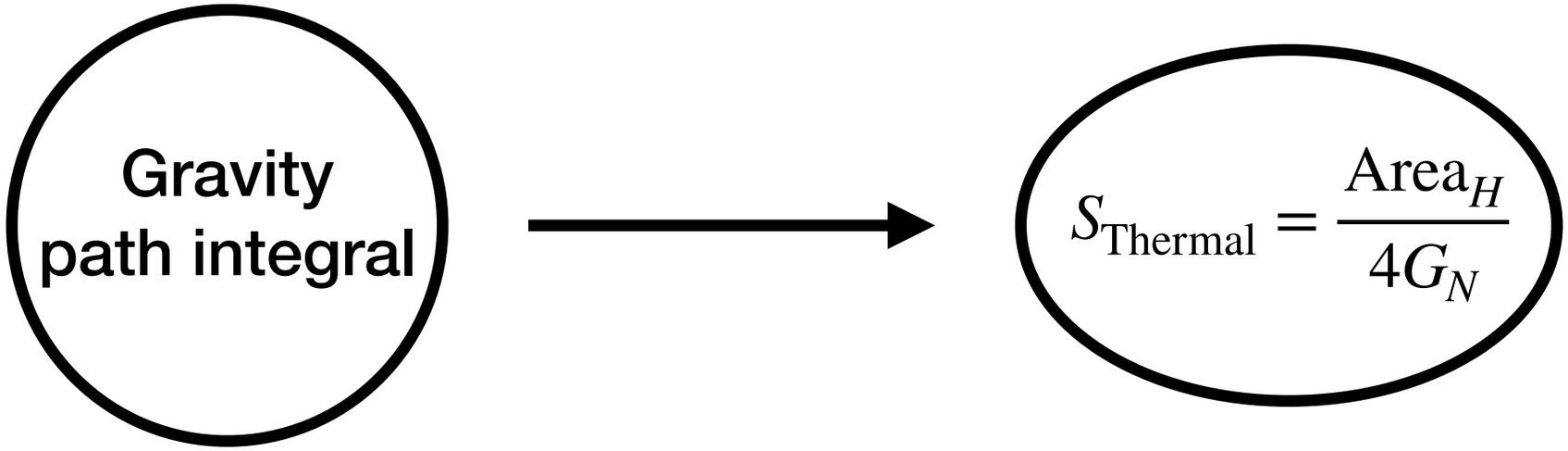
$$Z[\beta] = \mathrm{Tr}[e^{-\beta H}] = \int Dg_{\mu\nu} e^{-I[g]} \times Z_{\mathrm{m}}[g] \supset e^{-I[g_{cg}]} \times Z_{\mathrm{m}}[g_{\mathrm{cg}}]$$

Boundary condition: $M_{d+1} \sim S^1 \times R^d$



$$ds^2 = \left(1 - \frac{r_s}{r}\right) d\tau^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

- Thermal entropy: $S_{\mathrm{Th}} = (1 - \beta\partial_\beta) \ln Z[\beta] = \dfrac{\mathrm{Area}_H}{4G_N} + S_{\mathrm{m,out}}[\mathrm{bh}]$



- GPI suggests that a black hole has $\frac{\text{Area}_H}{4G_N}$ number of d.o.f.

Quantum black holes

- Hilbert space of a quantum black hole would have dimension

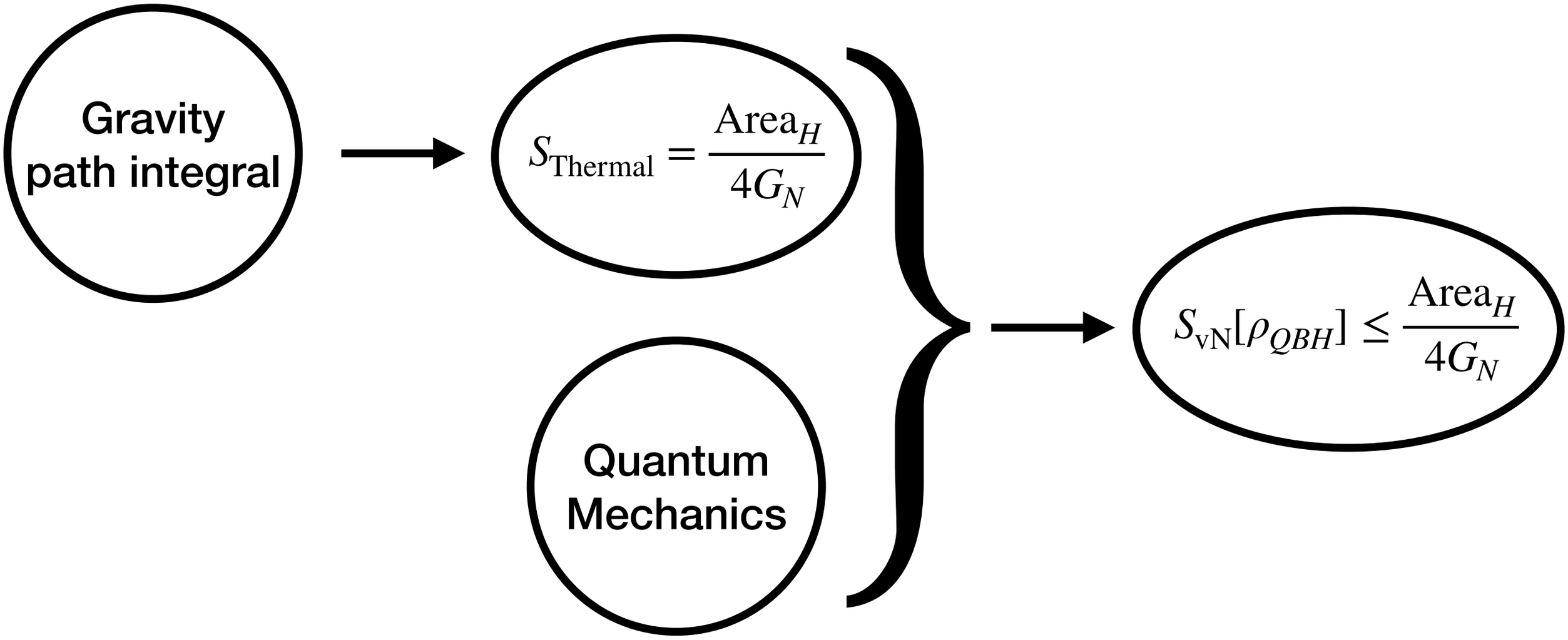
$$|\mathscr{H}_{QBH}| = \frac{\text{Area}_H}{4G_N}$$

- This bounds its fine-grained (von Neumann) entropy

$$S_{\text{vN}}[\rho_{QBH}] \equiv -\,\text{Tr}\left[\rho_{QBH} \ln \rho_{QBH}\right] \leq \frac{\text{Area}_H}{4G_N}$$

