Supergravity in LQG

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Section 1

Introduction

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Supersymmetry

Coleman-Mandula: most general Lie algebra of symmetries of the S-matrix has form

$$\mathfrak{iso}(\mathbb{R}^{1,3}) \oplus \text{internal sym.} \tag{1}$$

Only way around this seems to be through new form of symmetry:

Haag-Łopuszański-Sohnius theorem

Going away from (1) in an interacting QFT with mass gap requires **super Lie algebras**, i.e. $\mathbb{Z}_2$-graded algebras $(\mathfrak{g}, [\cdot,\cdot])$ of the form

$$\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \quad \text{with} \quad [\cdot,\cdot] : \text{(anti) commutator on } \mathfrak{g}_0\ (\mathfrak{g}_1)$$

such that $[\mathfrak{g}_i, \mathfrak{g}_j] \subseteq \mathfrak{g}_{i+j}$ (+ graded Jacobi identity)

Supersymmetry

$\Rightarrow$ smallest possible superalgebra containing spacetime symmetries:

super Poincaré/super anti-de Sitter $\mathfrak{osp}(1|4)$

$$\mathfrak{iso}(\mathbb{R}^{1,3|4}) = \underbrace{\mathbb{R}^{1,3} \ltimes \mathfrak{so}(1,3)}_{\mathfrak{g}_0} \oplus \underbrace{S_{\mathbb{R}}}_{\mathfrak{g}_1}$$

generators: $P_I,\ M_{IJ},\ Q^{\alpha} = (Q^A, Q_{A'})^T$ (Majorana spinor)

$$[P_I, Q_\alpha]_- = 0 - \frac{1}{2L} Q_\beta (\gamma_I)^\beta{}_\alpha$$

$$[M_{IJ}, Q_\alpha]_- = \frac{1}{2} Q_\beta (\gamma_{IJ})^\beta{}_\alpha$$

$$[Q_\alpha, Q_\beta]_+ = \frac{1}{2}(C\gamma_I)_{\alpha\beta} P^I + \frac{1}{4L}(C\gamma^{IJ})_{\alpha\beta} M_{IJ}$$

$AdS_4 := \{x \in \mathbb{R}^5 | -(x^0)^2 + (x^1)^2 + \ldots + (x^3)^2 - (x^4)^2 = -L^2\}$

Supersymmetry

Since new generators Q^α...

- transform as spinors, they relate particles of integer, half-integer spin,
- are anti-commuting, they relate bosons and fermions.

Can include further fermionic generators Q^i_α, $i = 1, \ldots, \mathcal{N}$. In present talk mostly $\mathcal{N} = 1$.

Application to gravity

Most ambitious use of this kind of symmetry:

local supersymmetry $\Rightarrow$ **supergravity** (SUGRA)

For $\mathcal{N} = 1, D = 4$ contains gravitational field and a spin $\frac{3}{2}$ fermion.

Supergravity

Why interesting?

- $\Rightarrow$ leads to unified description of force and matter $\rightarrow$ GUT
- (max.) $\mathcal{N} = 8$, $D = 4$ SUGRA may be **perturbatively finite** [Green et al '07,Berne et al '18]

Specifically for LQG

- Matter as part of gravity
- contact to boundary description in string theories and AdS/CFT conjecture [Mikhaylov + Witten '14]
- $\leftrightarrow$ black hole entropy calculations ($\rightarrow$ BPS states)?

LQG and supergravity

What has been done?

- SUGRA in Ashtekar variables [Jacobson '88, Tsuda '00]
- enhanced gauge symmetry $\mathfrak{osp}(1|2)$ [Fülöp '94, Gambini + Obregon + Pullin '96]
- formal quantization [Gambini + Obregon + Pullin '96, Ling + Smolin '99]
- $\mathfrak{osp}(1|2)$ spin networks, inner boundaries [Ling + Smolin '99]
- Spinfoam models in D=3 [Livine + Oeckl '03, L+Ryan '07, Baccetti+L+R '10]
- Canonical theory for higher D, quantization of RS fields, p-form fields [Bodendorfer+Thiemann+Thurn '11]

LQG and supergravity

Open questions, tasks:

- dynamics quantum theory ($\rightarrow$ quantization SUSY constraint)
- relations/differences to standard quantization Hamilton constraint in classical LQG
- symmetry reduced models $\rightarrow$ cosmological implications

Where does enlarged $\mathfrak{osp}(1|2)$-gauge symmetry come from

- which **Immirzi parameters**? Reality conditions?
- generalizations to $\mathcal{N} > \mathbf{1}$? [Ezawa '95, Tsuda '00, KE '21 in prep.]
- boundary theory ($\longrightarrow$ BPS states, **black holes**)?

