Avenue to spin foam phenomenology —

Generalized spacetime geometries and continuum effective dynamics

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Motivation

- spin foams = covariant path integrals for quantum gravity

$$\int \mathscr{D}\mu(\text{geometry})\exp(iS[\text{geometry}])$$

geometry classical: $[g]$
geometry quantum: fuzzy

- phenomenon relevant for large class of spin-foam models:

(primary) simplicity constraints classical: {1st class}
(primary) simplicity constraints quantum: {1st class} $\cup$ {2nd class, $\{\,\cdot\,,\cdot\,\} \sim \gamma$} — imposed weakly

- consequence: path integral taken over extended configuration space

geometry semi-classical (effective): ?

Hypothesis: Semi-classical ($\leftrightarrow$ extended classical) configuration space is captured by area metrics.

plan:
- area metric
- evidence for area metrics in semi-classical limit of spin foams
- area-metric dynamics from "classical constraints imposed weakly"
- effective length-metric action
- geometric interpretation of non-metric d.o.f.

$\Rightarrow$ prospect for phenomenology & …

Area metrics

area metric [Schuller & Wohlfahrt, 2005; Punzi, Schuller & Wohlfarth, 2006]

$G_p : \Lambda^2 T_p\mathscr{M} \times \Lambda^2 T_p\mathscr{M} \to \mathbb{R} \quad \in \mathscr{T}_p^{(0,4)}(\mathscr{M})$

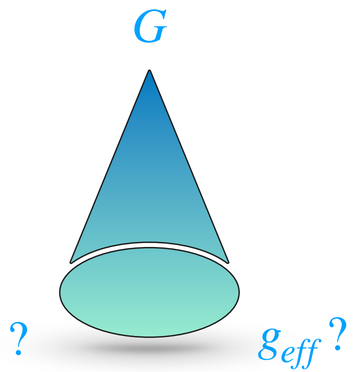
i) $G_{\mu\nu\rho\sigma} = G_{\rho\sigma\mu\nu}$
ii) $G_{\mu\nu\rho\sigma} = -\,G_{\nu\mu\rho\sigma} \;\; (= -\,G_{\mu\nu\sigma\rho})$
iii) $G_{IJ}\,,\;\; I,J = [\mu\nu] = 1,\ldots,d(d-1)/2$ non-degenerate

$G(A,A) = \|A\|^2$ area of parallelogram A (viewed as simple bivector $A = l \wedge r$)
$G(l \wedge t, r \wedge t) \leftrightarrow 3d$ dihedral angle between planes ($\perp$ bivectors) intersecting at a line

cyclic area metrics

iv) $G_{\mu[\nu\rho\sigma]} = 0$

$d = 4 : 21 - 1 = 20$ d.o.f.



length metric

$g_p : T_p\mathscr{M} \times T_p\mathscr{M} \to \mathbb{R} \quad \in \mathscr{T}_p^{(0,2)}(\mathscr{M})$

i) $g_{\mu\nu} = g_{\nu\mu}$

ii) $g_{\mu\nu}\,,\;\; \mu,\nu = 1,\ldots,d$ non-degenerate

$g(l,l) = \|l\|^2$ length of vector l
$g(r,l) \leftrightarrow 2d$ angle between vectors l, r intersecting at a point

induced area metric

$G^g_{\mu\nu\rho\sigma} = g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}$

$d = 4 : 10$ d.o.f.

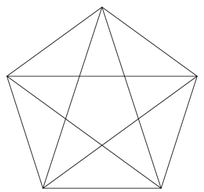
Area Regge action — continuum limit

l_e: length of edge e
A_t: area of triangle t
$\theta_{\sigma,t}$: dihedral angle at triangle t in simplex σ
$\epsilon_t = 2\pi k - \sum_{\sigma \supset t} \theta_{\sigma,t}$: deficit angle at triangle t

length Regge calculus [Regge, 1961]

$$S_{LR}[l_e] = \sum_t A_t(l_e)\epsilon_t(l_e) \quad \Leftrightarrow \quad \textbf{vacuum GR} \text{ on piecewise flat } \triangle^{(d=4)}, \quad \epsilon_t \leftrightarrow \text{curvature}$$

change of variables $l_e \to A_t$ invertible only locally



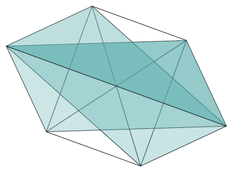
one simplex:
$n_{l_e} = n_{A_t} = 10$

area Regge calculus [Rovelli, 1993; Barret, Rocek & Williams, 1999; Asante, Dittrich & Haggard, 2018]

$$S_{AR}[A_t] = \sum_t A_t\epsilon_t(A_t) \quad \Leftrightarrow \quad \textbf{dynamics of many more d.o.f.}, \quad \epsilon_t \leftrightarrow \text{curvature + shape mismatch}$$