Theorem: $S_{\text{vN}} \leq S_{\text{Thermal}}$

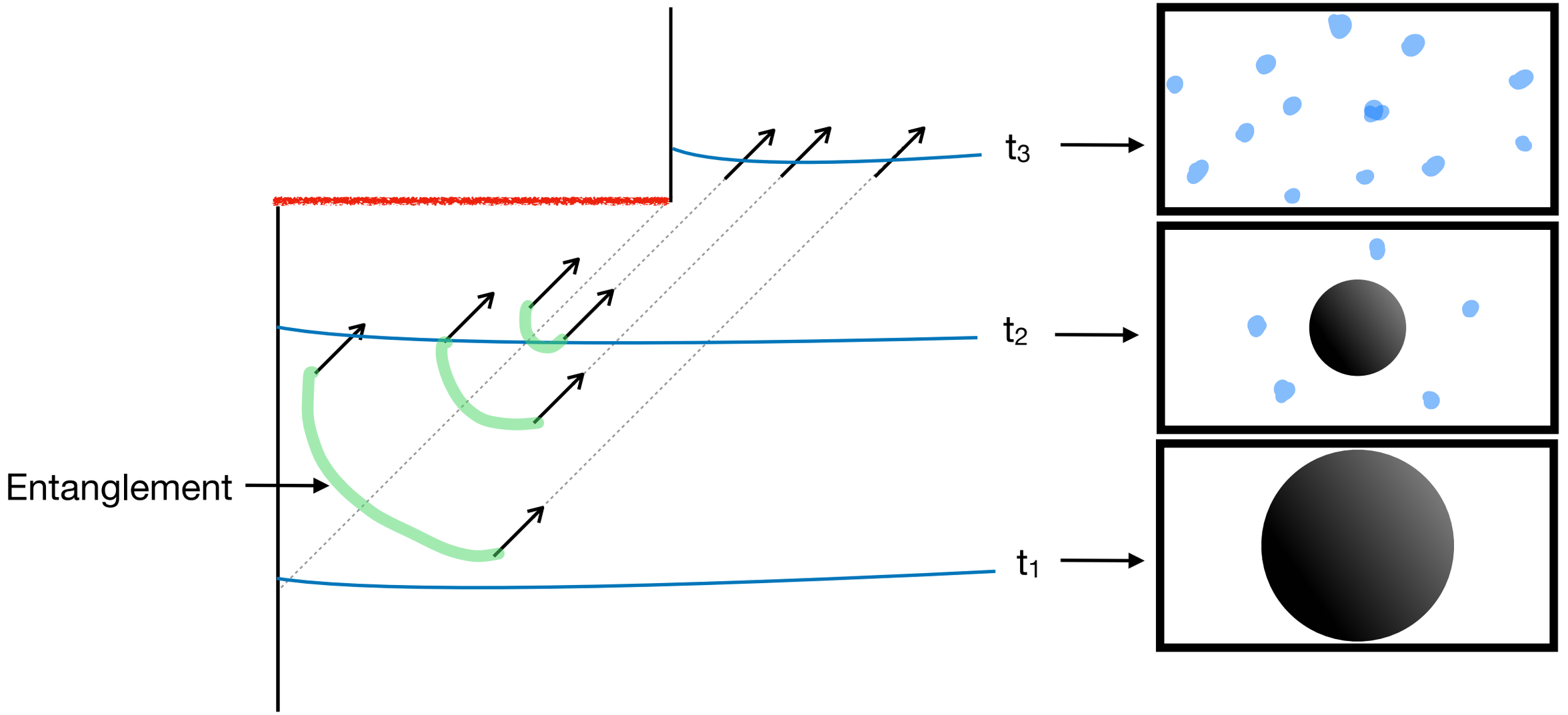
- Von Neumann entropy measures uncertainty in a quantum state
  => Maximum uncertainty is the total number of possible states.


Gravity
path integral
$S_{\text{Thermal}} = \frac{\text{Area}_H}{4G_N}$
Quantum
Mechanics
$S_{\text{vN}}[\rho_{QBH}] \leq \frac{\text{Area}_H}{4G_N}$

Black holes *evaporate*

…by emitting *Hawking radiation*

- Pair-creation at the event horizon

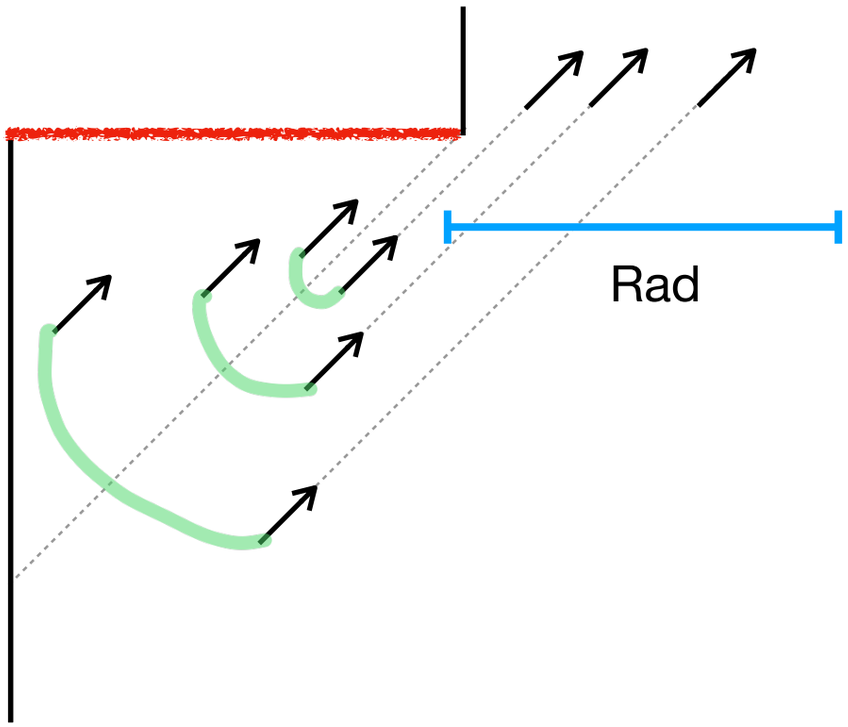


- Vacuum at the horizon = entangled state in Hawking mode basis

Hawking's *assumption*

- Two steps involved in Hawking's calculation:

