Cosmic inflation prevents black hole singularity formation

Michał Bobula

University of Wrocław

based on MB, arXiv:2404.12243.

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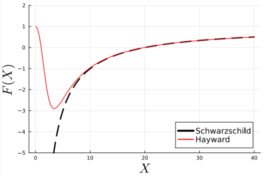
In this talk

- modifications to Oppenheimer-Snyder collapse different than in Francesco's talk, **Exterior $\rightarrow$ regular Hayward BH**
- What is the interior? How well the corresponding FRW geometry describe Universe?

Quantum-corrected Schwarzchild metric – Hayward black hole (Hayward, 2005), **No BH information paradox**:

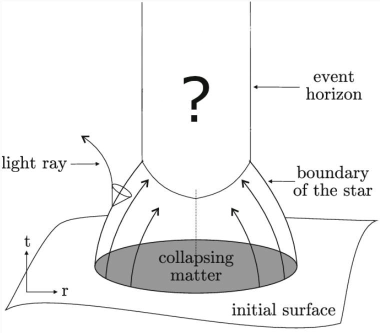
$$\mathrm{d}s^2_{\mathrm{ext}} = -F(X)\mathrm{d}t^2 + F(X)^{-1}\mathrm{d}X^2 + X^2\mathrm{d}\Omega^2 \quad (1)$$

$$F(X) = 1 - \frac{2MX^2}{X^3 + 2l^2 M}.$$



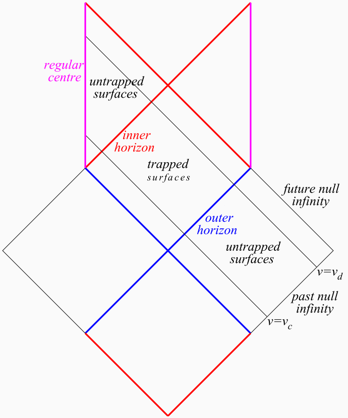
What is the FRW geometry modelling the interior of the dust ball?

$$\mathrm{d}s^2_{\mathrm{int}} = -\mathrm{d}T^2 + a(T)^2\,\mathrm{d}r^2 + r^2 a(T)^2\,\mathrm{d}\Omega^2. \quad (2)$$

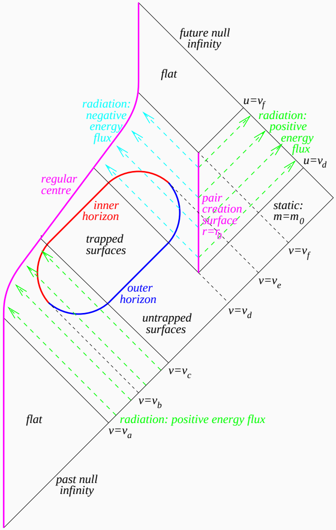


Two descriptions delivering Hayward black hole as a vacuum solution:
i) $D \geq 5$ Quasi-topological Gravity $\rightarrow$ E-H action supplemented with infinite tower of higher-curvature corrections. Eq. of motion of second order in sph. symm. (Bueno, et al. 2024)
ii) Giesel-Liu-Rullit-Singh-Weigl framework for polymerized LTB spacetimes (2024). Specific unbounded polymerization — different from the one present in the model discussed by Francesco.

Remarkably, for Hayward geometry the information paradox is (probably) non-existent. This is why it got so much attention in the literature.



Static Hayward black hole (2005)



Formation and evaporation of Hayward black hole within single asymptotic region (Hayward, 2005). Consistentency with Ashtekar-Bojowald paradigm (2005).

This talk: what are the details of Hayward black hole formation (without backreaction)?

A universe out of Hayward black hole

The assumptions for the modified Oppenheimer-Snyder collapse scenario are following

- the exterior vacuum is given by Hayward geometry
- interior FRW metric is smoothly joined with the exterior one. Its exact form will be determined by the junction conditions.
- the conservation of EM tensor holds. In particular $\rho = M / \left(\frac{4}{3}\pi r_b^3 a^3\right)$

These assumptions are compatible with Quasi-Topological gravity and the framework for polymerized LTB spacetimes.

The exterior is given by

$$\mathrm{d}s^2_{\mathrm{ext}} = -F(X)\mathrm{d}t^2 + F(X)^{-1}\mathrm{d}X^2 + X^2\mathrm{d}\Omega^2 \tag{3}$$

$F(X) = 1 - \frac{2MX^2}{X^3 + 2l^2 M}$.
The interior geometry is described by

$$\mathrm{d}s^2_{\mathrm{int}} = -\mathrm{d}T^2 + a(T)^2\ \mathrm{d}r^2 + r^2 a(T)^2\ \mathrm{d}\Omega^2. \tag{4}$$

with yet-to-be-determined (via junction conditions) scale factor a.

