Hierarchy of Entanglement Laws in LQG

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QISS THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

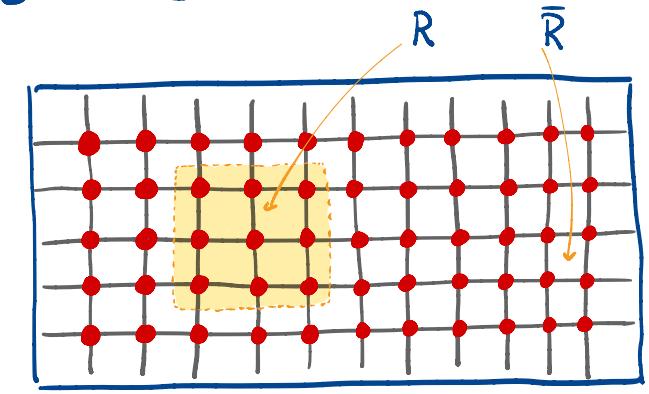
1 Hierarchy of States in LQG

- Model System: Spin-network with
  - fixed graph Γ = cubic lattice (N nodes)
  - fixed spins $j_e = j_0$

Intertwiner d.o.f.

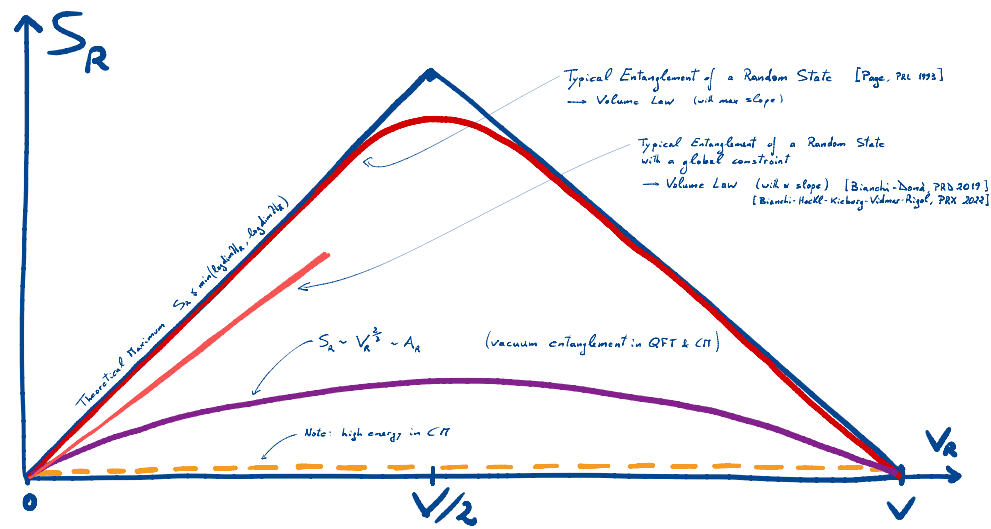
- Hilbert Space and Geometric Subsystems

$$\boxed{|\psi\rangle = \sum_{i_1,\dots,i_N} \psi_{i_1\dots i_n} \, |i_1\rangle \otimes \dots \otimes |i_N\rangle} \in \mathcal{H} = \bigotimes_{n=1}^{N} \mathcal{H}_n = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$$



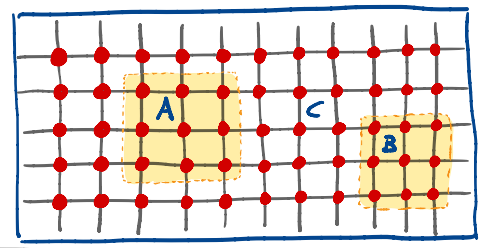
$$S_R(|\psi\rangle) = -\mathrm{Tr}_R(\rho_R \log \rho_R) \quad , \quad \rho_R = \mathrm{Tr}_{\bar{R}}(|\psi\rangle\langle\psi|)$$

- **Zero Law**: $S_R(|\psi\rangle) = 0$
- **Volume Law**: $S_R(|\psi\rangle) = \alpha V_R + \dots$
- **Area Law**: $S_R(|\psi\rangle) = \beta A_R + \dots$



[Bianchi-Livine "LQG & QI", in QG Handbook 2022]