    1. Matter state prepared on top of black hole saddle: $\Psi_{\mathrm{m}}[g_{BH}]$
    2. Compute matter entropy

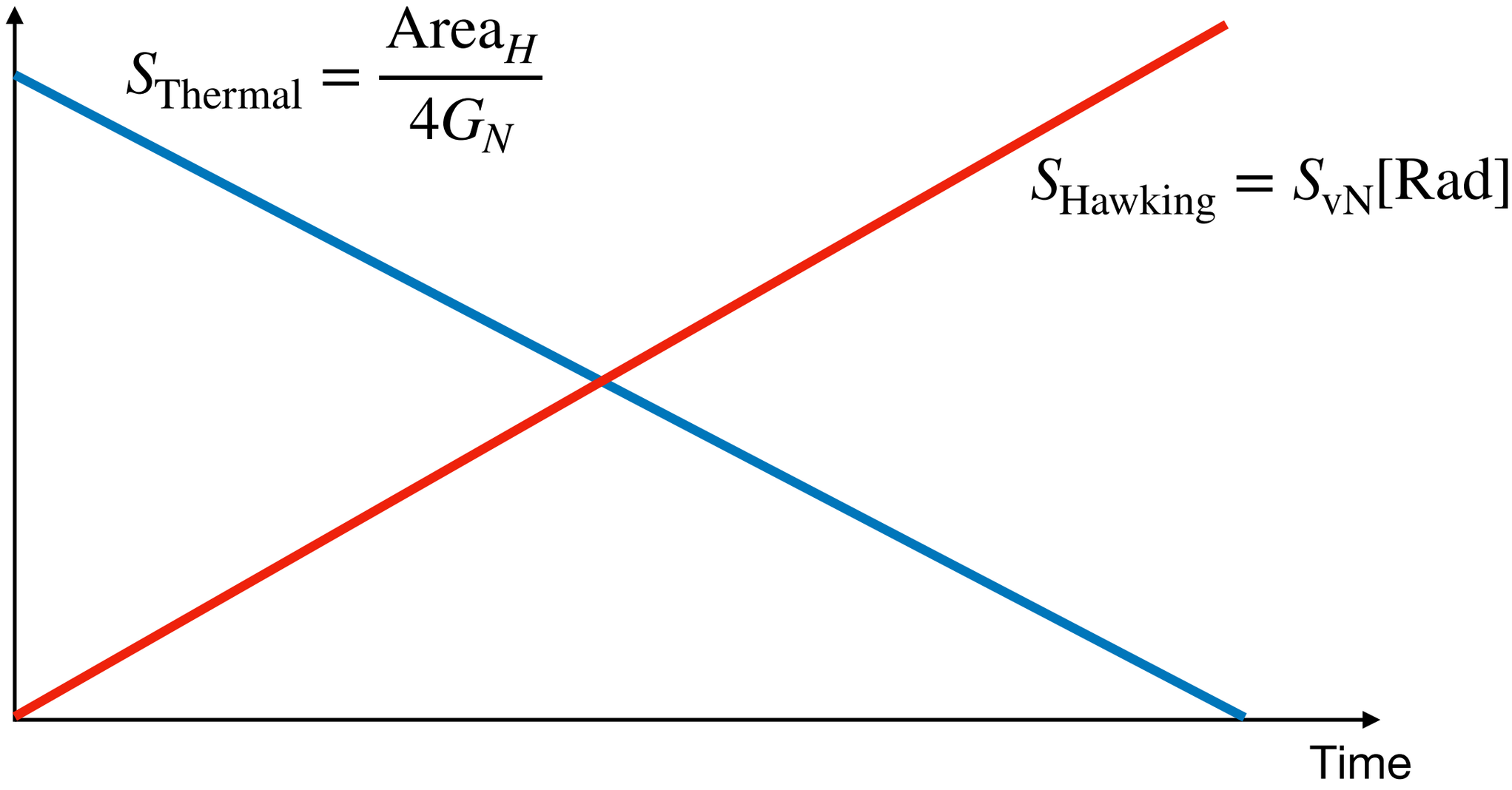


$$S_{\mathrm{Hawking}}[\mathrm{Rad}] = S_{\mathrm{m}}[\mathrm{Rad}]$$

$$\rho_{\mathrm{Rad}} \to \langle \rho_{\mathrm{Rad}} \rangle_{g_s} \qquad S_{\mathrm{Hawking}} = -\,\mathrm{tr}\left[\langle \rho_{\mathrm{Rad}} \rangle_{g_s} \ln \langle \rho_{\mathrm{Rad}} \rangle_{g_s} \right]$$

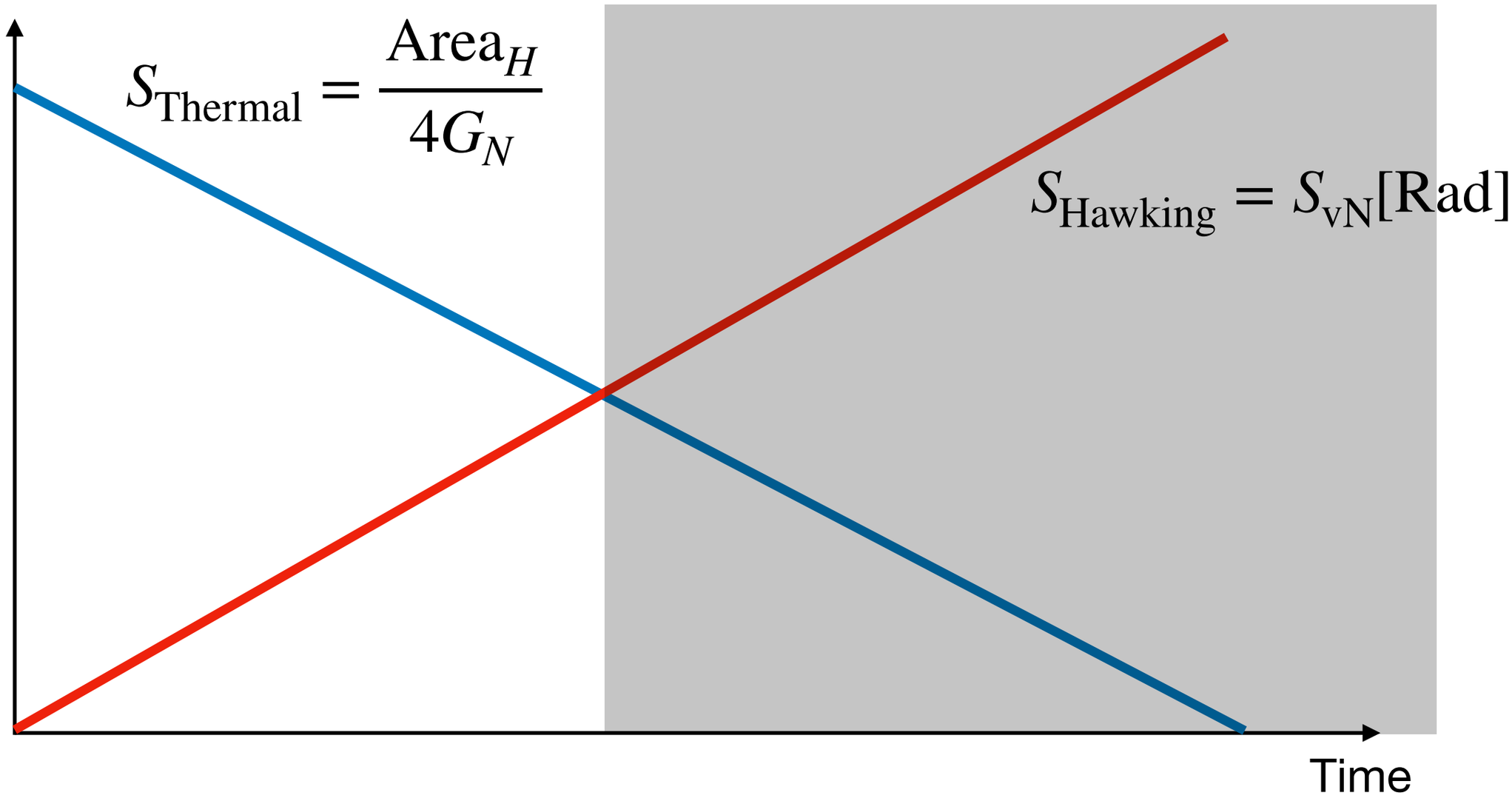
Black holes get *over-filled*

- Entanglement between black hole and radiation gets too large



Black holes get *over-filled*

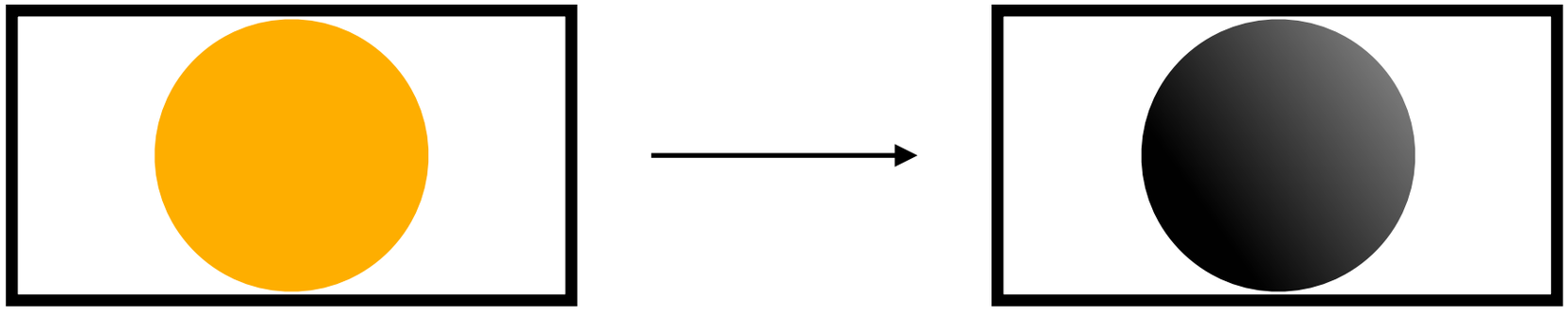
- Entanglement between black hole and radiation gets too large



