Quantum Isolated Horizons:

The Planck Scale Regime

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MOTIVATION

- **How do we characterize black holes in equilibrium?**
- **What are quantum horizon states?**
- **Which states should we count?**
- **How does the entropy behave?**
- **What happens when we look at small BH's?**
- **What is the domain of applicability of the model?**

PLAN OF THE TALK

1. Some History
2. Classical Preliminaries
3. Quantum Horizon Geometry
4. Counting and Entropy
5. New Results: Counting by Numbers

1. SOME HISTORY

- **94' The area operator is defined (Smolin & Rovelli)**
- **96' Krasnov and Rovelli consider punctures as horizon degrees of freedom.**
- **97' Isolated Horizon boundary conditions understood.**
- **99' Quantum Horizon Geometry fully understood (ABK).**
- **02' Possible relation to QNM proposed (SO(3) *vs* SU(2))**
- **04' Error in original ABK computation found. A new counting proposed (DLM)**
- **05'- Several new countings proposed (GM, Dreyer et al, ...)**
- **06' This seminar**

The Beginning

Physically, one is interested in describing black holes in equilibrium. That is, equilibrium of the horizon, not the exterior. Can one capture that notion via boundary conditions?

Yes! Answer: Isolated Horizons

Isolated horizon boundary conditions are imposed on an inner boundary of the region under consideration.

The interior of the horizon is cut out. In this a physical boundary?
No! but one can ask whether one can make sense of it:

What is then the physical interpretation of the boundary?

• The boundary Δ, the 3-D isolated horizon, through its boundary conditions, will provide an effective description of the degrees of freedom of the inside region, that is cut out in the formalism.

• The boundary conditions are such that they capture the intuitive description of a horizon in equilibrium and allow for a consistent variational principle.

• The quantum boundary degrees of freedom are then responsible for the entropy.

• The entropy thus found can be interpreted as the entropy assigned by an 'outside observer' to the (2-dim) horizon $S = \Sigma \cap \Delta$.

• Interpretational issues: is this to be regarded as the entropy contained by the horizon? Is there some 'holographic principle' in action? Can the result be associated to entanglement entropy between the interior and the exterior?, etc.

ISOLATED HORIZONS

An isolated horizon is a null, non-expanding horizon Δ with some notion of translational symmetry along its generators. There are two main consequences of the boundary conditions:

- **The gravitational degrees of freedom induced on the horizon are captured in a $U(1)$ connection,**
  $W_a = -\frac{1}{2}\,\Gamma^i_a\, r_i$

- **The total symplectic structure of the theory (and this is true even when matter is present) gets split as, $\Omega_{\rm tot} = \Omega_{\rm bulk} + \Omega_{\rm hor}$ with**

$$\Omega_{\rm hor} = \frac{a_0}{8\pi\, G} \oint_S \mathrm{d}W \wedge \mathrm{d}W'$$

- **The 'connection part' and the 'triad part' at the horizon must satisfy the condition, $F_{ab} = -\frac{2\pi\,\gamma}{a_0}\, E^i_{ab}\, r_i$, the 'horizon constraint'.**

CONSTRAINTS

The formalism tells us what is gauge and what not. In particular, with regard to the constraints we know that:

- **The relation between curvature and triad, the horizon constraint, is equivalent to Gauss' law.**

- **Diffeomorphims that leave S invariant are gauge (vector field are tangent to S).**

- **The scalar constraint must have $N|_{\mathrm{hor}} = 0$. Thus, the scalar constraint leaves the horizon untouched; any gauge and diff-invariant observable is a Dirac observable.**

In the quantum theory of the horizon we have to implement these facts.

QUANTUM THEORY

The total Hilbert Space is of the form:

$$\mathcal{H} = \mathcal{H}_{\mathrm{V}} \otimes \mathcal{H}_{\mathrm{S}}$$

where $\mathcal{H}_{\mathrm{S}}$, the surface Hilbert Space, can be built from Chern Simons Hilbert spaces for a sphere with punctures.

The conditions on $\mathcal{H}$ that we need to impose are: Invariance under diffeomorphisms of S and the quantum condition on Ψ:

$$\left(\mathrm{Id} \otimes \hat{F}_{ab} + \frac{2\pi\,\gamma}{a_0}\,\hat{E}^i_{ab}\,r_i \otimes \mathrm{Id} \right) \cdot \Psi = 0$$

Then, the theory we are considering is a quantum gravity theory, with an isolated horizon of fixed area a_0. Physical state would be such that, in the bulk satisfy the ordinary constraints and, at the horizon, the quantum horizon condition.