$\epsilon_t(A_t) = 0$ e.o.m. do not imply flatness!

describes semi-classical limit of spin foams



glued simplices:
$n_{A_t} > n_{l_e}$

lattice continuum limit of Area-Regge action: [Dittrich, 2021; Dittrich & Kogios, 2022]

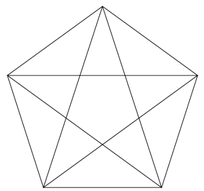
$S_{AR}[A_t] \xrightarrow{\lim_{\lambda\to 0}} S[G]$ **dynamics of area metric G associated to hypercube,** $G \leftrightarrow g$ (massless graviton) + massive d.o.f.

$$S_{eff}[g] = S_{EH}[g] + \text{Weyl}^2 + \mathcal{O}(\lambda^4)$$

Semi-classical simplex geometry

τ: tetrahedron
σ: 4-simplex

[Dittrich & Padua-Argüelles, 2023]



microscopic identification: semi-classical spin foam d.o.f. $\leftrightarrow$ area-metric G d.o.f.

- **classical σ:**
  10 d.o.f. = 10 x lengths or 10 x areas

- **quantum σ:** 5 quantum τ glued as dictated by quantum uncertainty
  quantum τ: 4 x norms of normal vectors (4 x areas) + 1 x inner product between two normals (1 x $3d$ dihedral angle)
  only areas of the 10 triangles shared by pairs of tetrahedra can be identified
  15 d.o.f. = 5 x 5 − 10

- **semi-classical/coherent/twisted σ:** 5 classical τ with quantum glue
  classical τ: 6 x lengths or 4 x areas + 2 x $3d$ dihedral angles at non-opposite edges
  only areas of the 10 triangles shared by pairs of tetrahedra are identified
  20 d.o.f. = 5 x 6 − 10 $\leftrightarrow$ 10 x areas + 10 x $3d$ dihedral angles (2 x $3d$ dihedral angles per τ, at non-opposite edges)

20 d.o.f. of a semi-classical σ can be arranged into 20 d.o.f. for an area metric G associated to σ [Dittrich & Padua-Argüelles, 2023]

⇒ Microscopic & macroscopic area metrics appear in the semi-classical regime of spin foams.

Goal:

Parametrize the continuum classical extended configuration space by area metrics, starting from the configuration variables in the Plebanski formulation of GR (↔ origin of spin-foam quantization).

Reproduce the continuum effective dynamics of spin foams by weakening a subset of the classical constraints.

Plebanski formulation of general relativity

non-chiral Plebanski action [Plebanski, 1977; Capovilla, Jacobson, Dell & Mason, 1991, Reisenberger, 1995; Pietri & Freidel, 1999]

B field: so(4) valued 2-form
ω connection: so(4) valued 1-form
ϕ: Lagrange multiplier
γ: Barbero-Immirzi parameter

$$S_P[B,\omega,\phi] = \int \underbrace{\delta_{IJKL} B^{IJ} \wedge F^{KL}(\omega)}_{\textbf{BF}} \underbrace{+ \frac{1}{2\gamma} \epsilon_{IJKL} B^{IJ} \wedge F^{KL}(\omega)}_{\textbf{Holst}} \underbrace{- \frac{1}{2} \phi_{IJKL} B^{IJ} \wedge B^{KL}}_{\delta\phi\ :\ \textbf{constraints}}, \qquad \phi_{IJKL} = \phi_{KLIJ} = -\phi_{JIKL}, \ \phi_{IJKL}\epsilon^{IJKL} = 0$$

simplicity constraints

$$B^{IJ} \wedge B^{KL} = \nu \epsilon^{IJKL}, \quad \nu = \frac{1}{4!} B^{AB} \wedge B^{CD} \epsilon_{ABCD} \quad (\nu \neq 0)$$

4 solution sectors

$B^{IJ} = \pm (\star e \wedge e)^{IJ}$ gravitational sector (—Palatini action for GR)