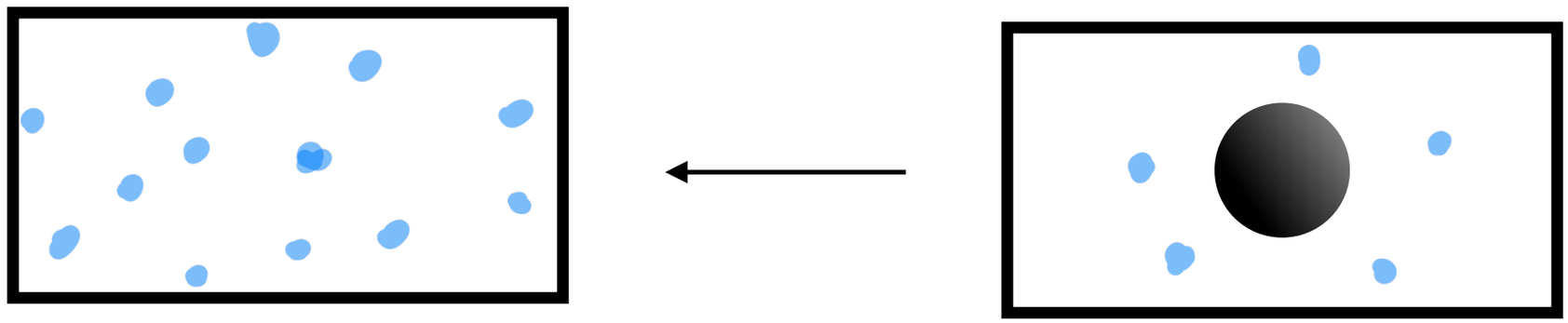
- Predicts light objects with arbitrarily large number of internal states

Pure to *Mixed*

Pure



Mixed



Information *Paradox*

- Why is it a paradox?

  Trusted assumption lead to unpalatable conclusions

- Does quantum mechanics break down?

- Where are those high degeneracy small mass objects?

Black hole evaporation

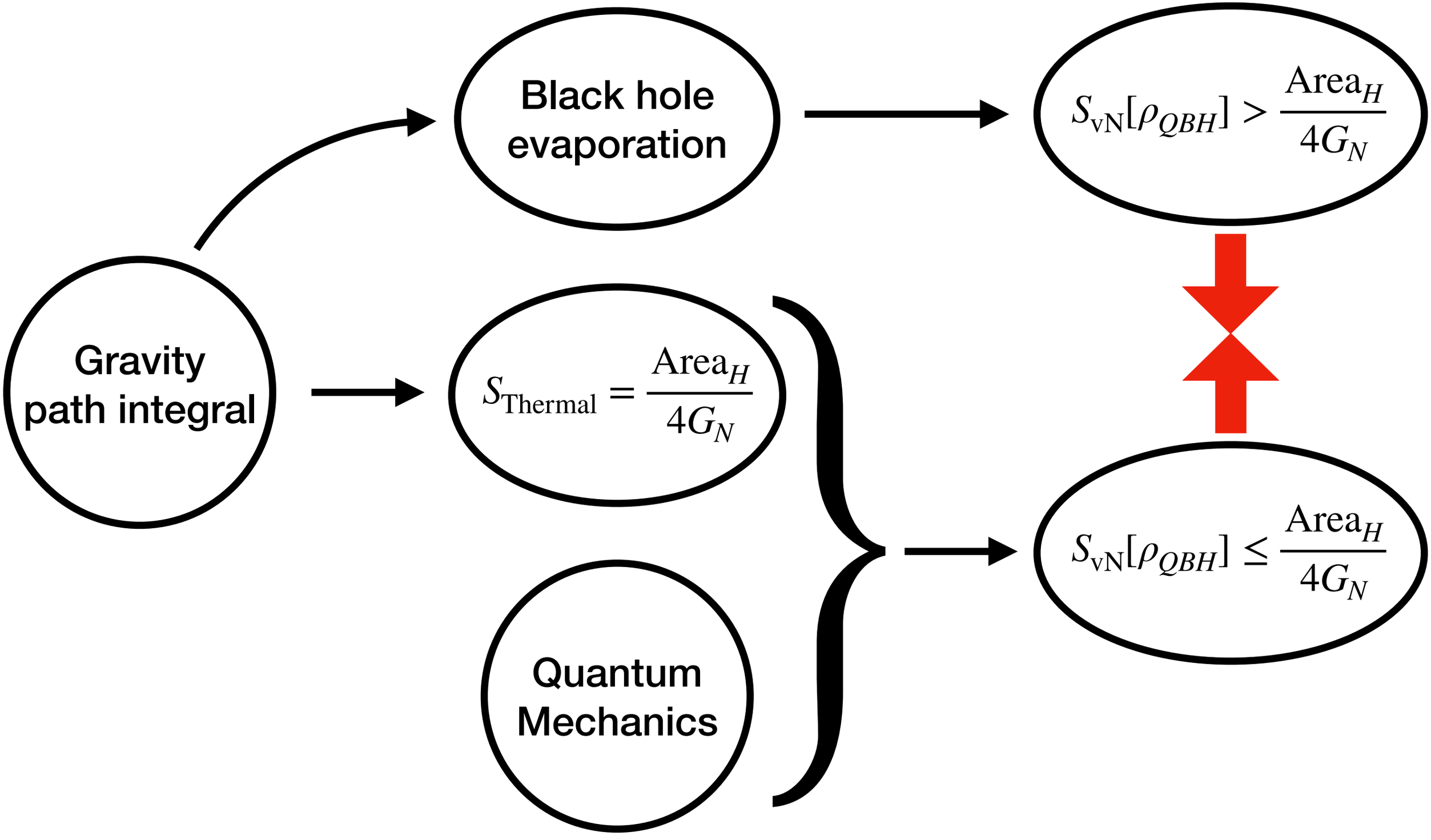
$S_{\mathrm{vN}}[\rho_{QBH}] > \dfrac{\mathrm{Area}_H}{4G_N}$