ENTROPY

We are given a black hole of area a_0. What entropy can we assign to it? Let us take the microcanonical viewpoint. We shall count the number of states $\mathcal{N}$ such that they satisfy:

- **The area eigenvalue $\langle\hat{A}\rangle \in [a_0 - \delta, a_0 + \delta]$**
- **The quantum horizon condition.**

The entropy $\mathcal{S}$ will be then given by

$$\mathcal{S} = \ln \mathcal{N}.$$

The challenge now is to identify those states that satisfy the two conditions, and count them.

CHARACTERIZATION OF THE STATES

There is a convenient way of characterizing the states by means of the spin network basis. If an edge of a spin network with label j_i ends at the horizon S, it creates a puncture, with label j_i. The area of the horizon will be the area that the operator on the bulk assigns to it: $A = 8\pi\gamma\ell_p^2 \sum_i \sqrt{j_i(j_i+1)}$.

Is there any other quantum number associated to the punctures p_i? Yes! the eigenstates of $\hat{E}_{ab}$ that are also half integers m_i, such that $-|j_i| \leq m_i \leq |j_i|$. The quantum horizon condition relates these eigenstates to those of the Chern-Simons theory. The requirement that the horizon is a sphere (topological) then imposes a 'total projection condition' on $m's$:

$$\sum_i m_i = 0$$

A state of the quantum horizon is then characterized by a set of punctures p_i and to each one a pair of half integer (j_i, m_i).

If we are given N punctures and two assignments of labels (j_i, m_i) and (j'_i, m'_i). Are they physically distinguishable? or a there some 'permutations' of the labels that give indistinguishable states?

That is, what is the statistics of the punctures?

As usual, we should let the theory tell us. One does not postulate any statistics. If one treats in a careful way the action of the diffeomorphisms on the punctures one learns that when one has a pair of punctures with the same labels j and m, then the punctures are indistinguishable and one should not count them twice. In all other cases the states are distinguishable.

THE COUNTING

We start with a small isolated horizon, with an area a_0 of the order of several Planck areas and ask how many states are there compatible with the two conditions, and taking into account the distinguishability of the states. We tell a computer how to count. We vary the area a_0 and count again.

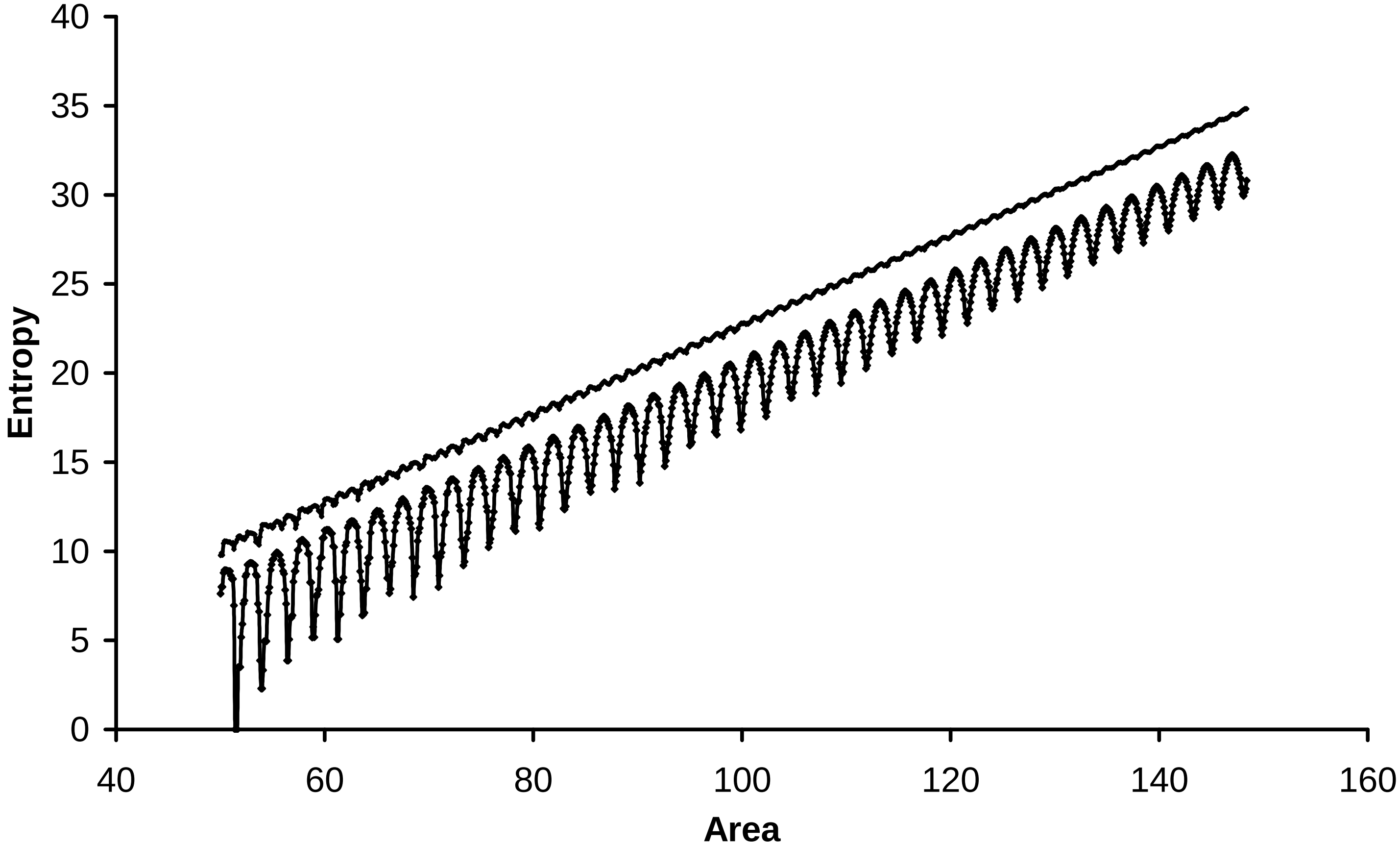
Once he have the program to do this we can ask all sorts of questions:

- **How does the incorporation or not of the projection constraint $\sum_i m_i = 0$ affect the number of states?**
- **Can we see for such small black holes that the entropy tends to be a linear function of the area?**
- **Can we say anything about the Barbero-Immirzi parameter?**
- **Is the entropy area relation for such small black holes like anything we had imagined?**

QUESTIONS

In the analytical calculations, done in the large area limit, it has been shown that the dominant term (linear) is unaffected by the imposition of the projection constraint $\sum_i m_i = 0$.

We did the counting with and without the constraint and the plot is as follows:

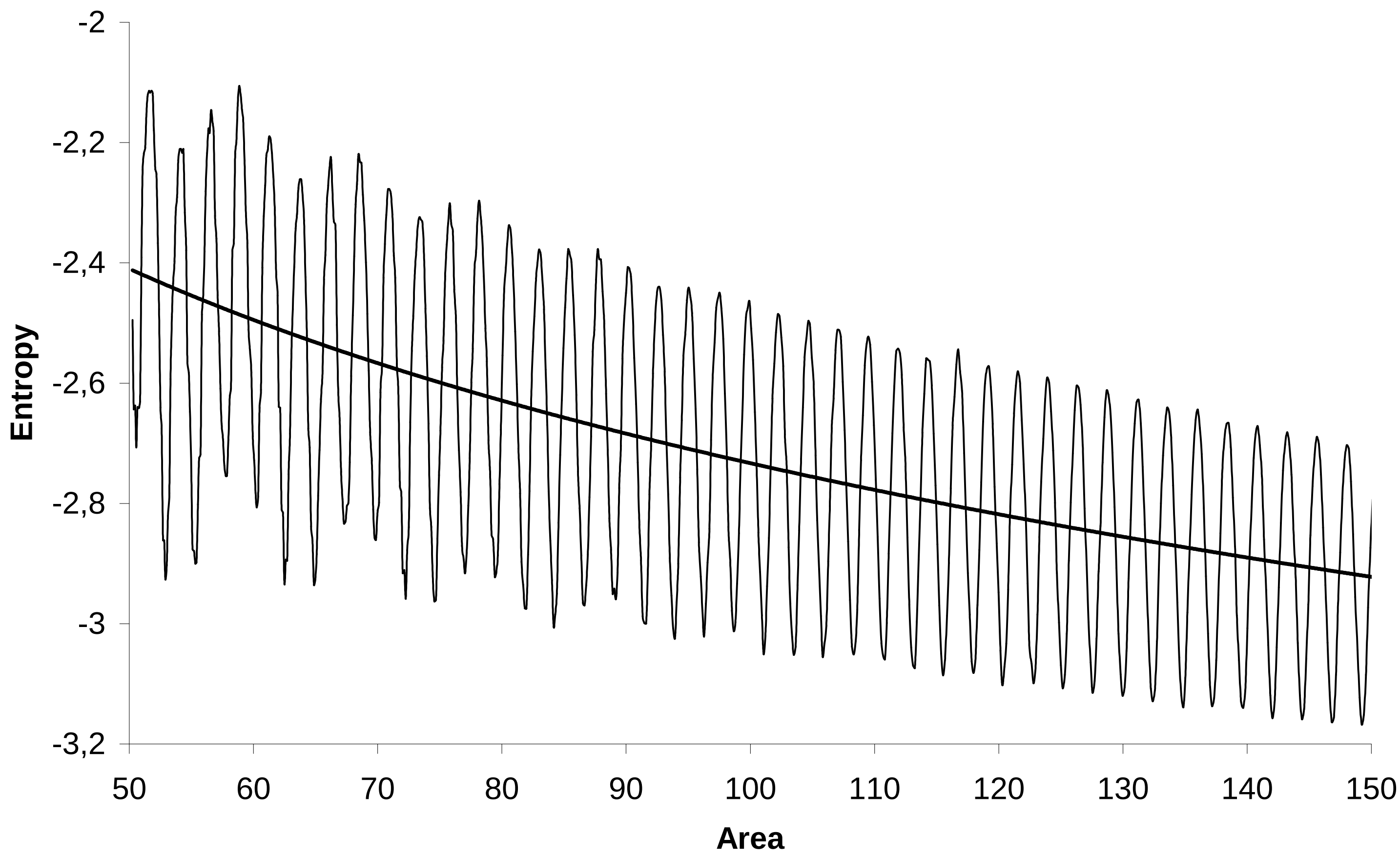

Entropy
40
35
30
25
20
15
10
5
0
40
60
80
100
120
140
160
Area

.

In the analytical computations, the introduction of the projection constraint introduces a first correction to the entropy area relation as

$$S = \alpha A - \frac{1}{2}\ln(A) + \dots$$

We then subtracted the two plots and found:


-2
-2,2
-2,4
-2,6
-2,8
-3
-3,2
Entropy
50
60
70
80
90
100
110
120
130
140
150
Area

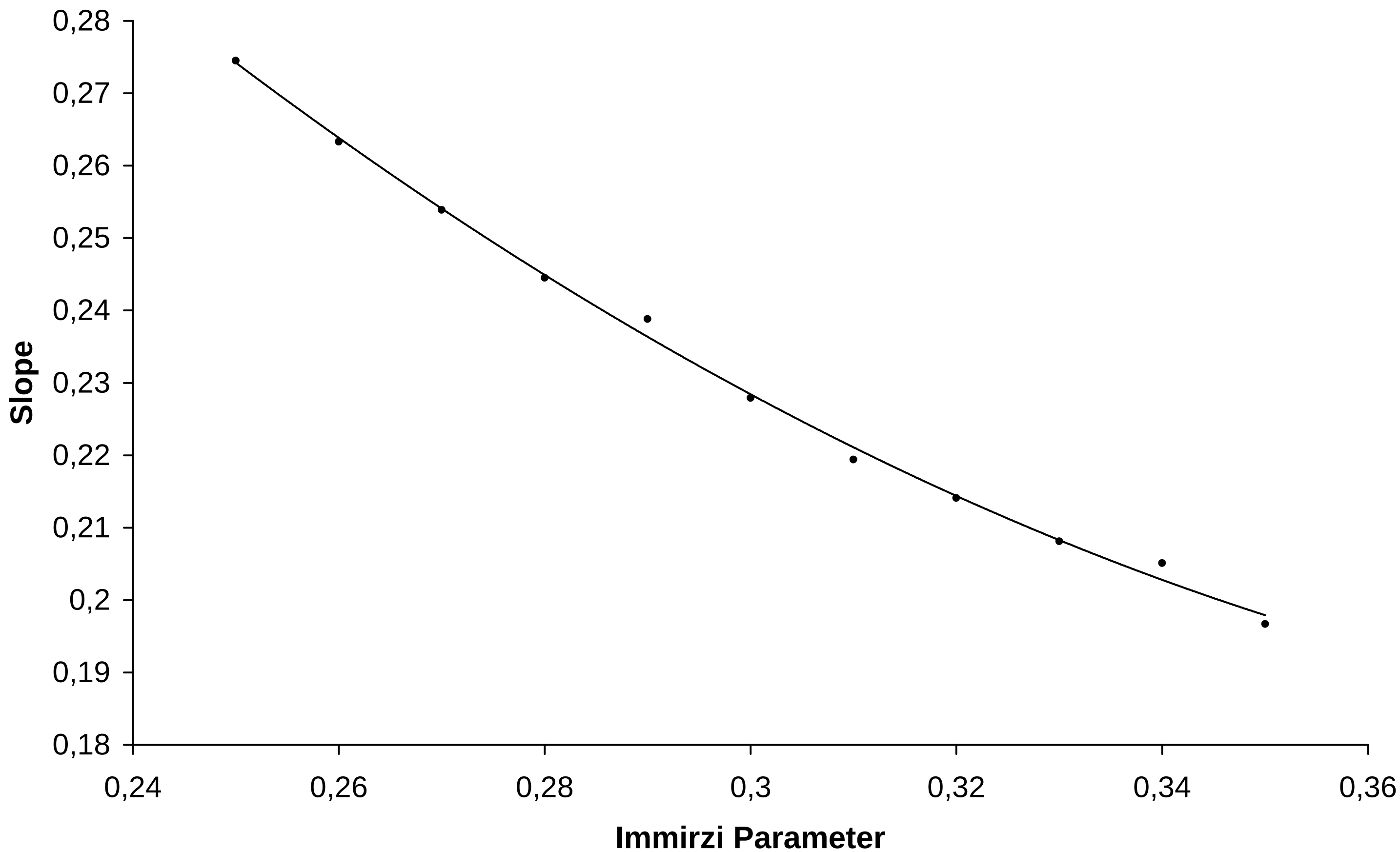
What we see in that, on average, the entropy tends to the analytical relations.

