Loop Quantum Gravity à la Aharonov-Bohm

Eugenio Bianchi

Centre de Physique Théorique de Luminy, Marseille

International LQG Seminar – October 6th, 2009

- In LQG, the state space $\mathcal{K}^{\text{Diff}}$ admits an orthogonal decomposition into subspaces $\mathcal{K}^{\text{Diff}}_{\{\Gamma\}}$ ($\{\Gamma\} =$ diff. equiv. class of spin-network graphs $\Gamma \subset \Sigma$)
- Remarks: [Rovelli-Smolin, Ashtekar-Lewandowski, $\sim$ '90]

  (i) The state space is obtained via
  - loop quantization of the Ashtekar-Barbero connection
  - group averaging over Diff(Σ)

  (ii) States in $\mathcal{K}^{\text{Diff}}_{\{\Gamma\}}$ admit a physical interpretation in terms of quantum geometry

- Aim of the talk:

  show that the state space $\mathcal{K}^{\text{Diff}}_{\{\Gamma\}}$, together with its interpretation (ii), can be obtained via standard QFT quantization of a theory of locally-flat Ashtekar-Barbero connections with magnetic defects

[based on `arXiv:0907.4388`]

Plan of the talk

- Motivation and the idea
- The framework:
  - defects, locally-flat connections and diff-invariant states
  - the QFT scalar product
- An example: the state space for a single line defect
- Relation with canonical LQG and with Spin Foams

Motivation

- The mathematical structure of the Hilbert space $\mathcal{K}_{\{\Gamma\}}^{\text{Diff}}$ and its physical interpretation in terms of quantum geometry are the ones proper to a system with a finite (but possibly large) number of degrees of freedom
- Therefore, while the full theory has an infinite number of degrees of freedom (as classical GR does), each subspace captures only a finite number of them

Question:

- is there a classical stystem with a finite number of degrees of freedom such that, when quantized, leads directly to the state space $\mathcal{K}_{\{\Gamma\}}^{\text{Diff}}$?
- which degrees of freedom of General Relativity are we capturing?

Notice that *Spin Foam quantization* offers an answer to these questions

- discretization of GR, then quantization of the discrete system
- boundary state space: spin networks with abstract graph dual to the discretization

However, due to the discretization and to the fact that the graph is not embedded, the role of diffeomorphisms is rather obscure

- Here we look for a system defined on a manifold, so that diffeomorphisms act on local fields in the standard way, via pullback

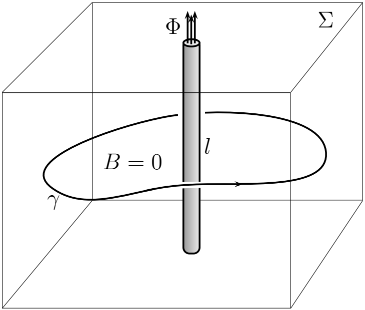
The idea: Aharonov-Bohm setting

Consider a configuration of the connection A such that its magnetic field B vanishes in the region $\Sigma - l$.

Despite the vanishing of B, there are observable effects associated to the holonomy of the connection along non-contractible closed paths.

Such effects depend only on the flux Φ of the magnetic field enclosed by the path.

In particular, the effect does not change if the path is deformed continuously.



- Idea: $\mathcal{K}^{\text{Diff}}_{\{\gamma\}}$ = space of Diff-invariant states $\Psi_{\gamma,\eta}[A] = \eta(h_\gamma[A]) = f(\Phi)$

  Scalar product via QFT path integral measure on the local field $A(x)$
- Similarly, the Aharonov-Bohm effect for electrons depends only on Φ and not on the specific path γ

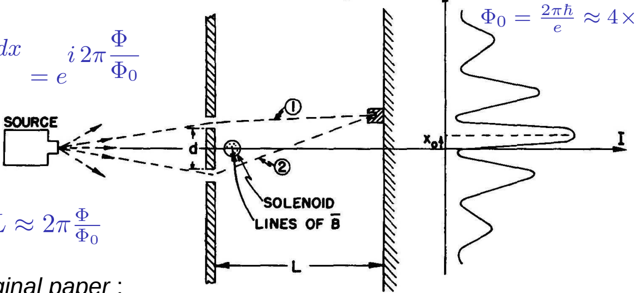
- *From Feynman Lectures* :