2 Zero Law vs Volume Law



Correlation Bound [Wolf et al, PRL 2008]

$$\boxed{\frac{1}{2\|O_A\|^2\|O_B\|^2}\left(\langle\psi|O_A O_B|\psi\rangle - \langle\psi|O_A|\psi\rangle\langle\psi|O_B|\psi\rangle\right)^2 \leq S_A(|\psi\rangle) + S_B(|\psi\rangle) - S_C(|\psi\rangle)}$$

- **Product State:** no correlations at space-like separation

  e.g.: coherent intertwiners $|\psi\rangle = \sum_{i_1 \dots i_N} \Phi_{i_1} \cdots \Phi_{i_N} |i_1\rangle \cdots |i_N\rangle$

  $\longrightarrow$ twisted geometry, collection of polyhedra with no correlations

- **Random State:** exp-small correlations at space-like separation

  $|\psi\rangle = \sum_{i_1 \dots i_N} \psi_{i_1 \dots i_N} |i_1\rangle \cdots |i_N\rangle$ with random coeff $\psi_{i_1 \dots i_N}$

$f_A = \frac{V_A}{V} < \frac{1}{2}$

$d_n = \begin{cases} 5 & j_0 = \frac{1}{2} \\ \sim \frac{8}{\pi} j_0^3 & j_0 \gg 1 \end{cases}$

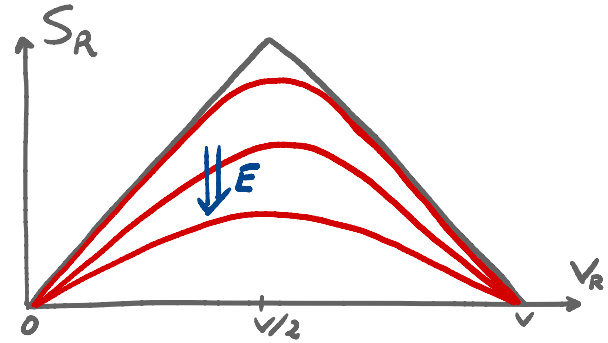
Typical Entropy Volume Law

$$\begin{cases} \langle S_A \rangle = f_A V \dfrac{\log d_n}{v_0} - \dfrac{1}{2} d_n^{-(1-2f_A)V/v_0} + O(d_n^{-V/v_0}) \\[2ex] \Delta S_A = \dfrac{1}{2} d_n^{-(1-f_A)V/v_0} + O(d_n^{-V/v_0}) \end{cases}$$

$\Rightarrow$ **Vanishing Angle-Angle Correlations**

$$\mathcal{G} = \langle \theta_{n\ell\tilde{\ell}} \, \theta_{n'\ell'\tilde{\ell}'} \rangle - \langle \theta_{n\ell\tilde{\ell}} \rangle \langle \theta_{n'\ell'\tilde{\ell}'} \rangle = O(d_n^{-V/2v_0})$$

3 Area Law & Semiclassicality



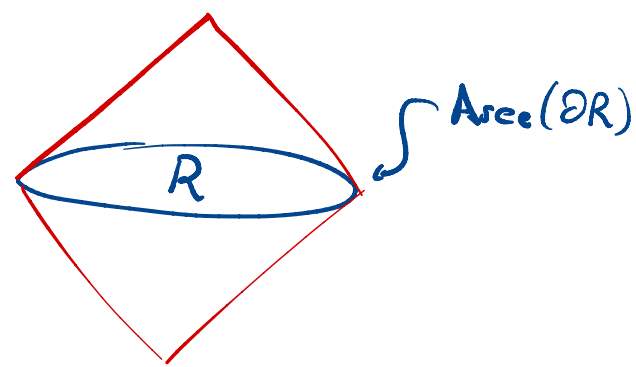
- In CM & QFT, as we lower the energy, we transition from volume-law to area-law
- In CM, zero law states are high-energy (not Fock in QFT)
- In QG, we don't have an immediate notion of energy or energy-density

