Gravity mediated entanglement growth: spacetime superpositions in the lab and the possibility to probe Planck time.

"On the possibility of laboratory evidence for quantum superposition of geometries", Phys. Lett. B, Vol. 792 64-68, arXiv:1808.05842, C.Rovelli and MC

"On the possibility of experimental detection of time discreteness", arXiv:1808.05842 , C.Rovelli and MC

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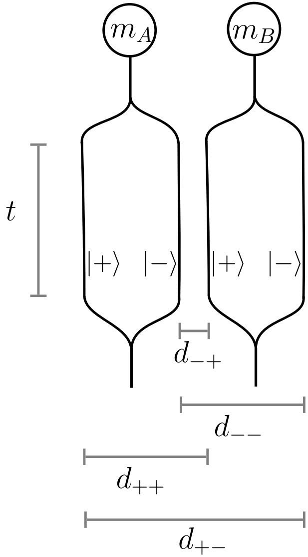
A proposal for a feasible bench-top quantum gravity experiment from the quantum information community

- Two works simultaneously appeared: "*Spin entanglement witness for quantum gravity*", by Bose et al, and "*Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity*", by Marletto and Vedral (PRL Dec 2017).
- The idea is that if we observe entanglement growth in two particles due to their gravitational interaction alone we are compelled to conclude that the medium, the gravitational field, also has quantum features. In particular, the authors show that the joint state of the system cannot be separable into gravity and matter sectors throughout the duration of the experiment.
- In this talk we see what is learned by taking into account general relativity, where the effect is understood as taking place because the gravitational field is set in a superposition of distinct macroscopic (semiclassical) geometries. Particularly intriguing is the role played by the Planck mass and the implication that this interference experiment is probing extraordinarily small proper time scales.

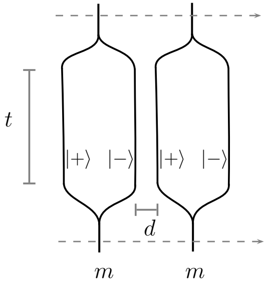
The experiment



$m_A = m_B = m$

- Two masses with embedded spin each set in path superposition through a Stern-Gerlach type apparatus. Say each mass A and B prepared in $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$ initially.
- Work in static limit: relevant dynamics is evolution by $e^{i\frac{Et}{\hbar}}$ of the quantum state when in path superposition (during t).
- There are four quantum branches : $++$,$--$,$-+$, $+-$. The simplest case is when the gravitational interaction of the masses can be neglected in all but the $-+$ branch where they are at closest approach.
- Then, the state $|-+\rangle \equiv |-\rangle_A \otimes |+\rangle_B$ picks up a relative phase difference $\delta\phi$ wrt to the other branches because of an interaction mediated by gravity alone.

The experiment

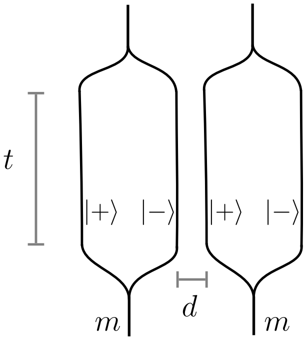


$$|\Psi_f\rangle \propto \left(|{+}{+}\rangle + |{-}{-}\rangle + |{+}{-}\rangle + e^{i\delta\phi(m,d,t)}\,|{-}{+}\rangle\right) \otimes |g\rangle$$

$$\begin{aligned} |\Psi_i\rangle &= |\psi_A\rangle \otimes |\psi_B\rangle \otimes |g\rangle \\ &\propto \left(|{+}{+}\rangle + |{-}{-}\rangle + |{+}{-}\rangle + |{-}{+}\rangle\right) \otimes |g\rangle \end{aligned}$$

At input the two particles are not entangled. If $\delta\phi \bmod 2\pi \neq 0$ the output is an entangled state. The phase is a function of the experimental parameters $\delta\phi(m, d, t)$. The feasibility study by Bose et al proposes specific experimental parameters m,t,d for which $\delta\phi(m, d, t) \sim 1$.

Newtonian approximation



The gravitational energy of two masses m at distance d is

$$E_g = \frac{Gm^2}{d}$$

Evolution of the $|-+\rangle$ branch when in path superposition picks up a phase difference

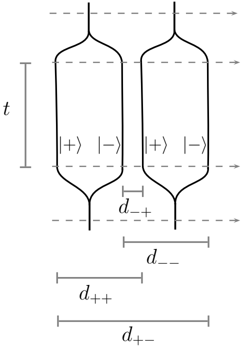
$$e^{i\delta\phi} = e^{i\frac{E_g\,t}{\hbar}}$$

This gives

$$\boxed{\delta\phi = \frac{Gm^2 t}{d\,\hbar}}$$

The above relation can be derived using time dilation.