What is the nature of the oscillations? What we see is that the frequency of the oscillations is independent of the size δ of the interval used in the counting, but its amplitude decreases as one increases δ.

What about the Immirzi parameter? In order to analyze the linear term, we computed the entropy without the projection constraint (oscillations disappear), for several values of γ, and looked for the value that reproduced, for the largest horizons we can compute, the Hawking-Bekenstein relation:

$$S = A/4$$

We found:


Slope
0,28
0,27
0,26
0,25
0,24
0,23
0,22
0,21
0,2
0,19
0,18
0,24
0,26
0,28
0,3
0,32
0,34
0,36
Immirzi Parameter

That is, we found that the Barbero-Immirzi parameter from our counting (by interpolating the curve) is very close to the analytical value:

$$\gamma_0 = 0.27398\ldots$$

Then, even when considering Planck scale horizons, we see that we can say something about the Barbero-Immirzi parameter and the logarithmic correction.

But can we learn something from the oscillations that we see?

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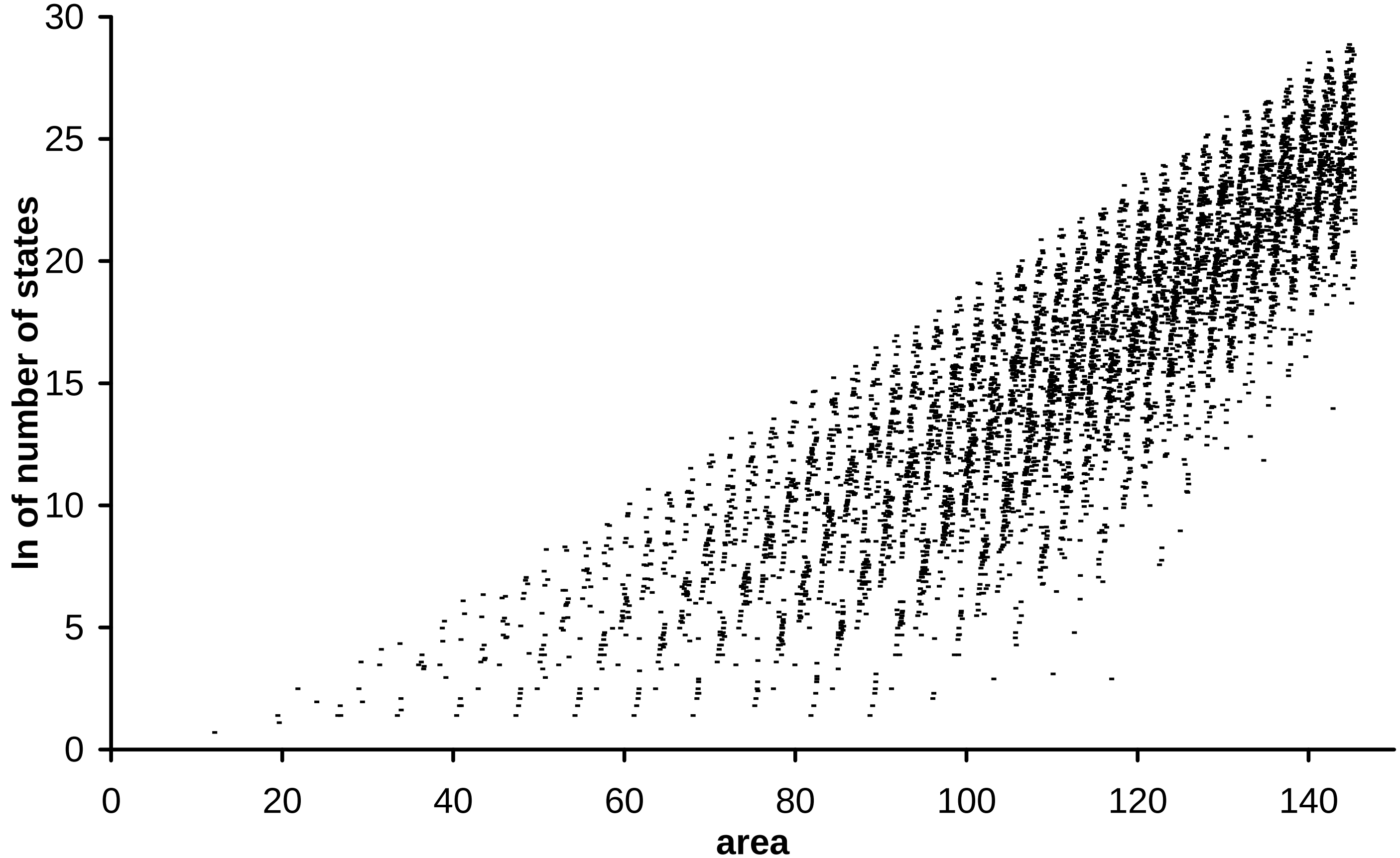
For that, we decided to explore the 'black hole entropy spectrum' by taking an interval very small