Quantization while keeping $\mathfrak{osp}(1|2)$ symmetry manifest:

- precise mathematical setting
- structure and properties of graded holonomies
- structure of Hilbert spaces $\leftrightarrow$ relation to standard quantization in LQG with fermions

In this talk

Mathematically clean formulation **studying enriched categories** (not part of this talk!)

Classical theory:

- Supergravity via Cartan geometry
- Holst-MacDowell-Mansouri SUGRA action for any β
- special properties of self-dual theory
- boundary theory

Quantum theory:

- graded connections, -holonomies, and -group integration
- applications: LQC, black holes (WIP)

Section 2

Gravity as Cartan geometry

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Klein geometry

F. Klein: "Classify geometry of space via group symmetries".

Example: Minkowski spacetime $\mathbb{M} = (\mathbb{R}^{1,3}, \eta)$

- isometry group $\mathrm{ISO}(\mathbb{R}^{1,3}) = \mathbb{R}^{1,3} \ltimes \mathrm{SO}_0(1,3)$
- event $p \in \mathbb{M}$: $G_p = \mathrm{SO}_0(1,3)$ (isotropy subgroup)

$$\mathrm{ISO}(\mathbb{R}^{1,3})/\mathrm{SO}_0(1,3) \cong \mathbb{M}$$

Definition

A *Klein geometry* is a pair (G, H) where G is a Lie group and $H \subseteq G$ a closed subgroup such that G/H is connected.

Cartan geometry

- flat spacetime $\leftrightarrow$ Klein geometry
- $\Rightarrow$ Cartan geometry as deformed Klein geometry

Definition: Cartan geometry

A **Cartan geometry** modeled on a Klein geometry (G, H) is a principal H-bundle

$$\begin{array}{ccc} P & \overset{r}{\longleftarrow} & H \\ {\scriptstyle\pi}\downarrow & & \\ M & & \end{array}$$

together with a **Cartan connection** $A \in \Omega^1(P, \mathfrak{g})$ s.t.

1. $r_g^* A = \mathrm{Ad}(g^{-1})A \; \forall g \in H$
2. $A(\widetilde{X}) = X, \; \forall X \in \mathfrak{h} \; \left(\widetilde{X}: \text{ fundamental vector field}\right)$
3. $A: \; T_p P \to \mathfrak{g}$ isomorphism $\forall p \in P$

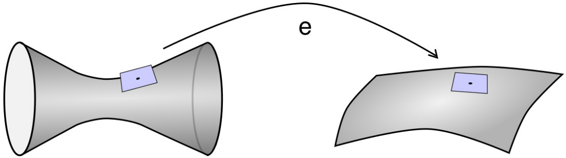
Gravity as Cartan geometry

Example: Cartan geometry modeled over **AdS₄** $(\mathrm{SO}(2,3), \mathrm{SO}_0(1,3))$

Cartan connection

$$A = \mathrm{pr}_{\mathbb{R}^{1,3}} \circ A + \mathrm{pr}_{\mathfrak{so}(1,3)} \circ A =: e + \omega$$

- ω: **Lorentz-connection**, e: **soldering form** (co-frame)



Holst as MacDowell-Mansouri

Holst action via Mac-Dowell-Mansouri [A. Randono '06, D. Wise '09]
via perturbed BF-Theory [Freidel+Starodubtsev '05]

Here: AdS_4: can use $\mathfrak{so}(2,3) \cong \mathfrak{sp}(4) \supset \mathfrak{so}(1,3)$, basis $M_{IJ} = \frac{1}{4}[\gamma_I, \gamma_J]$

Definition

$$\mathcal{P}_\beta := \frac{\mathbb{1} + i\beta\gamma_5}{2\beta} : \mathfrak{so}(1,3) \to \mathfrak{so}(1,3), \quad \beta : \textit{Immirzi}$$

$\to$ yields inner product on $\mathfrak{sp}(4)$:

$$\langle \cdot, \cdot \rangle_\beta := \mathrm{tr}(\,\cdot\, \mathbf{0} \oplus \mathcal{P}_\beta \,\cdot\,)$$