The effect as due to spacetime set in a superposition



$$|\Psi_f\rangle \propto \left(|{++}\rangle + |{--}\rangle + |{+-}\rangle + e^{i\delta\phi(\delta\tau,m)}\,|{-+}\rangle\right) \otimes |g\rangle$$

$$|\Psi_2\rangle \propto |{++}\rangle\,|g_{++}\rangle + |{--}\rangle\,|g_{--}\rangle + |{+-}\rangle\,|g_{+-}\rangle + e^{i\delta\phi(\delta\tau,m)}\,|{-+}\rangle\,|g_{-+}\rangle$$

$$|\Psi_1\rangle \propto |{++}\rangle\,|g_{++}\rangle + |{--}\rangle\,|g_{--}\rangle + |{+-}\rangle\,|g_{+-}\rangle + |{-+}\rangle\,|g_{-+}\rangle$$

$$|\Psi_i\rangle \propto |\psi_A\rangle \otimes |\psi_B\rangle \otimes |g\rangle$$

Assume that the gravity state space contains semiclassical states and superposition of those. Assume validity of GR at these mass scales.

When the masses are in a superposition, the gravitational field is not in a semiclassical state, but its state is a sum of four semiclassical states. Each is peaked on a diffeomorphically inequivalent metric.

Entanglement growth as due to relative time dilation between quantum branches

The classical spacetime sourced by the two masses is approximately a static weak field configuration. That is, in some coordinate system the metric takes the form:

$$\begin{aligned} \mathrm{d}s^2 &= -\left(1 + 2\phi(\vec{x})\right)\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 \\ \phi &= \phi_A + \phi_B \\ \phi_{A,B} &= -Gm/r \quad r > R \\ \phi_{A,B} &= -Gm/R \quad r < R \end{aligned}$$

where R is the mass radius. Inside each particle

$$\mathrm{d}s^2 \approx -\left(1 - r_S/R - r_S/d\right)\mathrm{d}t^2 + \mathrm{d}\vec{x}^2$$
$$r_S = 2Gm/c^2$$

$$\begin{aligned} \tau &= \int_0^t \mathrm{d}\tau \\ &\approx \sqrt{1 - \frac{2Gm}{c^2 R} - \frac{2Gm}{c^2 d}} \int_0^t \mathrm{d}t \\ &\approx \left(1 - \frac{r_S}{R} - \frac{r_S}{d}\right) t \ , \ r_S = \frac{Gm}{c^2} \end{aligned}$$

$$\delta\tau = \frac{r_S}{d} t = \frac{Gmt}{dc^2}$$

Relative time dilation $\delta\tau$ due to the gravitational interaction between the two masses. Since proportional to ratio r_S/d it corresponds to a tiny time interval, which however can be picked up by interference.

Gravity mediated entanglement growth as due to gravitational redshift.

In the previous slide we derived the relative time dilation in the quantum branch where the masses are at closest approach:

$$\delta\tau = \frac{Gmt}{dc^2}$$

The branch of closest approach develops a relative phase

$$\delta\phi = \frac{E\delta\tau}{\hbar} = \frac{mc^2\delta\tau}{\hbar} = \frac{Gm^2t}{d\hbar}$$

This is the same relation as in the Newtonian treatment. Notice that c^2 cancels above as the effect is non-relativistic in the sense that it survives the $c \to \infty$ limit.

The role of Planck mass

We saw that $\delta\phi$ in terms of the mass m and relative time dilation $\delta\tau$ reads:

$$\delta\phi = \frac{mc^2\delta\tau}{\hbar}$$

Rearrange constants:

$$\boxed{\delta\phi = \frac{m}{m_P}\frac{\delta\tau}{t_P}}$$

By directly manipulating quantum superpositions of Planck mass particles that interact gravitationally, perhaps we can indirectly probe time intervals at the Planck time scale by detecting interference. That is, fix m and measure $\delta\phi$ to deduce $\delta\tau$.

Plug in numbers for $\delta\tau$

Experimental parameters considered feasible with current technological capabilities: $d \sim 10^{-4}m$, $t \sim 1s$, $m \sim 10^{-6}m_P$

$$\delta\tau = \frac{Gmt}{dc^2} = 10^{-38}s = 10^6 t_P$$

For comparison, precision direct measurments of time with atomic clocks are currently at $\sim 10^{-19}s$.