You remember that for a long solenoid carrying an electric current there is a ***B***-field inside but none outside, while there is lots of ***A*** circulating around outside, as shown in Fig. 15–6. If we arrange a situation in which electrons are to be found only *outside* of the solenoid—only where there is ***A***—there will still be an influence on the motion, according to Eq. (15.33). Classically, that is impossible. Classically, the force depends only on ***B***; in order to know that the solenoid is carrying current, the particle must go through it. But quantum-mechanically you can find out that there is a magnetic field inside the solenoid by going *around* it—without ever going close to it! What will be the effect on our interference experiment?

$\Phi =$ flux of B

$\Phi_0 = \frac{2\pi\hbar}{e} \approx 4\times10^{-7}$G $\cdot$ cm^2

$$e^{\frac{i}{\hbar} e \int_\gamma A dx} = e^{i\, 2\pi \frac{\Phi}{\Phi_0}}$$



$$\theta \approx x_0/L \approx 2\pi \frac{\Phi}{\Phi_0}$$

- *The original paper* :

Y. Aharonov and D. Bohm, "*Significance of electromagnetic potentials in the quantum theory*", Phys. Rev. **115** (1959) 485.

The framework ◔: manifold Σ' and locally-flat connections

1. Start with a 3-manifold Σ connected and without boundary

   Consider a network $\mathcal{D}$ of curves embedded in Σ

   Regard $\mathcal{D}$ as a network of defects and introduce a new manifold Σ'

   $$\Sigma' = \Sigma - \mathcal{D}$$

   Σ' is path connected, but not simply connected

Example: - consider a cellular decomposition $\mathcal{C}(\Sigma)$ of the original manifold

- regard its 1-skeleton as a defect-network, $\mathcal{D} = \mathcal{C}_1(\Sigma)$

closed paths around edges of the 1-skeleton are non-contractible

$\Rightarrow$ non-trivial first homotopy group $\pi_1(\Sigma')$

2. Introduce a locally-flat connection $A(x)$ on Σ', $\quad \mathcal{A}_f = \{A : \Sigma' \to G \mid F(A) = 0\}$

   Despite being locally-flat, the connection can have non-trivial holonomy around non-contractible loops in Σ' $\rightsquigarrow$ AB effect

3. The space $\mathcal{A}_f/\mathcal{G}$ of locally-flat connections modulo gauge transformations is finite dimensional. We call $\mathcal{N}$ this *moduli* space

   $$\mathcal{N} = \mathcal{A}_f/\mathcal{G} = \mathrm{Hom}(\pi_1(\Sigma'), G)/G$$

   and $\{m_r\}$ coordinates (moduli) on $\mathcal{N}$

4. Locally-flat connection $A^{m_r, g}$: labeled by its moduli m_r and by a gauge transf $g(x)$

The framework ◔: the state space $\mathcal{K}'$ and functions of moduli

- We consider the kinematics of General Relativity in Ashtekar-Barbero variables
  The configuration variable is a $SU(2)$ connection $A(x)$ on a 3-manifold Σ

  In Loop Quantum Gravity: kinematical Hilbert space $\mathcal{K}$ of functionals of the connection invariant under
  - $SU(2)$-gauge transformations
  - Diff(Σ) of the manifold Σ

- Here we follow a similar procedure but we require also
  - *topological* invariance, $\hat{F}\,\Psi[A] = 0$

  We introduce a state space $\mathcal{K}'$ of gauge invariant functionals of a locally-flat connection on Σ'

∗ Notice that: in a topological gauge theory, gauge- and diff-invariance are related
  $\Rightarrow$ once we have gauge-invariance, Diff-invariance comes for free