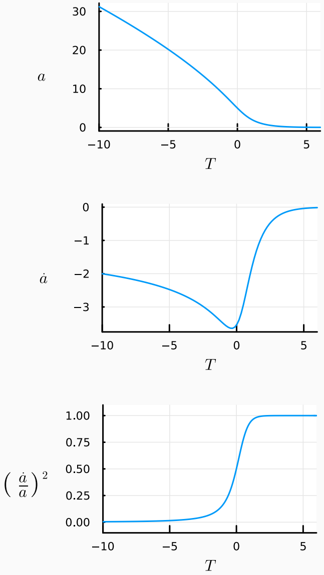
Junction conditions: minimal *geometric* requirement — metrics are C^1 smooth at the junction surface:
i) continuity of the induced metrics and
ii) continuity of extrinsic curvatures (first derivatives of metrics)
at the surface of the collapsing dust ball

The features of the derived universe

- Dynamics is governed by the modified Friedmann equations
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3+8\pi\ell^2\rho}$$
where RHS can exanded for small ρ (or small $\ell^2\rho$) as (infinite tower of higher-order corrections)

$$\frac{8\pi\rho}{3+8\pi\ell^2\rho} = \frac{8\pi\rho}{3} - \frac{64}{9}\left(\pi^2\ell^2\right)\rho^2 + \frac{512}{27}\pi^3\ell^4\rho^3 \ldots$$

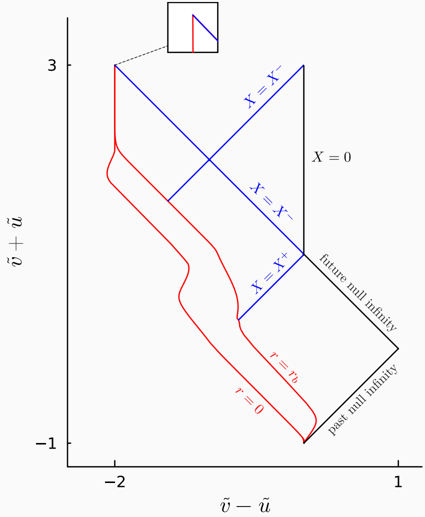
- No curvature singularities, the Kretschmann scalar (full contraction of Riemann tensor) is bounded everywhere. In particular, $\lim_{a\to 0} K = 24/\ell^4$.
- Universe is timelike geodesically complete. Free-falling observers "fall" for infinite proper time $T \in (-\infty, \infty)$
- Smooth transition from power-law contraction to the inflationary phase, with the graceful entrance.
- The energy density $\rho = M/\left(\frac{4}{3}\pi r_b^3 a^3\right)$ diverges in the final collapsing point $T \to \infty$ (equivalently $a \to 0$). However, neither timelike observer nor light ray reaches this point in finite affine time



Conformal diagram for quantum corrected OS collapse

- The collapse lasts forever as measured by the comoving observers
- Two horizons: X^+ and X^-. In general, inner horizon instabilities might be cured by Hawking radiation (Bonanno et al. 2022)
- There is only one radial light (null) ray reaching the final collapsing point (arriving there in infinite affine time)
- Hawking radiation included $\rightarrow$ infinite density $\rho \rightarrow \infty$ not reached. Black hole should evaporate in finite time.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3 + 8\pi\ell^2\rho}$$



Discussion

- Quasi-Topological gravity (EH action with infinite tower of higher curvature corrections) and polymerized LTB models $\rightarrow$ Hayward black hole as unique vacuum solution in sph. sym..
  I derived FRW dust dynamics viable for the interior in the modified OS collapse. Planck scale corrections $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3+8\pi l^2\rho}$

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- A lesson for cosmology: finite universes surrounded by black hole vacuum may circumvent Borde–Guth–Vilenkin theorem: *any spacetime where* $H_{av} = \frac{1}{\tau_{\mathrm{f}} - \tau_{\mathrm{i}}}\int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} H(\tau)d\tau > 0$ *is past incomplete*

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Thank you for your attention!

Backup slides

Very recently Hayward black hole was derived as a unique spherically symmetric vacuum solution within

- $D \geq 5$ Quasi-topological gravity – Einstein-Hilbert action supplemented with infinite tower of higher-curvature corrections (Bueno et al., 2024)

$$S_{\mathrm{QT}} = \frac{1}{16\pi G} \int \mathrm{d}^D x \sqrt{|g|} \left[R + \sum_{n=2}^{n_{\max}} \alpha_n \mathcal{Z}_n \right]$$

$\alpha_n = l^{n-1}$, $n_{\max} \to \infty$, $\mathcal{Z}_n$ are contractions of Riemann tensor (def. property: eq. of motion of the second order in sphericall symmetry). $\mathcal{Z}_2$ corresponds to Gauss-Bonnet density. Different, specific choices of α_n can deliver other regular black holes e.g. Bardeen, Dymnikova ones. Such an infinite tower is what one could naively expect as an ultraviolet completion (renormalisation) of GR.

- Giesel-Liu-Rullit-Singh-Weigl framework for polymerized LTB spacetimes (Giesel, et al., 2024). Contrary to Husain-Kelly-Santacruz-Wilson-Ewing model (the model's extension to include pressure was discussed by Francesco), the 'polymerization' function of gravitational Hamiltonian is unbounded.