$$\delta = 0.005\,\ell_p^2,$$

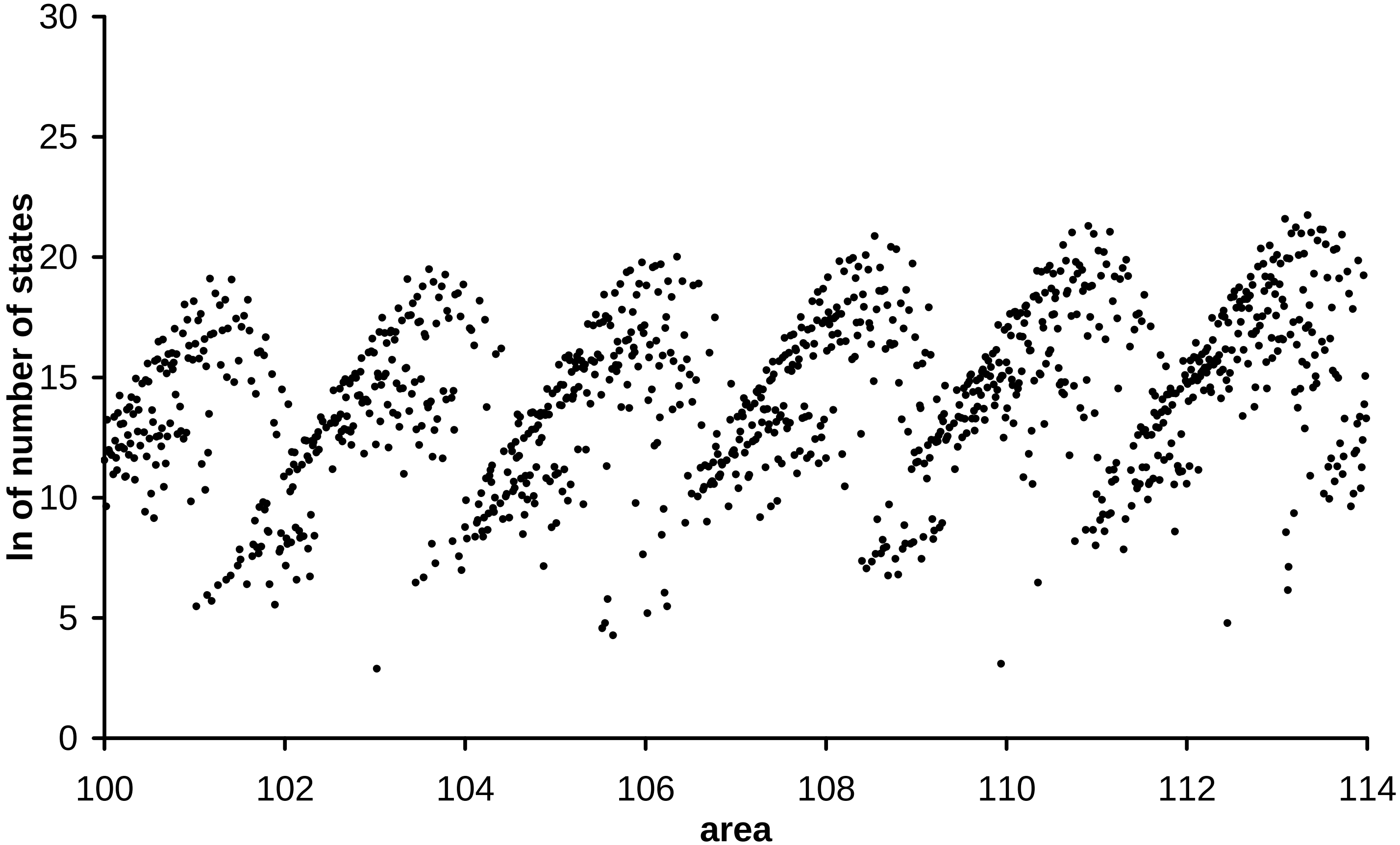
as opposed to the 'large values' used so far: $\delta = 2\,\ell_p^2$.

With increments of a_0 in steps of $0.01\,\ell_p^2$, we covered all possible values of a_0 without overlap. In this way we can know exactly how many black holes states are there in such small intervals.