- As states in $\mathcal{K}'$ are gauge-invariant, they can depend on the connection $A^{m_r,g}$ only through its moduli m_r

$$\Psi_f[A^{m_r,g}] = f(m_1, .., m_R) \qquad [a]$$

$\Rightarrow$ states labeled by a function f on moduli space

$$f : \mathcal{N} \to \mathbb{C}$$

The framework ◔: states in terms of dual graph and holonomies

- Can we introduce an embedded graph $\Gamma \subset \Sigma'$ and write states as *cylindrical* functions? $\Psi_{\Gamma,\eta}[A] = \eta(h_{\gamma_1}[A], .., h_{\gamma_L}[A])$ [*b*]

1. Notice that contractible loops of the graph evaluate to the identity
2. Choose minimal graph Γ': embedded in Σ' and "dual" to the defect-network $\mathcal{D}$
3. Property the dual graph satisfies: edge-path-group $\pi(\Gamma')$ isomorphic to $\pi_1(\Sigma')$
4. Construction in a specific case: $\pi(\Gamma') = \pi_1(\Sigma - \mathcal{D})$
   - Focus on a defect-network arising from a cell decomposition, $\mathcal{D} = \mathcal{C}_1(\Sigma)$
   - Consider an embedded graph Γ' dual to the decomposition, $\Gamma' = \big(\mathcal{C}(\Sigma)^*\big)_1$
5. Properties:
   - Gauge-invariance, $\Psi_{\Gamma',\eta}[A^{m_r,g}] = \Psi_{\Gamma',\eta}[A^{m_r,h\cdot g}]$
   - Diff-invariance from local-flatness: $\phi \in$ Diff(Σ') connected to the identity,

$$\Psi_{\Gamma',\eta}[\phi^* A] = \Psi_{\phi^{-1}\circ\Gamma',\eta}[A] \stackrel{!}{=} \Psi_{\Gamma',\eta}[A]$$

   - $\Psi_{\Gamma',\eta}[A]$ depends on Γ' only via its 'knotting' with the defect-network $\mathcal{D}$
6. Spin-network states with graph Γ' as cyl functions labelled by Γ' and η_{j_l,i_n}

$$\eta_{j_l,i_n}(h_1, .., h_L) = \Big(\bigotimes_{n\subset\Gamma'} v_{i_n}\Big) \cdot \Big(\bigotimes_{\gamma_l\subset\Gamma'} D^{(j_l)}(h_l)\Big)$$

The framework ◑: summarizing

- There are two equivalent characterizations of diff-invariant states in $\mathcal{K}'$:

  [a] in terms of a function f on moduli space $\mathcal{N} = \text{Hom}\big(\pi_1(\Sigma'), SU(2)\big)/SU(2)$

  $$\Psi_f[A^{m_r,g}] = f(m_1, .., m_R) \qquad\qquad \mathcal{N} = \{m_1, .., m_R\}$$

  [b] in terms of a dual graph Γ' and a cylindrical function $\eta : SU(2)^L \to \mathbb{C}$

  $$\Psi_{\Gamma,\eta}[A] = \eta(h_{\gamma_1}[A], .., h_{\gamma_L}[A])$$

- The characterization of states $[\,b\,]$ in terms of (Γ', η) is slightly redundant, but leads to a picture definitely more clear than the one in terms of moduli $[a]$

- We can go from description [b] to description [a] gauge-fixing the $SU(2)$ invariance at the nodes of Γ'
  - once chosen coordinates $\{m_1, .., m_R\}$ on $\mathcal{N}$, the couple (Γ', η) determines a unique function $f : \mathcal{N} \to \mathbb{C}$
  - e.g.: choose a maximal tree of Γ' and fix to the identity the group elements associated to it

The framework ◔: the scalar product

- In order to promote the linear space described before to a Hilbert space we need to introduce a scalar product

  A topologically invariant functional measure $\mathcal{D}[A]$ has to reduce to an ordinary measure on moduli space $\mathcal{N}$

$$\langle f|g\rangle = \int_{\mathcal{A}_f/\mathcal{G}} \mathcal{D}[A]\, \overline{\Psi_f[A]}\, \Psi_g[A] = \int_{\mathcal{N}} d\mu(m_r)\, \overline{f(m_1,..,m_R)}\, g(m_1,..,m_R)$$

- Making sense of the formal functional measure $\mathcal{D}[A]$ amounts to a choice of measure $d\mu(m_r)$ on the moduli space $\mathcal{N}$

Problem: Is there a principle that can guide us in the choice of the measure $d\mu(m_r)$?