Holst-MacDowell-Mansouri action

$$S_{H-MM}[A] = \int_M \langle R[A] \wedge R[A] \rangle_\beta = S^\beta_{Holst} + \textit{boundary term}$$

$R[A] = \mathrm{d}A + \frac{1}{2}[A \wedge A]$: Cartan curvature

Section 3

Supergravity as super Cartan geometry

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Supergravity as super Cartan geometry

- starting point for **geometric approach** to SUGRA initiated by Ne'eman and Regge [N+R '76]
- developed further by Castellani-D'Auria-Fré [A+F '76, C+A+F '91]
- extension to higher *D* reveals **higher categorial structure** of max. D=11 SUGRA [Fiorenza+Sati+Schreiber '14, S+S '17]
- studied proper mathematical realization incorporating **anticommutative nature** of fermionic fields [KE '20+'21]
- $\rightarrow$ work in **enriched** category of supermanifolds [Schmitt '96, Deligne '99, Keßler+Jost+Tolksdorf '17, K+Sheshmani+Yau '20]
- $\rightarrow$ describe (AdS) SUGRA as *enriched* **super Cartan geometry** modeled over **super AdS$_4$** $(\mathrm{OSp}(1|4), \mathrm{Spin}^+(1,3))$ [KE '20+'21]

Supergravity as super Cartan geometry

$$\begin{array}{ccc} \mathcal{P} & \longleftarrow & \mathrm{Spin}^+(1,3) \\ {\scriptstyle\pi}\big\downarrow & & \\ \mathcal{M} & & \end{array}$$

Super Cartan connection: $\mathcal{A} \in \Omega^1(\mathcal{P}, \mathfrak{osp}(1|4))_0$

$$\mathcal{A} = \underbrace{\mathrm{pr}_{\mathfrak{g}_1} \circ \mathcal{A}}_{\psi} + \underbrace{\mathrm{pr}_{\mathbb{R}^{1,3}} \circ \mathcal{A}}_{e} + \underbrace{\mathrm{pr}_{\mathfrak{spin}(1,3)} \circ \mathcal{A}}_{\omega}$$

$\psi = \psi^\alpha Q_\alpha$ (Rarita-Schwinger field)

Super soldering form

$$E := e + \psi : \; T_p\mathcal{M} \xrightarrow{\sim} \mathbb{R}^{1,3} \oplus S_{\mathbb{R}}, \qquad \forall p \in \mathcal{M}$$

$\rightarrow$ locally identifies $\mathcal{M}$ with super AdS_4
$\rightarrow$ metric on $\mathfrak{osp}(1|4)$ induces metric structure on $\mathcal{M}$ (Killing-isometries, Killing spinors $\rightarrow$ BHs)

Supergravity and LQG

Holst action for (extended) Poincaré SUGRA [Tsuda '00, Kaul '07]
via MM ($\mathcal{N} = 1$, β as θ-ambiguity) [Obregon+Ortega-Cruz+Sabido '12]

Here: via Holst projection ($\rightarrow$ extension $\mathcal{N} > 1$ & discussion boundary theory [KE '21])

- $\mathfrak{osp}(1|4) \cong \mathfrak{sp}(4) \oplus S_{\mathbb{R}}$
- $S_{\mathbb{R}}$ Clifford module $\rightarrow$ can naturally extend $\mathcal{P}_\beta$

Definition

$$\mathbf{P}_\beta := \mathbf{0} \oplus \mathcal{P}_\beta \oplus \mathcal{P}_\beta : \mathfrak{osp}(1|4) \rightarrow \mathfrak{osp}(1|4)$$

$\rightarrow$ yields inner product on $\mathfrak{osp}(1|4)$:

$$\langle \cdot, \cdot \rangle_\beta := \mathrm{str}(\,\cdot\,\mathbf{P}_\beta\,\cdot\,)$$

Supergravity and LQG

super Holst-MacDowell-Mansouri action

$$S_{sH-MM}[\mathcal{A}] = \int_M \langle R[\mathcal{A}] \wedge R[\mathcal{A}] \rangle_\beta$$

Cartan curvature

$R[\mathcal{A}] = \mathrm{d}\mathcal{A} + \frac{1}{2}[\mathcal{A} \wedge \mathcal{A}]$