Gravity path integral

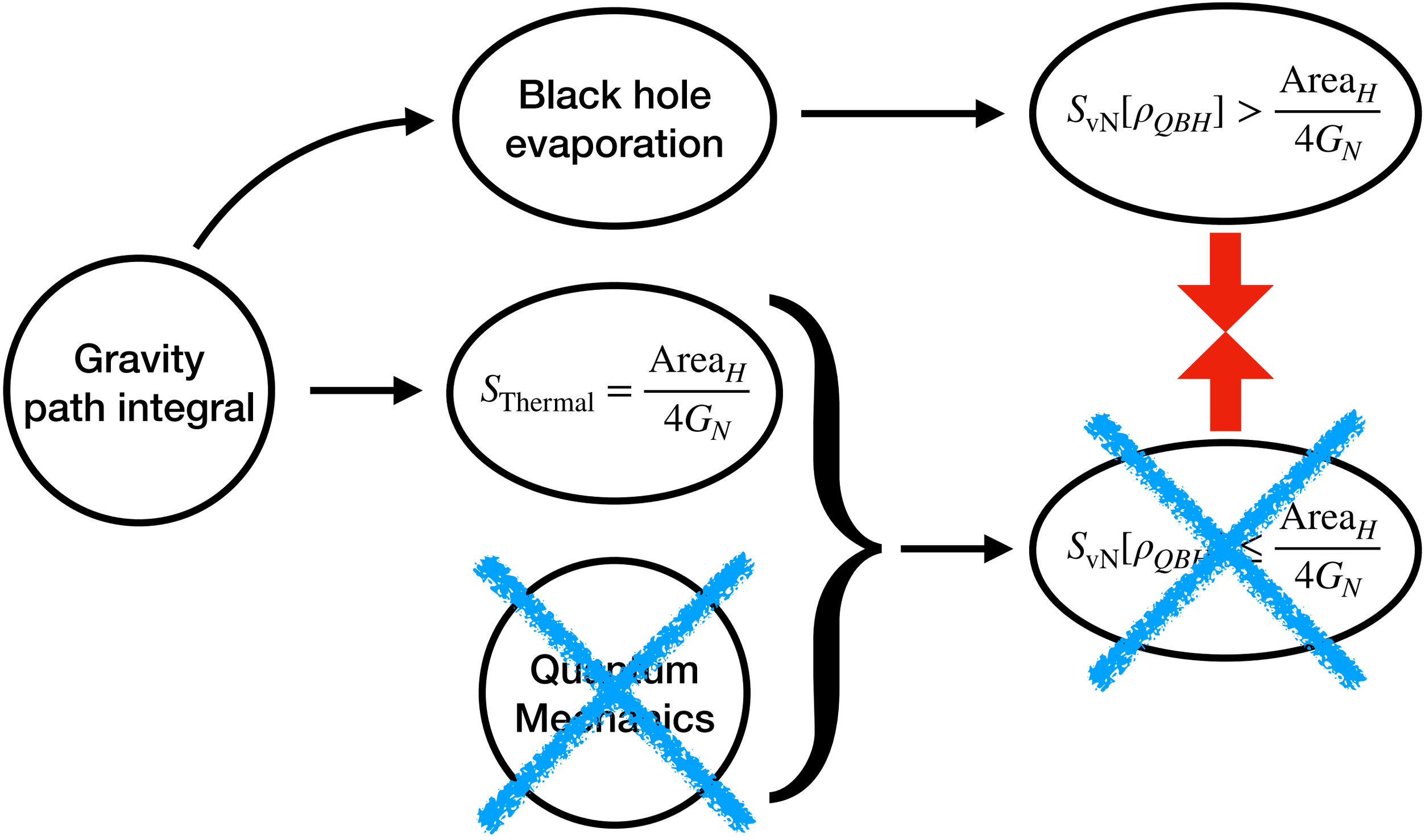
$S_{\mathrm{Thermal}} = \dfrac{\mathrm{Area}_H}{4G_N}$

Quantum Mechanics

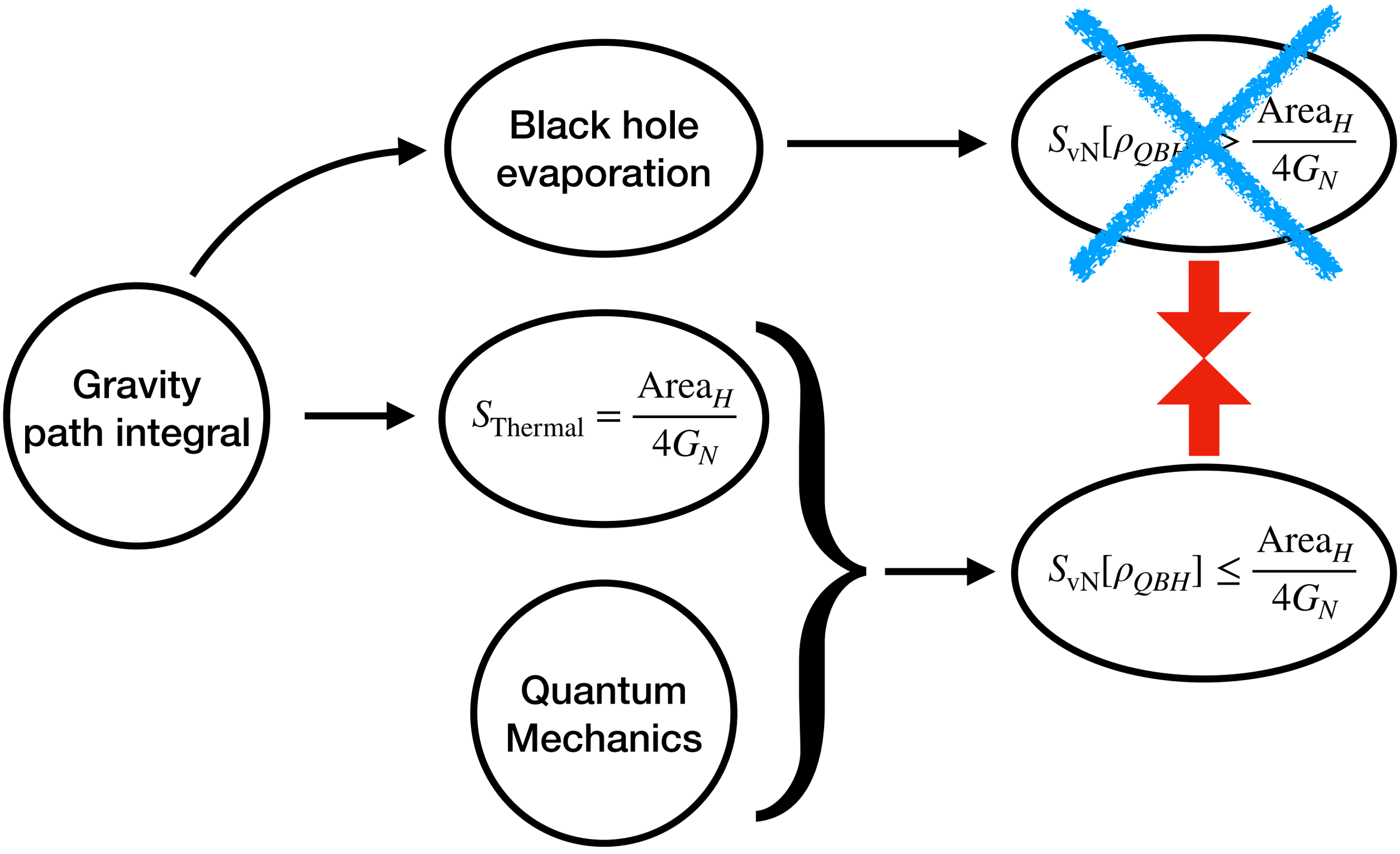
$S_{\mathrm{vN}}[\rho_{QBH}] \leq \dfrac{\mathrm{Area}_H}{4G_N}$



- What's the weakest link?



- What's the weakest link?