$B^{IJ} = \pm (e \wedge e)^{IJ}$ topological sector

Modified Plebanski theories

Plebanski action [Plebanski, 1977; Capovilla, Jacobson, Dell & Mason, 1991]
BF (+ Holst) + simplicity constraints (ϕ Lagrange multiplier)

modified Plebanski action [Krasnov, 2006-9; Bengtsson, 2007]
BF (+ Holst) + potential $V(\phi)$

e.o.m. from varying ϕ: $B \wedge B = \nu\left(\epsilon + \frac{\partial V}{\partial \phi}\right) \xrightarrow{\det\left(V^{(2)}\right) \neq 0} \phi(B)$

$$S_{P,mod.}[B,\omega] = \int \delta_{IJKL} B^{IJ} \wedge F^{KL}(\omega) \; + \; \frac{1}{2\gamma}\epsilon_{IJKL} B^{IJ} \wedge F^{KL}(\omega) \; + \; V(B), \quad \omega(B) \text{ solution to } d_\omega B = 0 \longrightarrow S_{P,mod.\,eff.}[B] \text{ second-order}$$

[Freidel, 2008; Speziale, 2010; Beke, Palmisano & Speziale, 2012]

non-degenerate case $\det\left(V^{(2)}\right) \neq 0 \Leftrightarrow$
full set of simplicity constraints $\mapsto$ potential for B
chiral (self-dual), $\mathscr{L}(G) = \mathrm{su}(2)$: 2 massless propagating d.o.f. = graviton, "deformations of GR" [Krasnov, 2006-9; Freidel, 2008]
non-chiral, $\mathscr{L}(G) = \mathrm{so}(4)$: $8 = 2 + 6$ d.o.f. [Alexandrov & Krasnov, 2008]

degenerate case $\det\left(V^{(2)}\right) = 0 \Leftrightarrow$
full set of simplicity constraints $\mapsto$ subset of simplicity constraints (sharp) + potential for rest of B (suppressed)

goal: use mechanism to derive area-metric action from modified non-chiral Plebanski action: $S_{P,mod.\,eff.}[B^{IJ}_{\mu\nu}] \to S[G]$ [JB & Dittrich, 2022]

Area-metric action from modified non-chiral Plebanski action

counting d.o.f.

$B^{IJ}_{\mu\nu}$: 36 = 30 + 6 (SO(4) gauge) d.o.f.
$\delta\phi$: 20 simplicity constraints (2,0) $\oplus$ (0,2) $\oplus$ (1,1) $\oplus$ (0,0)

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e^I_μ : 16 = 10 + 6 (SO(4) gauge) d.o.f.

$g_{\mu\nu}$: 10 d.o.f.

$B^{IJ}_{\mu\nu}$: 36 = 30 + 6 (SO(4) gauge) d.o.f.
$\delta\phi'$: 10 simplicity constraints — **which subset?**

$G_{\mu\nu\rho\sigma}$: 20 (gauge-invariant) d.o.f.

parametrization of B-field

i) so(4) = su(2)$_+$ $\oplus$ su(2)$_-$ isomorphism, $P^{IJ}_{\pm\, i} = \pm\, \delta^{IJ}_{0i} + \frac{1}{2}\epsilon^{IJ}_{0i}$ projectors: $B^{IJ}_{\mu\nu} = P^{IJ}_{+\, i} B^i_{+\, \mu\nu} + P^{IJ}_{-\, i} B^i_{-\, \mu\nu}$

ii) parametrization of su(2) B-field $B_\pm$ using Urbantke theorem: 4d metric $g_\pm$ & unimodular 3x3 matrix of scalars $b_\pm$

[Urbantke, 1984; Freidel, 2008]

$$B^i_{\pm\, \mu\nu} = b^i_{\pm\, a} \Sigma^a_{\pm\, \mu\nu}(e_\pm)$$

$\Sigma^a_{\pm\, \mu\nu}(e_\pm) = \pm\, e^0_{\pm\, [\mu} e^i_{\pm\, \nu]} + \epsilon^i_{jk} e^j_{\pm\, \mu} e^k_{\pm\, \nu}$ Plebanski 2-forms (self-/antiself-dual) — encode 2 x spacetime metrics: $g_{\pm\, \mu\nu} = e^I_{\pm\, \mu} e^J_{\pm\, \nu} \delta_{IJ}$
$b_\pm$ define 2 x unimodular internal metrics: $q_{\pm\, ab} = b^i_{\pm\, a} b^j_{\pm\, b} \delta_{ij}$

$B^{IJ}_{\mu\nu}(36) \quad \Leftrightarrow \quad g_{+\, \mu\nu}(10),\ g_{-\, \mu\nu}(10),\ b^i_{+\, a}(8),\ b^i_{-\, a}(8)$