- $R[\mathcal{A}]^I = \Theta^{(\omega)I} - \frac{1}{4}\bar{\psi} \wedge \gamma^I \psi$
- $R[\mathcal{A}]^{IJ} = R[\omega]^{IJ} + \frac{1}{L^2}\Sigma^{IJ} - \frac{1}{4L}\bar{\psi} \wedge \gamma^{IJ}\psi$
- $R[\mathcal{A}]^\alpha = D^{(\omega)}\psi^\alpha - \frac{1}{2L}e^I(\gamma_I)^\alpha{}_\beta \wedge \psi^\beta$

- $\rightarrow$ yields Holst action of $D = 4$, $\mathcal{N} = 1$ AdS-SUGRA + *bdy terms*

Section 4

Supersymmetry and the SUSY constraint

Supersymmetry

- D'Auria-Fré: supersymmetry as **superdiffeomorphisms**
- in Cartan-geometric framework:

Correspondence

$$\textbf{Cartan connections}\ \mathcal{A} \leftrightarrow \textbf{Ehresmann connections}\ \hat{\mathcal{A}}$$

on $\mathcal{P}$ $\qquad$ on $\mathcal{P} \times \mathrm{OSp}(1|4)$

$\rightarrow$ can study **odd** local gauge transformations

Gauge transformations

$$(\delta_\epsilon e, \delta_\epsilon \psi, \delta_\epsilon \omega) := \delta_\epsilon \hat{\mathcal{A}} = D^{(\hat{\mathcal{A}})}\epsilon$$

$\Rightarrow$ yields local symmetry $\leftrightarrow$ $R[\hat{\mathcal{A}}]^I = 0$ ($\leftrightarrow$ EOM of ω) [Nieuwenhuizen '03, Castellani '10, KE '20]

The SUSY constraint

- have seen: local SUSY as field dependent gauge transformation
- in canonical theory: any local symmetry induces constraint
  $\rightarrow$ Here: **SUSY constraint** S
- due to $[Q, Q] \propto P_0 + P_i + \mathit{Lorentz}$ generally expect:

Constraint algebra

$$\{S[\eta], S[\eta']\} = H[N(\eta, \eta')] + \text{kin. constraints}$$

- H: Hamilton constraint
- $N \equiv N[E_i^a]$: (generally field dependent) lapse function

- $\Rightarrow$ SUSY-constraint **superior** to the Hamilton constraint!

The SUSY constraint

SUSY constraint (full theory, real β)

$$S[\eta] := \int_\Sigma \mathrm{d}^3x\, \bar{\eta} i \epsilon^{abc} e_a^i \gamma_i \gamma_5 D_b^{(A^\beta)} \left(\frac{1}{\sqrt[4]{q}} e_c^j \phi_j \right) + \cdots$$

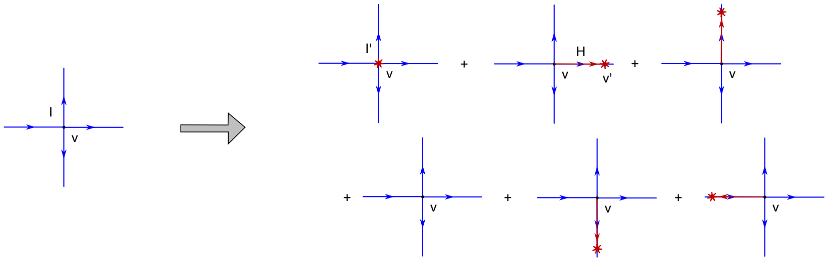
A^β: Ashtekar-Barbero connection

ϕ_j: half-densitized RS-field [Thiemann '98, Bodendorfer et al '11]

Quantization: [KE+HS '20]

- various possibilities $\rightarrow$ aim at simplest possible form
- try to avoid 'K-terms' involving Hamilton constraint [Thiemann '98] ($\leftrightarrow$ conistency check with quantization of Hamilton constraint)
- $\rightarrow$ choose Rovelli-Smolin volume operator [R+S '95](also proposed variant with Ashtekar-Lewandowski [A+L '98])
- $\Rightarrow$ turns out that operator is indeed finite!