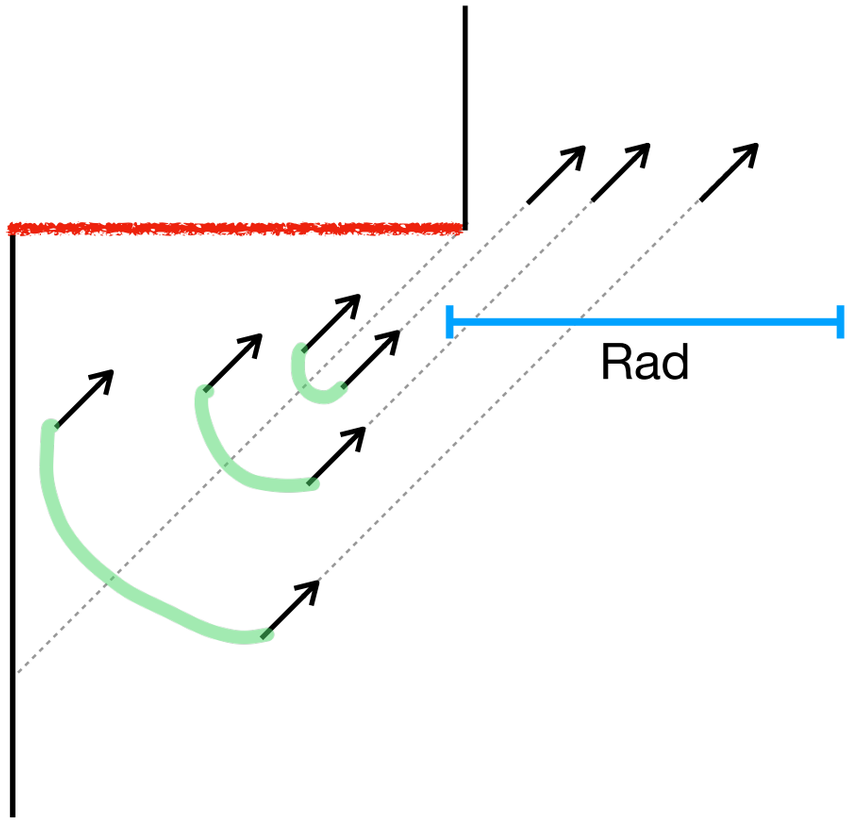


- What's the weakest link?