The formula in terms of Planck mass and Planck time makes clear that the relative time dilation probed is determined by the mass used alone:

$$\delta\phi = \frac{m}{m_P}\frac{\delta\tau}{t_P} = \pi$$
$$\Rightarrow \delta\tau \sim 10^6 t_P$$

Entanglement entropy

First trace out all dof except spins in the final state

$$|\psi\rangle = \frac{1}{4}\bigg(|{+}{+}\rangle + |{+}{-}\rangle + |{-}{+}\rangle + e^{i\delta\phi} |{-}{-}\rangle \bigg)$$

Trace out spin dof of mass B

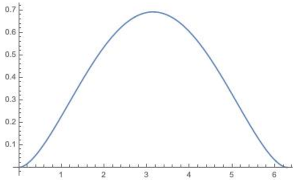
$$\rho_A = \mathrm{Tr}_B \, |\psi\rangle \langle\psi|$$

Density matrix

$$\rho_A = \frac{1}{2}\bigg(|+\rangle \langle+| + |-\rangle \langle-| \bigg) + \frac{e^{i\delta\phi}+1}{4} |+\rangle \langle-| + \frac{e^{-i\delta\phi}+1}{4} |-\rangle \langle+|$$

Diagonalize and calculate entanglement entropy

Entanglement entropy



The entanglement entropy for $\delta\phi \in \{0, 2\pi\}$. The maximum is for $\delta\phi = \pm\pi$ where $I = \log 2$.

$$\rho_1 = +\frac{\sqrt{1+\cos\delta\phi}}{2\sqrt{2}} + \frac{1}{2}$$
$$\rho_2 = -\frac{\sqrt{1+\cos\delta\phi}}{2\sqrt{2}} + \frac{1}{2}$$

When $\delta\phi = 0$, $\rho_1 = 1$ and $\rho_2 = 0$, thus $I(\rho_A) = 0$, i.e. there is no interference in the output. When $\delta\phi = \pi$, $\rho_1 = 1/2$ and $\rho_2 = 1/2$ the state is maximally entangled.

What happens if the superposed masses are Planckian?

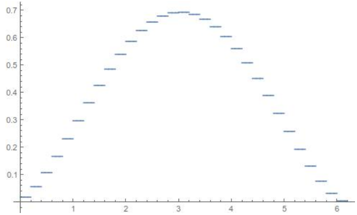
$$\boxed{\delta\phi = \frac{m}{m_P}\frac{\delta\tau}{t_P}}$$

If $\delta\phi \sim \pi$ and $m = m_P$

$$\delta\tau \sim t_P$$

Does proper time take continuous values in this regime? What happens to entanglement entropy if we assume that $\delta\tau$ takes discrete values?

Quantum levels in entanglement entropy?



The entanglement entropy for $\delta\phi \in \{0, 2\pi\}$ for the simple ansatz that $\frac{\delta\tau}{t_P} \in \mathbb{N}^+$ and mass $m = 0.2 m_P$.

Consider a simple ansatz, that $\delta\tau$ takes values as multiples of Planck time.

Quantum levels could appear in the entanglement entropy since $I(\rho_A) = I(\delta\tau)$.

The further we move to sub-Planckian masses, the number of quantum levels increases, becoming a continuous curve for $m << m_P$.

Summary

- General relativistic treatment of gravity mediated entanglement growth reveals that quantum superposition of geometries could be achieved in the lab and have observable effects.
- There are intriguing hints that the interference experiments with superposed masses approaching the Planck mass and interacting gravitationally could explore the structure of time at the Planck scale. The proper time intervals involved in the claimed technologically feasible setup lie at about a million Planck times.
- Perhaps the significant experimental developments aiming to explore quantum phenomena in macroscopic objects and the 'border' between quantum and classical world will provide a complementary avenue for QG phenomenology. The key seems to be to involve gravitational interaction between masses close to the Planck mass scale.

Directions

- To probe smaller $\delta\tau$, we need not necessarily increase the superposed mass. These experiments aim to near maximally entangled states in order to measure an entanglement witness ($\delta\phi \sim \pi$). Detection of small amounts of entanglement (small $\delta\phi$) would involve smaller $\delta\tau$ and explore the entanglement entropy curve at its minimum rather than at its maximum.
- The analysis for time discreteness is rough and not based on a specific quantum gravity model. Hypothesis of discrete time not falsified on general grounds if continuous entanglement entropy curve measured, the precise dependence on the time variable could be more involved (for instance $\delta\tau$ could be an expectation value rather than an eigenvalue).
- Less clear results than the area and volume spectrum for time discreteness in LQG. There are well developed tools for the low energy regime in which the effect takes place. It could be feasible to see if LQG predicts quantum levels in the entanglement entropy.