Quantum SUSY constraint



Section 5

The chiral theory

Chiral Theory

- SUSY constraint relatively complicated object
- in canonical theory: underlying SUSY seems completely hidden in the formalism
- (partial) resolution $\rightarrow$ **chiral theory** $(\beta = \mp i)$

Holst projection

$$\mathbf{P}_{-i} : \mathfrak{osp}(1|4)_{\mathbb{C}} \rightarrow \mathfrak{osp}(1|2)_{\mathbb{C}}$$

$M_{IJ} \rightarrow T_i^+ = \frac{1}{2}(J_i + iK_i),\ Q^\alpha \rightarrow Q^A$

Proposition: (T_i^+, Q_A) generate subalgebra

$$[T_i^+, T_j^+] = \epsilon_{ij}{}^k T_k^+$$
$$[T_i^+, Q_A] = Q_B(\tau_i)^B{}_A$$
$$[Q_A, Q_B] = 0 + \frac{1}{L}(\epsilon\sigma^i)_{AB} T_i^+$$

generate D=2 super Poincaré algebra $\mathfrak{sl}(2,\mathbb{C}) \ltimes \mathbb{C}^2$ (orthosymplectic Lie superalgebra $\mathfrak{osp}(1|2)_{\mathbb{C}}$)

Chiral Theory

Chiral Holst-MacDowell-Mansouri action

$$S_{sH-MM}[\mathcal{A}] = \int_M \mathrm{str}(R[\mathcal{A}] \wedge \mathbf{P}_{-i} R[\mathcal{A}])$$

- $\Rightarrow$ manifestly $\mathrm{OSp}(1|2)_{\mathbb{C}}$-gauge invariant
- $\Rightarrow$ SUSY partially becomes true gauge symmetry![Fülöp '94, Gambini + Obregon + Pullin '96, Ling + Smolin '99, KE '20]

Super Ashtekar connection

$$\mathcal{A}^+ := \mathbf{P}_{-i}\mathcal{A} = A^{+i} T_i^+ + \psi^A Q_A$$

- $\rightarrow$ defines **generalized super Cartan connection**
- induces gauge field via correspondence **Cartan** $\leftrightarrow$ **Ehresmann**

Super Ashtekar connection

What about **extended SUSY**?

- chiral generators still generate proper sub super Lie algebra $\rightarrow \mathfrak{osp}(\mathcal{N}|2)_{\mathbb{C}}$
- $\rightarrow \mathcal{A}^+$ can be even formulated for $\mathcal{N} > 1$

What about **real** β?

- Q^A and $Q_{A'}$ generate momentum P
- By Cartan: P encoded in dual electric field $E^a_i \Rightarrow$ no proper subalgebra

Super Ashtekar connection

- $\rightarrow \mathcal{A}^+$ natural candidate to quantize SUGRA à la LQG
- contains both gravity and matter d.o.f. $\rightarrow$ unified description, more fundamental way of quantizing fermions
- substantially simplifies constraints (canonical form of Einstein equations): partial solution via gauge invariance
- natural candidate to study inner boundaries in LQG ($\rightarrow$ BPS states, black holes)
- $\leftrightarrow$ boundary theories in string theories (see later)

Quantum theory: Overview

What do we need for quantum theory?

- holonomies (parallel transport map)
- Hilbert spaces
- spin network basis

Quantum theory: Overview

- Quantization: study $\mathcal{A}^+$ and associated holonomies

Super holonomies [KE '20+'21]

$$h_e[\mathcal{A}^+] = h_e[A^+] \cdot \mathcal{P}\exp\left(-\oint_e \mathrm{Ad}_{h_e[A^+]^{-1}}\psi^{(\tilde{s})}\right) : \mathcal{S} \to \mathcal{G}$$

- $\mathcal{A}^+ = A^+ + \psi$ and $e : [0,1] \to M \subset \mathcal{M}$
- $h_e[A^+]$: bosonic holonomy associated to A^+

Proposition [KE '20+'21]

- $h_e[\mathcal{A}^+]$ group object of a **generalized** super Lie group
- natural under reparametrization $\lambda^* h_e[\mathcal{A}^+] = h_e[\lambda^* \mathcal{A}^+]$
- **Wilson-loops** invariant under gauge transformations

Quantum theory: Overview

- $\Rightarrow$ SUSY holonomies yield 1d excitation of matter-geometry (fermions parallely transported along bosonic holonomy)
- $\rightarrow$ matter-geometry flux along these lines
- **Haar measure** of $\mathrm{OSp}(1|2)$ induces **Krein structure** on Hilbert space ($\leftrightarrow$ standard LQG with fermions)
- Issue: work with complex variables: need to recover ordinary real SUGRA $\rightarrow$ need to solve **reality conditions**