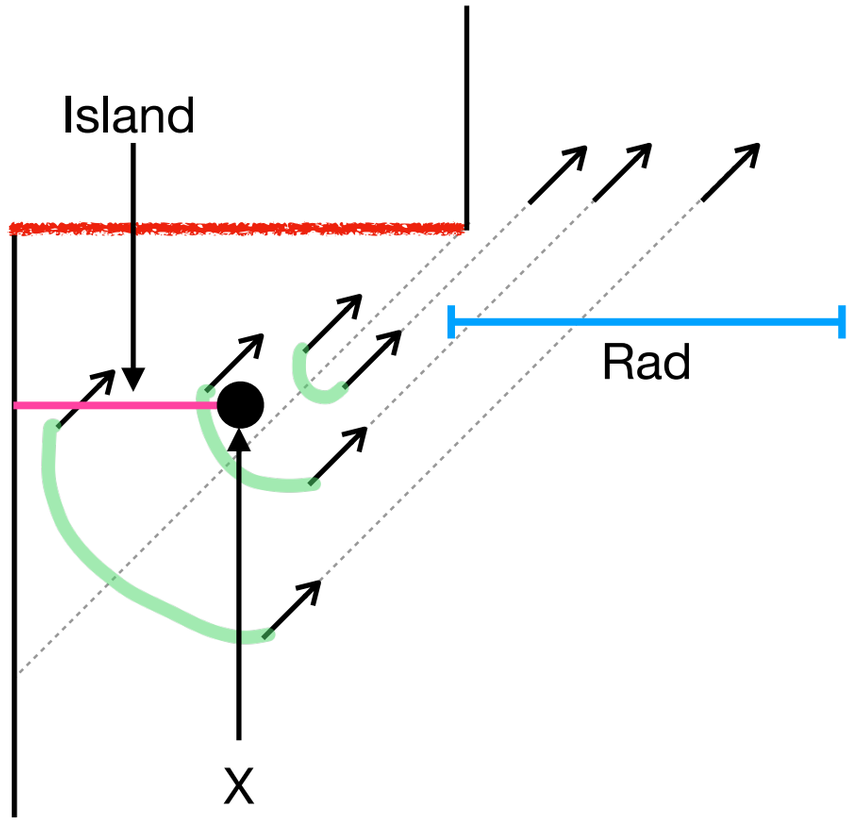
A *new formula* from the *GPI*

Gravitational fine-grained entropy

- There are new contributions from the gravity path integral



VS

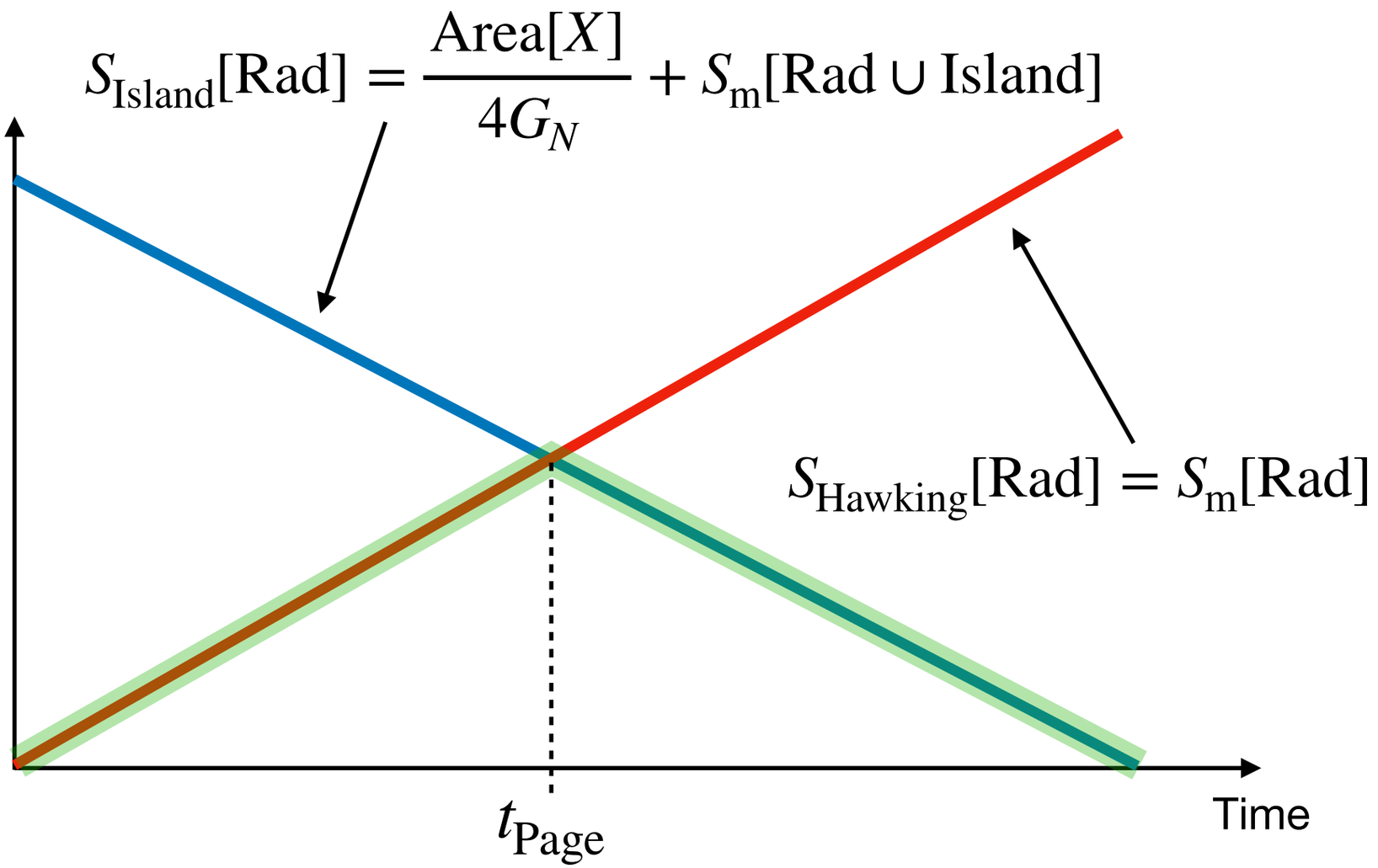


$$S_{\text{Hawking}}[\text{Rad}] = S_{\text{m}}[\text{Rad}]$$

$$S_{\text{Island}}[\text{Rad}] = \underbrace{\frac{\text{Area}[X]}{4G_N} + S_{\text{m}}[\text{Rad} \cup \text{Island}]}_{S_{\text{gen}}}$$

- X is fixed by extremizing S_{gen}

The Page Curve!



$$S_{\text{vN}}[\text{Rad}] = \min\left\{ S_{\text{Hawking}}[\text{Rad}],\ S_{\text{Island}}[\text{Rad}] \right\}$$

- Thermal entropy bound is satisfied

Relaxing Hawking's *assumption*

- Hawking's method:

$$1.\ \rho_{\mathrm{Rad}} \to \langle \rho_{\mathrm{Rad}} \rangle_{g_s} \quad 2.\ S_{\mathrm{Hawking}} = -\,\mathrm{tr}\left[\langle \rho_{\mathrm{Rad}} \rangle_{g_s} \ln \langle \rho_{\mathrm{Rad}} \rangle_{g_s}\right]$$

- New method:

$$\rho_{\mathrm{Rad}} \to \langle \rho_{\mathrm{Rad}} \rangle_{g_s}$$

$$1.\ S_{\mathrm{Rad}} = -\,\langle \mathrm{tr}\left[\rho_{\mathrm{Rad}} \ln \rho_{\mathrm{Rad}}\right] \rangle_{g_s}$$