- Here: we use the *methods proper to quantum field theory* to make sense of the functional integral over locally-flat connections modulo gauge transformations.

  Such methods, in the case of a finite dimensional moduli space, allow to fully determine a specific measure on the moduli space and answer our question

  used in string measure: Polyakov 1981, Alvarez 1983
  2+1 gravity with and w/o particles: Carlip 1995, Cantini Menotti 2002
  simplicial measure: Jevicki Ninomiya 1986, Menotti Peirano 1995

∗ The guiding principle here is locality, that is:

  the configurations we are integrating over are local fields

The framework ◕: the scalar product via QFT methods

1. In the integral over locally-flat connections $\mathcal{D}[A^{m_r,g}(x)]$, we can change variables to the moduli m_r *and* gauge transformations $g(x)$
   The integral over gauge transformations factorizes so that we are left with a finite dimensional integral over moduli, with a non-trivial measure on the moduli
2. Equivalently, we can use the more well-known Faddeev-Popov procedure:
   $$A^{m_r,g} = g^{-1}\bar{A}^{m_r}g + g^{-1}dg \;\text{ with } \bar{A}^{m_r} \text{ gauge-fixed to } \chi(\bar{A}^{m_r}) = 0$$
   measure on locally-flat connections modulo gauge transformations written as
   $$\int_{\mathcal{A}_f/\mathcal{G}} \mathcal{D}[A] = \int_{\mathcal{A}} \Big[\prod_x dA(x)\Big]\, \delta\big(F(A)\big)\, \delta\big(\chi(A)\big)\, \Delta_{\text{FP}}(A)$$
   where $\chi(A) = 0$ is a gauge-fixing condition and $\Delta_{\text{FP}}(A) = \left|\text{Det}\frac{\delta\chi}{\delta\xi}\right|$
3. Integrating and changing variables, we end up with a finite dimensional integral
   $$\int_{\mathcal{A}_f/\mathcal{G}} \mathcal{D}[A] = \int d\bar{A}^{m_r}\, \Delta_{\text{FP}}(\bar{A}^{m_r}) = \int_{\mathcal{N}} dm_1 \cdots dm_R\, J(m_r)\Delta_{\text{FP}}(m_r)$$
   where $J(m_r)$ is the Jacobian from gauge-fixed connections $\bar{A}^{m_r}$ to the moduli m_r
4. This is our field theoretical proposal for the measure $d\mu_{QFT}(m_r)$: states in the Hilbert space $\mathcal{K}'$ are assumed to be normalizable in the scalar product

$$\langle f|g\rangle = \int_{\mathcal{A}_f/\mathcal{G}} \mathcal{D}[A]\, \overline{\Psi_f[A]}\, \Psi_g[A] = \int_{\mathcal{N}} dm_1 \cdots dm_R\, J(m_r)\, \Delta_{\text{FP}}(m_r)\, \overline{f(m_r)}\, g(m_r)$$

$\longrightarrow$ explicit expression for the measure $d\mu_{QFT}(m_r)$ in our example

The framework ◕: the scalar product via Loop methods

- The field theoretical derivation of the measure $d\mu(m_r)$ described above can be compared to a different construction which is proper to Loop Quantum Gravity

1. Using the characterization of states $[\,b\,]$ in terms of the dual graph Γ', there is a natural choice of measure: the Haar measure on links of the graph

$$\langle\eta|\xi\rangle = \int_{\mathcal{A}_f/\mathcal{G}} \mathcal{D}[A]\, \overline{\Psi_{\Gamma',\eta}[A]}\, \Psi_{\Gamma',\xi}[A] = \int_{SU(2)^L} \prod_{l=1}^{L} d\mu_H(h_l)\, \overline{\eta(h_1,..,h_L)}\, \xi(h_1,..,h_L)$$