$S_{P,\, mod.\, eff.}[B^{IJ}_{\mu\nu}] = S_{P,\, mod.\, eff.}[e^I_{+\, \mu}, e^I_{-\, \mu}, q_{+\, ab}, q_{-\, ab}] = S_{BF\, eff.,\, su(2)_+}[e^I_{+\, \mu}, q_{+\, ab}] + S_{BF\, eff.,\, su(2)_-}[e^I_{-\, \mu}, q_{-\, ab}] + \int V[e^I_{+\, \mu}, e^I_{-\, \mu}, q_{+\, ab}, q_{-\, ab}]$ {Speziale, 2010]

Area-metric action from modified non-chiral Plebanski action

$B^{IJ}_{\mu\nu}(36) \quad \Leftrightarrow \quad g_{+\,\mu\nu}(10),\, g_{-\,\mu\nu}(10),\, b^{i}_{+\,a}(8),\, b^{i}_{-\,a}(8)$

area metric from B-field

define cyclic left- and right-handed area metrics:

$$G_{\pm\,\mu\nu\rho\sigma} \equiv \frac{1}{2}\Big((B^i_\pm \otimes B_{\pm\,i})_{\mu\nu\rho\sigma} - \frac{1}{4!}(B^i_\pm \otimes B_{\pm\,i})_{\alpha\beta\gamma\delta}\epsilon^{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\rho\sigma}\Big)$$

imposing equal left- and right-handed area metrics: [Reisenberger, 1995]

$G_{+\,\mu\nu\rho\sigma} \overset{!}{=} G_{-\,\mu\nu\rho\sigma} \quad \Leftrightarrow$ **20 simplicity constraints** $(0,0) \oplus (1,1) \oplus (2,0) \oplus (0,2) \quad \longrightarrow \quad S[g_{\mu\nu}]$, **GR**

imposing flat left- and right-handed internal metrics: [Speziale, 2010; Beke, Palmisano & Speziale, 2012]

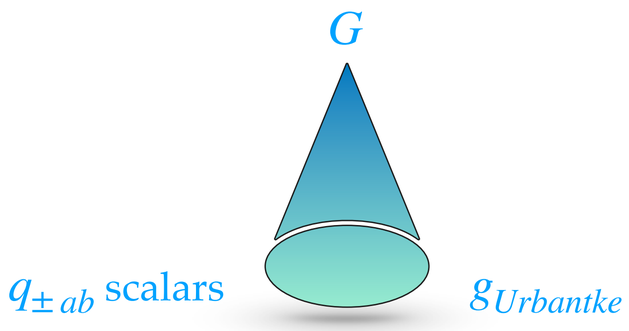
$q_{\pm\,ab} \overset{!}{=} \delta_{ab} \quad \Leftrightarrow$ **10 simplicity constraints** $(0,0) \oplus (1,1) \oplus (2,0) \oplus (0,2) \quad \longrightarrow \quad S[g_{+\,\mu\nu}, g_{-\,\mu\nu}]$, **bi-metric gravity** (unstable)

imposing equal left- and right-handed length metrics: [JB & Dittrich, 2022]

$g_{+\,\mu\nu} \overset{!}{=} g_{-\,\mu\nu} \quad \Leftrightarrow$ **10 simplicity constraints** $(0,0) \oplus (1,1) \oplus (2,0) \oplus (0,2) \quad \longrightarrow \quad S[g_{\mu\nu}, q_{+\,ab}, q_{-\,ab}] \mathrel{\hat{=}} S[G_{\mu\nu\rho\sigma}]$, **area-metric action**

$$G_{\mu\nu\rho\sigma} \equiv G_{+\,\mu\nu\rho\sigma} + G_{-\,\mu\nu\rho\sigma} = \frac{1}{2}\sum_{\pm} q_{\pm\,ab}\Sigma^a_{\mu\nu}(e)\Sigma^b_{\rho\sigma}(e) - \frac{1}{6}\mathrm{Tr}(q_+ - q_-)\det(e)\epsilon_{\mu\nu\rho\sigma}$$

$G_{\mu\nu\rho\sigma}(20) \quad \Leftrightarrow \quad g_{\mu\nu}(10),\, q_{+\,ab}(5),\, q_{-\,ab}(5)$