$\Longrightarrow$ Reverse Perspective: Entanglement as a Probe

Architecture Conjecture

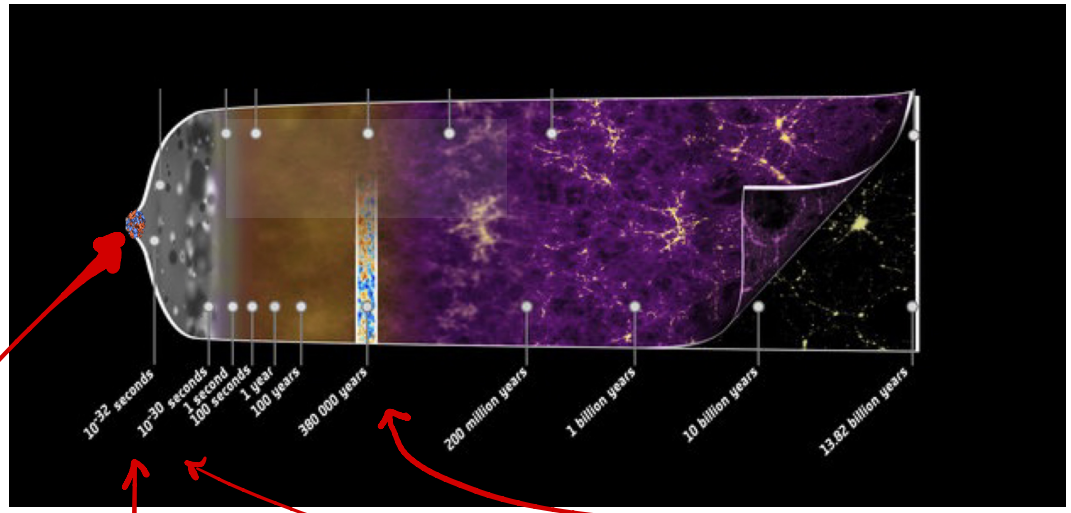
Semiclassical $|\Psi\rangle$ in QG belong to the area-law corner of $\mathcal{H}_{phys}$

$$S_R(|\Psi\rangle) = 2\pi \frac{\text{Area}(\partial R)}{\ell_p^2} + \dots$$

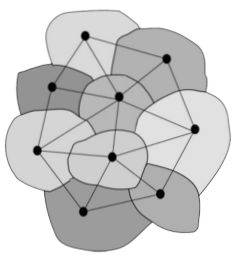


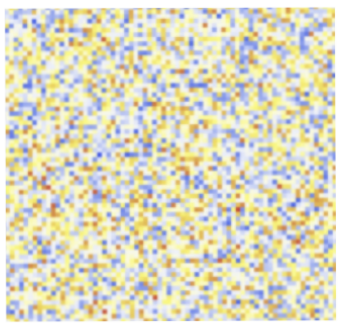
Bianchi-Myers [1212.5183] (CQG)
Bianchi-Guglielmon-Hackl-Yokomizo [1609.02219] (PRD)
Baytas-Bianchi-Yokomizo [1805.05856] (PRD)
Bianchi-Donà-Vilensky [1812.10996] (PRD)

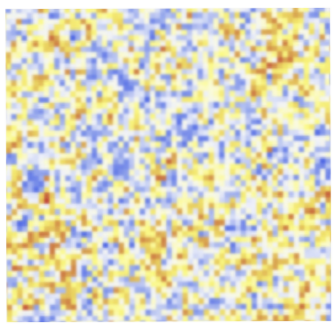
4 Primordial Entanglement: from Zero to Area Law ?

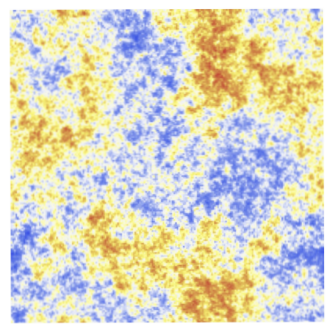


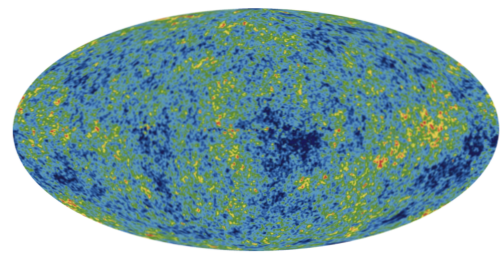
Planck Scale → Pre-Inflationary Phase → Inflation → Hot Big Bang & CMB











Zero Law ⟶ QFT Area Law

Scenario: BKL Phase, Cosmological Quantum Quench and the origin of correlations at space-like separation

Bianchi-Hackl-Yokomizo [1512.08959] (IJMPD)