2. With this choice of measure, spin-network states η_{j_l,i_n} provide an o.n. basis of $\mathcal{K}'$

$$\langle\eta_{j_l,i_n}|\eta_{j'_l,i'_n}\rangle = \Big(\prod_l \delta_{j_l,j'_l}\Big)\Big(\prod_n \delta_{i_n,i'_n}\Big)$$

3. Notice that, as the functions η and ξ are invariant under conjugation at nodes,

   the Haar measure $\prod_l d\mu_H(h_l)$ on $SU(2)^L$ reduces to a specific measure $d\mu(m_r)$ on moduli via the Weyl integration formula*

   Therefore it provides again a definition of the functional measure $\mathcal{D}[A]$ via a prescription of a specific measure $d\mu_{Loop}(m_r)$ on moduli space.

*[equivalent to gauge-fixing of $SU(2)$ invariance at nodes via a choice of maximal tree]

The framework ●: the scalar product – a conjecture

- While the group theoretical choice of measure à la Loop is certainly natural, the field theoretical choice is a well-motivated one

  What is the relationship between the two?

- We investigate these two proposals in a rather simple case and provide an explicit expression for the measure $d\mu(m_r)$
- The result supports the following conjecture:

  the QFT measure on the moduli space $\mathcal{N}$ obtained via the field theoretical construction coincides with the one obtained from the product of Haar measures via Weyl integration formula

$$d\mu_{\text{QFT}}(m_r) = d\mu_{\text{Loop}}(m_r)$$

- If such conjecture turns out to be robust, then spin network states with embedded graph Γ' are not just a good tool for describing Diff-invariant functionals of the connection but provide an o.n. basis of the Hilbert space $\mathcal{K}'$ built via the standard field theoretical construction

An example: Σ' with a single line defect

1. Original manifold with trivial topology $\Sigma \approx \mathbb{R}^3$

   Defect-network consisting of a single line $l \approx \mathbb{R}$ (assumed to be unknotted in Σ)

   $\longrightarrow$ the new manifold $\Sigma' = \Sigma - l$ has non-trivial $\pi_1(\Sigma')$ generated by the homotopy class $[\gamma]$ of loops which encircle once the line l

2. $SU(2)$ locally-flat connection A on Σ'

3. In order to identify a representative in each gauge orbit, we introduce a gauge fixing condition $\longrightarrow$ break the local symmetries of the problem:

   $\longrightarrow$ a convenient choice is to introduce an auxiliary metric $q_{ab}(x)$ on Σ and consider the Coulomb-like gauge $\chi = q^{ab}\partial_a A_b$

   we assume $q_{ab} = \delta_{ab}$, choose Cartesian coord $x^a = (x, y, z)$, l along the z-axes

4. Gauge-fixed locally-flat connection in Σ'

$$A_a^i(x) = \frac{\Phi^i}{2\pi}\alpha_a(x) \quad \text{with} \quad \alpha_a(x) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$

5. its holonomy along a loop γ is simply given by

$$h_\gamma[A] = \exp[i\Big(\int_\gamma \alpha_a dx^a\Big)\frac{\Phi^i}{2\pi}\tau_i\,] \quad \text{where} \quad \int_\gamma \alpha_a dx^a = 2\pi n$$

   $n =$ winding number of the loop γ around the line l

   $\longrightarrow$ the holonomy $h_\gamma[A]$ provides a homomorphism from $\pi_1(\Sigma')$ to $SU(2)$