Area-metric action from modified non-chiral Plebanski action

$$S[G_{\mu\nu\rho\sigma}] \mathrel{\hat{=}} S_{P,\,mod.\,eff.}[e^I_\mu, q_{+\,ab}, q_{-\,ab}] = S_{BF\,eff.,\,su(2)_+}[e^I_\mu, q_{+\,ab}] + S_{BF\,eff.,\,su(2)_-}[e^I_\mu, q_{-\,ab}] + \int V[e^I_\mu, q_{+\,ab}, q_{-\,ab}]$$

length metric and internal metrics from area metric

invert $G(g, q_+, q_-) \quad \longrightarrow \quad g(G), q_+(G), q_-(G)$ (20 polynomial equations)

perturbative inversion in background expansion: [JB & Dittrich, 2022]

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$$
$$q_{\pm\,ab} = \delta_{ab} + \chi_{\pm\,ab}\,, \quad \mathrm{Tr}(\chi_\pm) = 0$$

$$G_{\mu\nu\rho\sigma} = 2\delta_{\mu[\rho}\delta_{\sigma]\nu} + a_{\mu\nu\rho\sigma}\,, \quad a_{\mu\nu\rho\sigma} = \mathbb{L}^{\lambda\tau}_{\mu\nu\rho\sigma} h_{\lambda\tau} + 2\mathbb{P}^{ab}_{+\,\mu\nu\rho\sigma}\chi_{+\,ab} + 2\mathbb{P}^{ab}_{-\,\mu\nu\rho\sigma}\chi_{-\,ab}$$

invert $a(h, \chi_+, \chi_-) \quad \longrightarrow \quad h(a), \chi_+(a), \chi_-(a)$

$$S^{(2)}[a_{\mu\nu\rho\sigma}] \mathrel{\hat{=}} S^{(2)}_{P,\,mod.\,eff.}[h_{\mu\nu}, \chi_{+\,ab}, \chi_{-\,ab}] = S^{(2)}_{BF\,eff.,\,su(2)_+}[h_{\mu\nu}, \chi_{+\,ab}] + S^{(2)}_{BF\,eff.,\,su(2)_-}[h_{\mu\nu}, \chi_{-\,ab}] + \int V^{(2)}[\chi_{+\,ab}, \chi_{-\,ab}]$$

Area-metric action from modified non-chiral Plebanski action

quadratic potential introducing mass terms for the χ-fields: [JB & Dittrich, 2022]

$$S^{(2)}_{P,\,mod.\,eff.}[h_{\mu\nu},\chi_{+\,ab},\chi_{-\,ab}] = \frac{\gamma_+}{2}\int \mathscr{L}^{(2)}_{EH}(h_{\mu\nu}+\chi_{+\,\mu\nu}) + \frac{\gamma_-}{2}\int \mathscr{L}^{(2)}_{EH}(h_{\mu\nu}+\chi_{-\,\mu\nu}) + \frac{\gamma_+}{8}\int m_+^2\chi_{+\,\mu\nu}\chi_+^{\mu\nu} + \frac{\gamma_-}{8}\int m_-^2\chi_{-\,\mu\nu}\chi_-^{\mu\nu}\,, \qquad \gamma_\pm = 1 \pm \frac{1}{\gamma}$$

$\chi_{\pm\,\mu\nu} \equiv 4\mathbb{P}^{ab}_{\pm\,\mu\nu\rho\sigma}\dfrac{k^\rho k^\sigma}{\triangle}\chi_{\pm\,ab}$ isometric embedding $\{\chi_{ab}\,|\,\chi_{ab}=\chi_{ba}\,,\,\mathrm{Tr}(\chi)=0\} \to \{\chi_{\mu\nu}\,|\,\chi_{\nu\mu}=\chi_{\mu\nu}\,,\,\mathrm{Tr}(\chi)=0\}$, $\chi_{ab}\chi^{ab}=\chi_{\mu\nu}\chi^{\mu\nu}$, $\partial^\mu\chi_{\mu\nu}=0$

$\mathscr{L}^{(2)}_{EH} \supset$ Pauli-Fierz operator $\mathscr{E}^{\mu\nu\rho\sigma} \equiv \dfrac{\triangle}{4}\left({}^2P^{\mu\nu\rho\sigma} - 2\,{}^0P^{\mu\nu\rho\sigma}\right)$ (incl. spin-0 & spin-2 projectors)

symmetries:

- **linearized diffeomorphisms**

- **BF shift symmetry broken by mass terms — propagating d.o.f.**
  chiral: $(h,\chi_\pm) \to (H_\pm \equiv h+\chi_\pm,\chi_\pm)$ non-local field redefinition; integrate out $\chi_\pm$ $\longrightarrow$ $S^{(2)}_{EH}[H_\pm]$ with "shifted graviton"
  non-chiral: existence & type of potential additional d.o.f. not settled