This is what we found:



ln of number of states
30
25
20
15
10
5
0
0
20
40
60
80
100
120
140
area


30
25
20
15
10
5
0
100
102
104
106
108
110
112
114
area
ln of number of states

PERIODICITY

Both the oscillations found with a large value of δ as well as these structures in the 'spectrum' posses the same periodicity

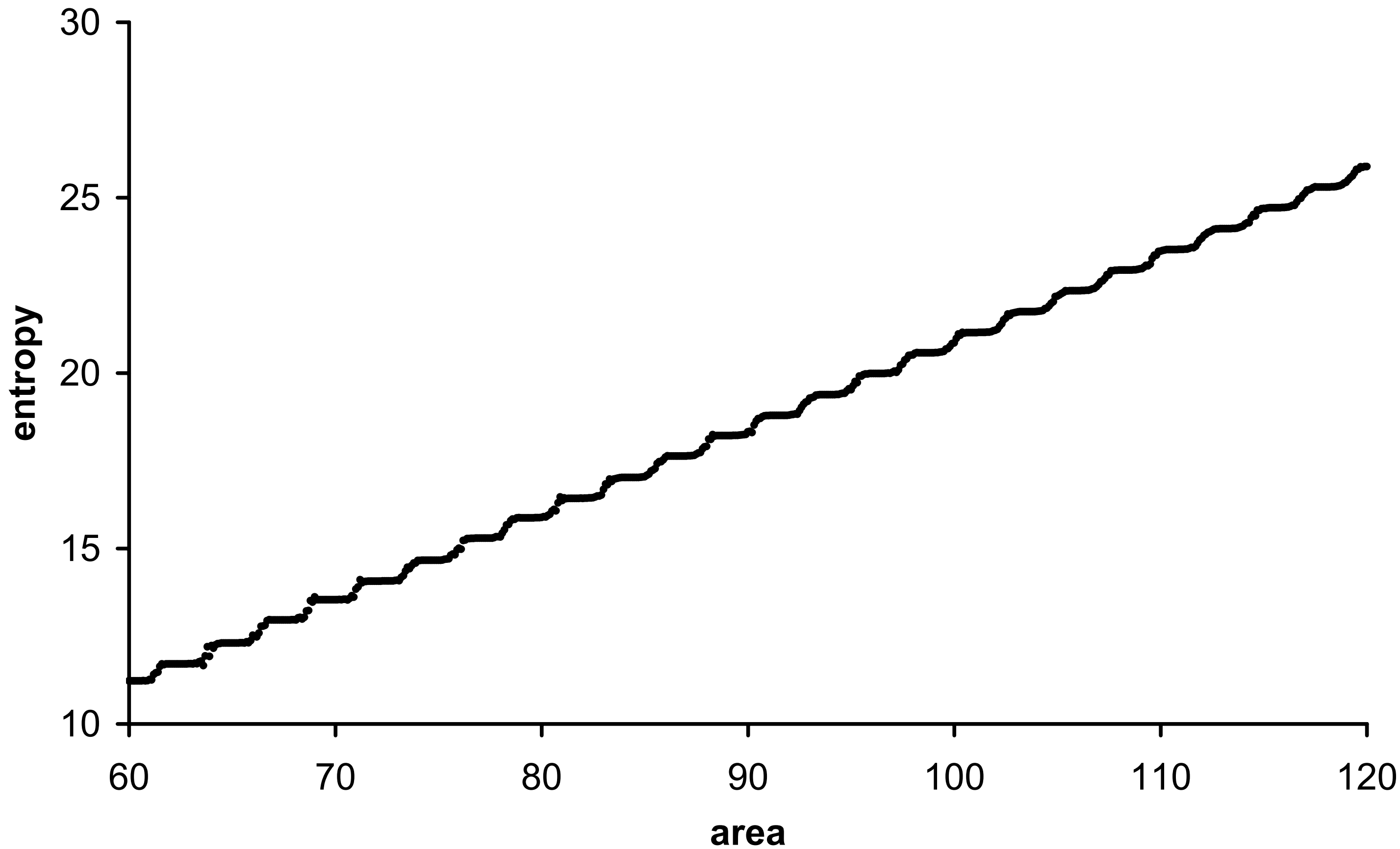
$$\delta A_0 \approx 2.41 \, \ell_p^2$$

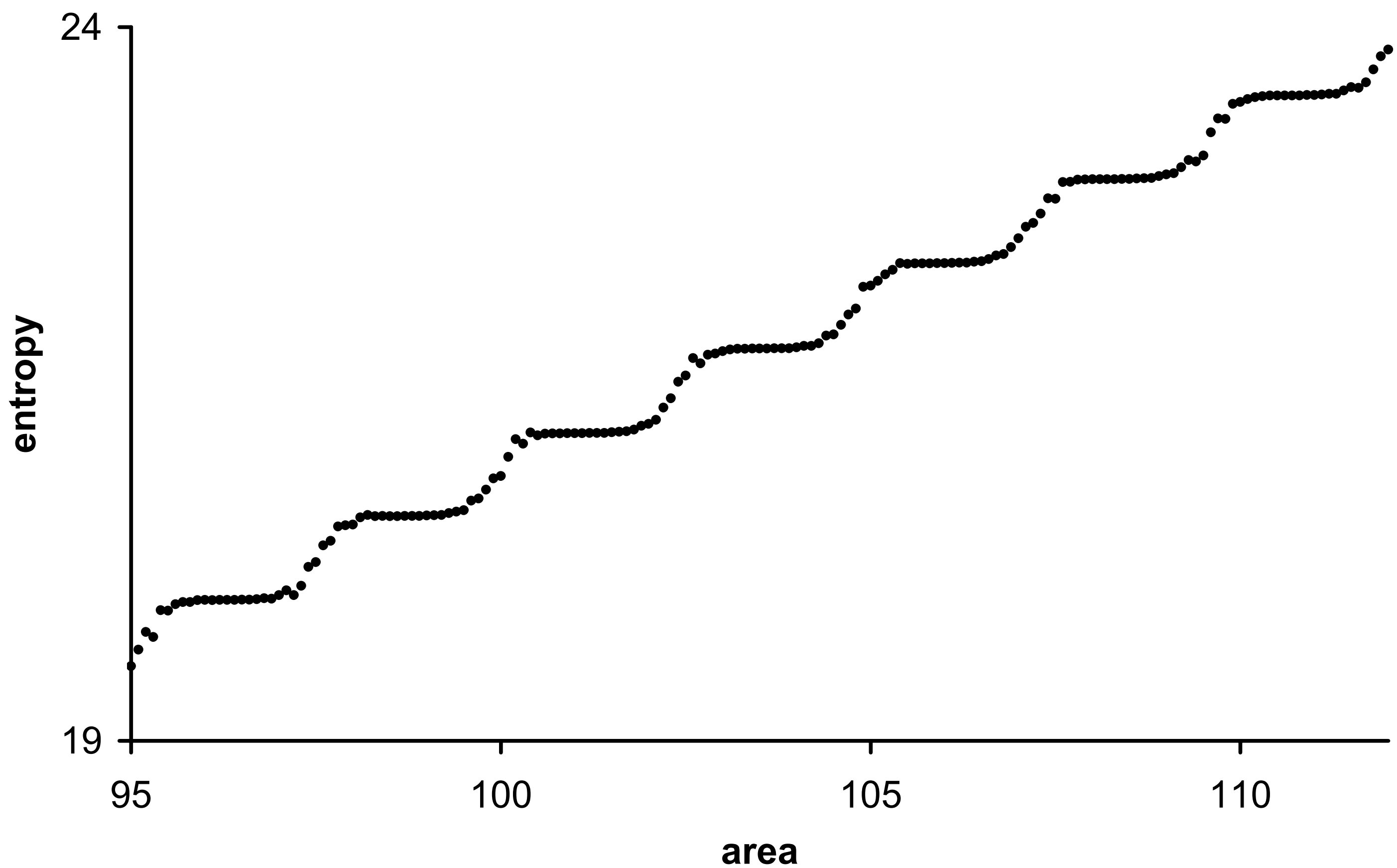
Is there any physical significance to this periodicity?

In order to answer this question we chose the interval of the area in the entropy computation to be dictated by the periodicity found. Thus we chose:

$$2\,\delta = \Delta A_0$$

With this choice, the plot of the entropy *vs* area becomes:


30
25
20
15
10
entropy
60
70
80
90
100
110
120
area


24
19
entropy
95
100
105
110
area

WHAT DOES THIS MEAN?

The first observation is that the entropy has a completely different behavior for this particular choice of interval: instead of oscillations, it seems to increase in discrete steps.

Furthermore, the height of the steps seems to approach a constant value as the area of the horizon grows, thus implementing in a rather subtle way the conjecture by Bekenstein that entropy should be equidistant for large black holes.

Is there any magic number appearing here? Maybe

When varying γ we find that the periodicity in oscillations very approximates

$$\Delta A \approx 8\,\gamma\, l_p^2\, \ln(3)$$

While the constant number in which the entropy of large black holes 'jumps' is:

$$\Delta S \mapsto 2\,\gamma_0\,\ln(3)$$

Are these simple coincidences or the tip of an iceberg?

STAY TUNED!

CONCLUSIONS

- **Somehow, unexpected features appear by considering Planck size horizons.**
- **We can say something about BI parameter and logarithmic correction.**
- **Oscillations found with a periodicity that is puzzling**
- **Unexpected contact with Bekenstein's heuristic model but in a subtle manner**
- **Is there more?**

BIBLIOGRAPHY

More details can be found in:

gr-qc/0605014

and

gr-qc/0609122