$$2.\qquad = S_{\mathrm{Hawking}} + S_{\mathrm{Island}}$$

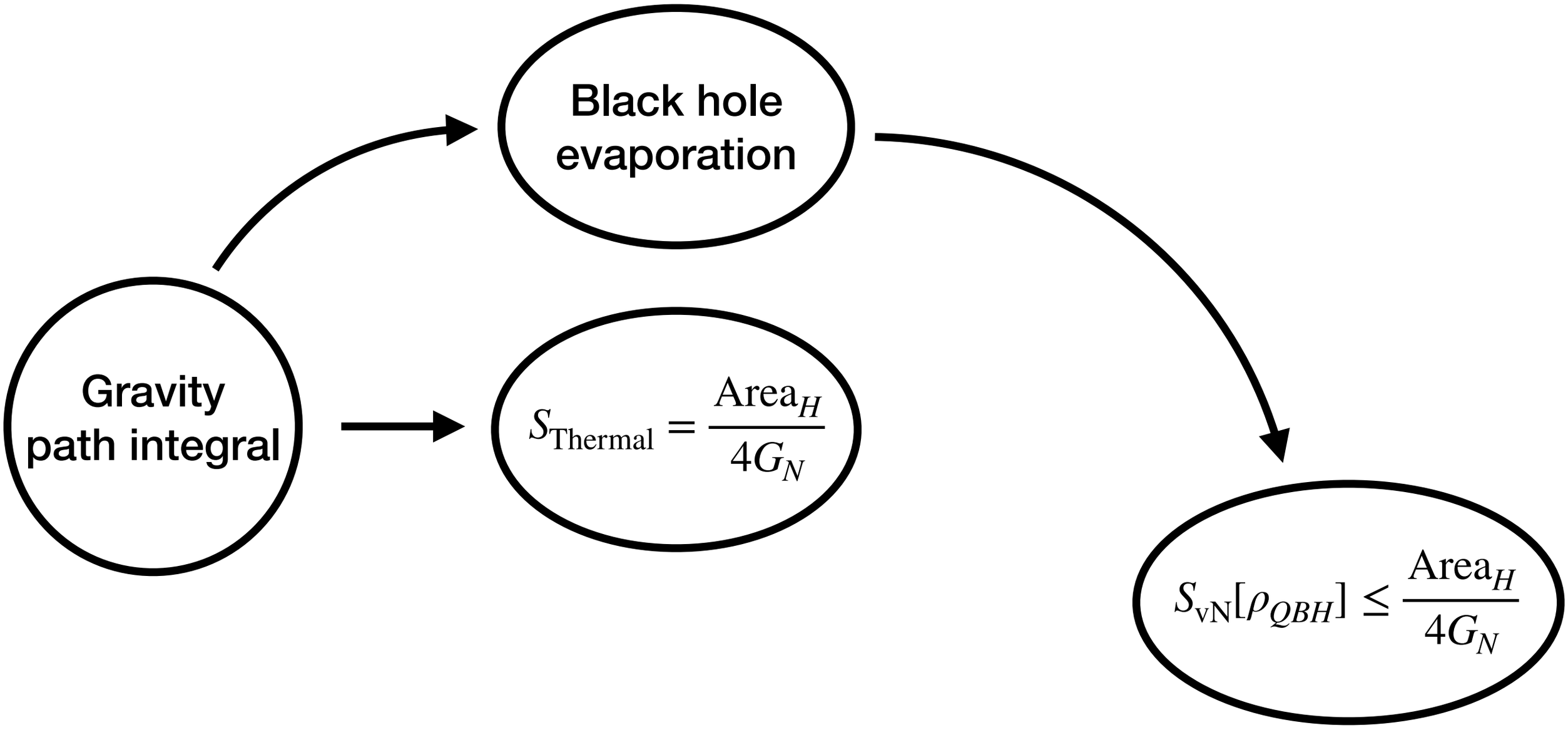
Black hole evaporation

Gravity path integral

$S_{\text{Thermal}} = \dfrac{\text{Area}_H}{4G_N}$

Quantum Mechanics

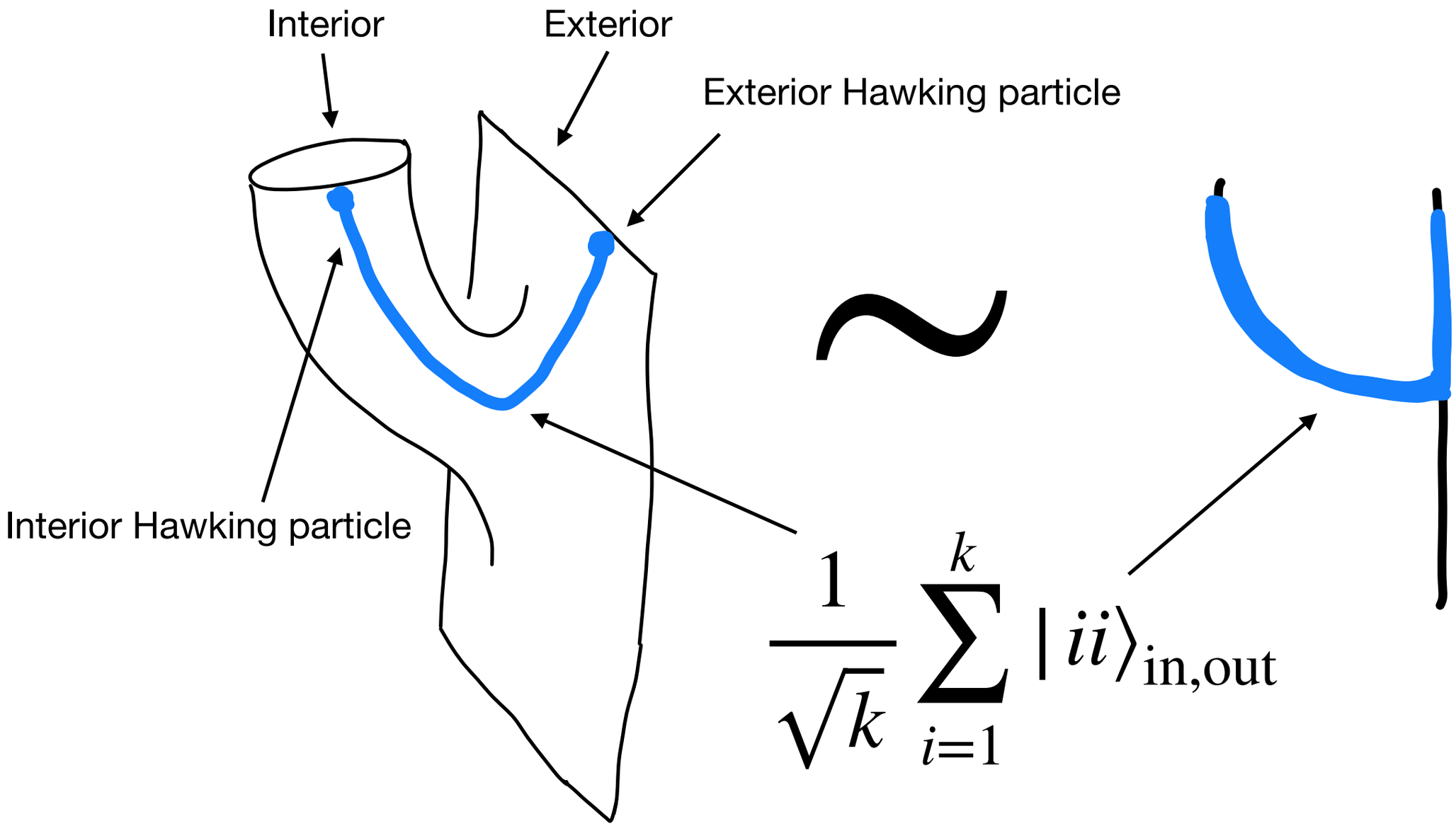
$S_{\text{vN}}[\rho_{QBH}] \leq \dfrac{\text{Area}_H}{4G_N}$



- The gravity path integral is self-consistent, so far.

The “How”

- From the gravitational path integral
- Abstract version of a completely evaporated black hole

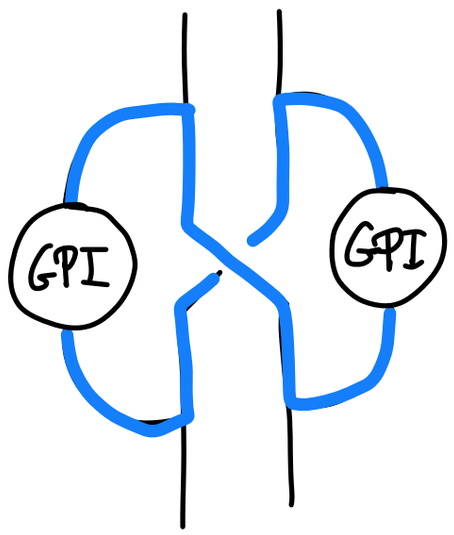


Compute the Purity

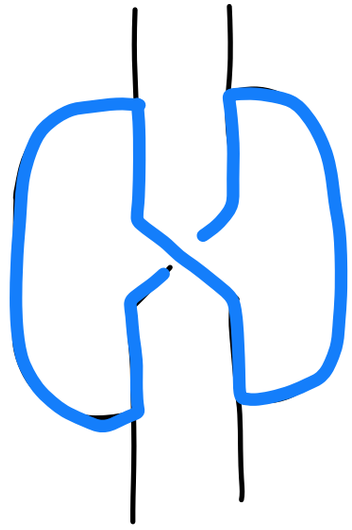
- External density matrix: $\rho_{\text{ext}} = \text{Tr}_{\text{int}} |\Psi\rangle\langle\Psi| =$

- Purity:

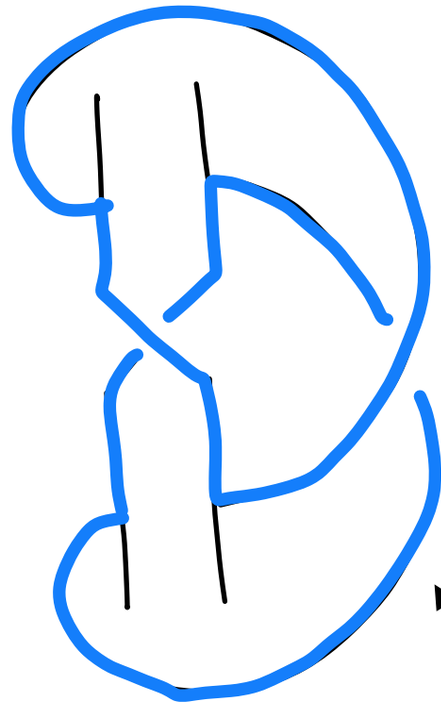
$$\text{Tr}[\rho_{\text{ext}}^2] =$$



$=$



$+$

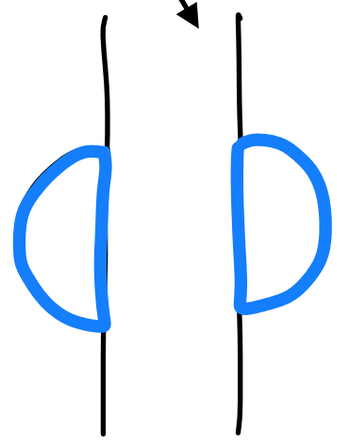


Hawking

$$\text{Tr}[\rho_{\text{ext}}^2] = \mathcal{O}(1/k) + (\text{Tr}[\rho_{\text{ext}}])^2$$

$$= 1/k \quad + \quad 1$$

Island
or Wormhole



Comments

- Origin of the area term is the same as that of Gibbons-Hawking — It comes from a Euclidean section of the path integral.
- Just as with Gibbons-Hawking, there is no microscopic understanding of the formula.
- The gravity path integral is good at computing numbers, like the entropy, but not is telling us what the exact state of the Hawking radiation is.
- Results consistent with the Central Dogma

Comments

- Origin of the area term is the same as that of Gibbons-Hawking — It comes from a Euclidean section of the path integral.
- Just as with Gibbons-Hawking, there is no microscopic understanding of the formula.
- The gravity path integral is good at computing numbers, like the entropy, but not is telling us what the exact state of the Hawking radiation is.
- Results consistent with the Central Dogma

Thanks!