An example: physical interpretation of Φ^i and the moduli space

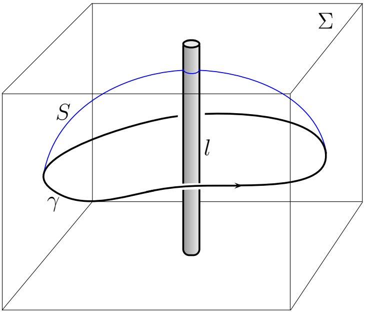
1. As the connection is locally-flat in Σ', the non-abelian magnetic field $B_i^a = \frac{1}{2}\varepsilon^{abc}F_{bc}^i$ vanishes everywhere except along the line l

   $$B_i^a(x) = \int_l ds\,\Phi_i\,\dot{x}^a(s)\,\delta^{(3)}(x - x(s))$$

   The line l can be thought as a thin *solenoid*

   $\{$Flux of B_i^a through S in fig$\}$ = Φ_i

   Via Stokes theorem, it determines $h_\gamma[A]$



2. The magnetic flux Φ^i provides a parametrization of the moduli space of locally-flat connections. To find its range we have to identify different magnetic fields which correspond to the same holonomy $\Rightarrow$ $\Phi^i \in S^3$
3. States $=$ functions of the flux Φ^i. Gauge invariance $\Rightarrow$ depend only on $\phi = |\Phi^i|$

   As a result, states in $\mathcal{K}'$ are labeled by a function $f(\phi)$,

   $$\Psi_f[A^{\phi,g}] = f(\phi)$$

   and the moduli space is given by

   $$\mathcal{N} = \{\Phi^i \in S^3\}/SU(2) = \{\phi \in [0, 2\pi]\}$$

An example: the QFT scalar product

1. QFT measure on moduli space $d\mu_{QFT}(\phi) = J(\phi)\, \Delta_{\text{FP}}(\phi)\, d\phi$
2. Jacobian easily determined: $d^3\Phi^i = \phi^2 d\phi d^2 v^i$, so that $J(\phi) = \phi^2$
3. Faddeev-Popov term: determinant of the operator $K(\Phi^i)$

$$K_{ij}(\Phi^i) = \frac{\delta \chi_i}{\delta \xi^j} = -\delta_{ij}\Delta - \varepsilon_{ijk}\frac{\Phi^k}{2\pi}$$

4. Eigenvalues $\lambda_n = n^2 + n\frac{\phi}{2\pi}$ with $n = \pm 1, \pm 2, \ldots$ and twice degenerate
5. Φ^i dependence extracted considering the appropriately regularized ratio

$$\Delta_{\text{FP}}(\phi) = c\,\frac{\text{Det}K(\Phi^i)}{\text{Det}K(0)} = c\,\frac{\prod_{n=1}^{\infty}\big(\lambda_n(\phi)\big)^2\big(\lambda_{-n}(\phi)\big)^2}{\prod_{n=1}^{\infty}\big(\lambda_n(0)\big)^2\big(\lambda_{-n}(0)\big)^2} =$$

$$= c\left(\prod_{n=1}^{\infty}\Big(1 - \big(\frac{\phi}{2\pi n}\big)^2\Big)\right)^2 = c\left(\frac{\sin\phi/2}{\phi/2}\right)^2$$

The constant c is undetermined and is fixed so that the measure of $\mathcal{N}$ is one

6. Therefore: $$d\mu_{QFT}(\phi) = \frac{1}{\pi}\sin^2\frac{\phi}{2}$$

An example: the QFT scalar product and the Loop one

1. Finally we have that the QFT scalar product on $\mathcal{K}'$ defined by this measure is

$$\langle f|g\rangle = \int_{\mathcal{A}_f/\mathcal{G}} \mathcal{D}[A] \overline{\Psi_f[A]} \Psi_g[A] = \int_{\mathcal{N}} d\phi\; J(\phi)\Delta_{\mathrm{FP}}(\phi)\; \overline{f(\phi)}\, g(\phi) = \frac{1}{\pi}\int_0^{2\pi} d\phi\; \sin^2\frac{\phi}{2}\;\; \overline{f(\phi)}\, g(\phi)$$

2. The QFT measure can be compared to the one obtained from the group theoretical construction via Weyl integration formula