Effective length-metric action

integrate out $\chi_\pm$: (in Fourier space, $S^{(2)} \equiv \sum_k \mathscr{L}^{(2)}(k) = \sum_k \phi(-k)H(k)\phi(k)\,, \quad \triangle = k_\mu k_\nu \delta^{\mu\nu}$) [JB & Dittrich, 2022]

$$\mathscr{L}^{(2)}_{eff.}(h_{\mu\nu}) = \mathscr{L}^{(2)}_{EH}(h_{\mu\nu}) - \frac{1}{4}{}^{(1)}C_{\mu\nu\rho\sigma}\left(\frac{1}{m_+^2+\triangle} + \frac{1}{m_-^2+\triangle}\right){}^{(1)}C^{\mu\nu\rho\sigma}$$

- GR to lowest order
- full corrections: quadratic in Weyl tensor, non-local, ghost-free (for $m_+ = m_-$)
- lattice continuum limit of Area-Regge action reproduced at next order: $\mathscr{L}^{(2)}_{eff.} = \mathscr{L}^{(2)}_{EH} + \mathrm{Weyl}^2$ [Dittrich, 2021; Dittrich & Kogios, 2022]

Geometric interpretation of non-metric d.o.f.

on-shell connection

Gauss law: $d_\omega B = 0$
splits into two independent su(2) Gauss laws: $d_{\omega_\pm} B^i_\pm = dB^i_\pm + \omega^i_{\pm\, j} \wedge B^j_\pm = 0$ (assuming non-degenerate tetrads)

two equivalent perspectives:

[Freidel, 2008; Speziale, 2010; Beke, Palmisano & Speziale, 2012]
$d_{\omega_\pm} B^i_\pm = 0 \quad \Leftrightarrow \quad d_{A_\pm} \Sigma_\pm(e) = 0\,, \quad d_{A_\pm} q_{\pm\, ab} = 0$ where $A_\pm \equiv b_\pm^{-1} db_\pm + b_\pm^{-1} \omega_\pm b_\pm$ **torsion-free & non-metric** w.r.t δ_{ij}

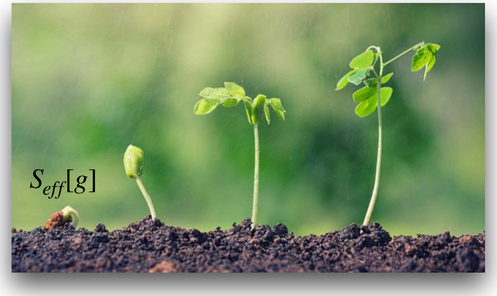
[JB & Dittrich, w.i.p.]
$d_{\omega_\pm} B^i_\pm = 0 \quad \Leftrightarrow \quad d_{\omega_\pm} \Sigma_\pm(e) \neq 0\,, \quad d_{\omega_\pm} \delta_{ij} = 0$ i.e. $\omega_\pm$ **torsion-full & metric** w.r.t δ_{ij}

Non-metric d.o.f. of area metric source torsion!

Summary & prospects

- spin foam path integral defined on extended configuration space
- semi-classical configuration space parametrized by area metrics
- indications: continuum limit of Area-Regge action; geometry of a twisted simplex
- continuum effective dynamics of spin foams reproduced by modified non-chiral Plebanski theories with area metric as configuration variable, obtained by imposing equal left-/right-handed length-metric geometries & replacing remaining half of simplicity constraints by potential
  G area metric $\leftrightarrow$ g length metric + scalars (—torsion)
- $S_{eff}[g] = S_{EH}[g]$ + Weyl^2 non-local, ghost-free correction

phenomenology:
- corrections to Newton potential & black hole entropy?
- hairy black holes à la Einstein-Weyl gravity & beyond?
- theoretical & observational bounds on scalar masses? constraints on γ?



conceptual:
- full non-linear GR from area-metric dynamics? (via alternative parametrization of B-field?)
- canonical counting of d.o.f.
- different choices of potential

fundamental:
- towards area metric geometry: area-metric compatible connection, curvature (continuum & discrete)
- "canonical" derivation of black hole entropy in language of area metrics?
- torsion scalar potential as dynamical or emergent cosmological constant? chiral vs non-chiral?
- possible relation to non-symmetric gravity & fundamental fields in string theory?