3. We can label states by the homotopy class $[\gamma]$ of loops encircling once the line l, and by a complex-valued function η on $SU(2)$,

$$\Psi_{\gamma,\eta}[A] = \eta(h_\gamma[A])$$

   Gauge invariance at the base point of γ requires that η is a class function

4. The Loop scalar product is given by the Haar measure on $SU(2)$ and reduces to an integral over the *class angle* $\phi/2$. Defining $f(\phi) = \eta(\exp i\phi\tau_3)$, we have

$$\langle \eta|\xi\rangle = \int_{\mathcal{A}_f/\mathcal{G}} \mathcal{D}[A]\; \overline{\Psi_{\gamma,\eta}[A]} \Psi_{\gamma,\xi}[A] = \int_{SU(2)} d\mu(h_\gamma)\; \overline{\eta(h_\gamma)}\, \xi(h_\gamma) = \frac{1}{\pi}\int_0^{2\pi} d\phi\; \sin^2\frac{\phi}{2}\;\; \overline{f(\phi)}\, g(\phi)$$

   which coincides with the field theoretical one derived above

5. This fact supports our conjecture and leads to a physical interpretation of the class angle as the modulus of the flux of the magnetic field through the defect line

Relation with canonical LQG

1. *In ordinary LQG, embedded spin networks are not diffeomorphism invariant*. Huge non-separable Hilbert space. Most states are identified when diffeomorphism invariance is imposed via group-averaging
2. Abstract spin networks (s-knots) depend only on the diffeomorphism equivalence class of the graph and on the $SU(2)$ labels
3. The diffeomorphism equivalence class of the graph knows about:
   - connectivity of the graph
   - its knotting with the non-trivial topology of the manifold
   - self-knotting of the graph
   - continuous parameters (Grot-Rovelli moduli) related to tangents to the links at each node (unless Diff* are considered [Fairbairn-Rovelli '04])
4. On the other hand, here we focus directly on diff-invariant states
5. Thanks to local-flatness, spin networks have embedded graph and nevertheless the state they define is diffeomorphism-invariant
6. No dependence on self-knotting, nor on moduli at nodes
7. States depend only on the knotting of the graph with the manifold Σ', i.e. with the network of defects and with the topology of the original manifold Σ
8. From this point of view, they are closer to the use that is made of spin networks as boundary states within the context of spin foams

Relation with Spin Foams

1. The construction discussed is purely kinematical. A natural tool for implementing its dynamics is provided by Spin Foams
2. Network of defects as the 1-skeleton of a cellular decomposition of Σ. Spin networks with graph dual to the decomposition
3. Dual picture of quantum geometry: operators build out of the flux of the electric field (through surfaces ending at defects) attach a geometric meaning to the cells of the decomposition (quanta of volume and of area) [in progress]
4. No non-local links [cfr. Markopoulou-Smolin '07]
5. Same structure of the Spin Foam boundary space [Lewandowski generaliz.'09]
6. Spin foam quantization [EPRL-FK, '08]: at the classical level, GR as a topological BF theory with constraints on the B field; at the quantum level, constraints are imposed *only* on the 2-boundary of the 4-cells of a simplicial complex
   - building-blocks have flat connection
   - 2-skeleton of defects (where the constraints are imposed) support curvature
7. A foliation reproduces a theory of locally-flat connections with magnetic defects
8. In ordinary LQG: Ashtekar-Lewandowski configuration space consists of distributional polymeric connections; then Diffs are taken into account.
   Here the degrees of freedom are associated to quantum configurations consisting of connections which are locally-flat but have *distributional polymeric magnetic field*

Conclusions

- We have considered a theory of locally-flat Ashtekar-Barbero connections with magnetic defects and quantized it via standard field theoretical methods
- The resulting state space $\mathcal{K}'$ is surprisingly close to the one of ordinary LQG

  Spin networks with embedded graph Γ' provide the appropriate tool for describing Gauge- and Diff-invariant states
- The construction leads to a physical interpretations of loops of the spin-network graph as a way to capture a distributional magnetic field, analogously to what happens in the Aharonov-Bohm effect
- The interest in the construction lies in its relation with LQG/Spin Foams and in the perspectives it may open