Comparison: standard LQG with fermions

- pre-Hilbert space at a vertex

$$\mathcal{H}_v = \mathcal{O}(\mathcal{G}),\ \mathcal{G} = \mathrm{SU}(2)^{|E(v)|} \rtimes \mathbb{C}_v^{0|4}$$

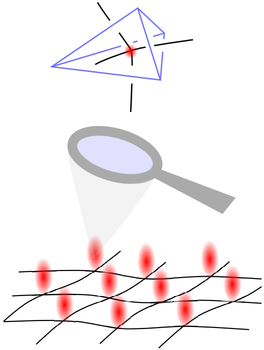
- gauge transformation

$$\mathrm{SU}(2) \ni g \triangleright f = f(\{g \cdot h\}_e, g^{\alpha}{}_{\beta}\, \theta_v^{\beta})$$

- Haar measure:

$$\int_{\mathcal{G}} = \int_{\mathrm{SU}(2)^{|E(v)|}} \int_B$$

yields **Krein space** $(\mathcal{H}_v, \int_{\mathcal{G}}) \overset{Krein\ compl.}{\rightarrow}$ [Thiemann '98]



Section 6

Applicaton: Loop quantum cosmology

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Symmetry reduction

- supersymmetric minisuperspace models [D'Eath + Hughes '88+'92, D'Eath + Hawking + Obregon '93]
- hybrid homogeneous isotropic ansatz for bosonic and fermionic d.o.f.
- in general: fermions not compatible with isotropy
- **But**: in (chiral) LQC can exploit enlarged gauge symmetry!
- $\rightarrow$ natural interpretation in terms of homogeneous isotropic super connection [KE '20, KE+HS '20]

Symmetry reduced connection

$$\mathcal{A}^+ = c\,\mathring{e}^i\,T_i^+ + \psi_A \mathring{e}^{AA'}\,Q_{A'}$$

$\mathring{e}^i$: fiducial co-triad

- **Also**: can show that this is the most general ansatz consistent with reality conditions (contorsion tensor isotropic)

Canonical theory

Phase space: bosonic: (c, p), fermionic: (ψ^A, π_A)

Poisson relations

$$\{p, c\} = \frac{i\kappa}{3} \qquad \{\pi^A, \psi_B\} = -\delta^A_B$$

Constraints:

- kinematical: (residual) Gauss constraint: $G_i = \pi\tau_i\psi$
- dynamical: **SUSY constraints** S^L, S^R and **Hamilton constraint** H

Poisson relation left and right SUSY constraint

$$\{S^{LA}, S^R_B\} = \left(\kappa H + f^C S^R_C\right) \delta^A_B - f^A S^R_B$$

Reality conditions: $c + c^* = k\ell_0 - \frac{i}{p}\pi^A\psi_A, \quad \pi^A = |p|^{\frac{1}{2}}\psi^A$

Quantization

- Construction of state space via super holonomies $h_e[\mathcal{A}^+]$ along straight edges of a fiducial cell
- $\rightarrow$ yields **graded holonomy-flux algebra** $\mathfrak{A} = \mathfrak{A}_0 \oplus \mathfrak{A}_1$
- Quantum theory: Representation of $\mathfrak{A}$ on a graded Hilbert space

Hilbert space

$$\mathcal{H} = \overline{H_{\mathrm{AP}}(\mathbb{C})}^{\|\cdot\|} \otimes \Lambda[\psi_A]$$

H_{AP}: bosonic vector space generated by

$$T(z) = \sum_i a_i e^{\mu_i z}$$ [Wilson-Ewing '15, KE+HS '20]

$e^{\mu z} \in \mathbb{C}^\times = \mathrm{U}(1)_{\mathbb{C}}$, $\mu \in \mathbb{R}$: subclass of generalized characters of $(\mathbb{C}, +)$

- **reality conditions**: use 'half-densitized' fermionic fields ϕ_A
  $\rightarrow$ can be **solved exactly**! [W.-E. '15, KE+HS '20]
  $\rightarrow$ requires distributional inner product

Quantum constraints and algebra

Quantum right SUSY constraint

$$\widehat{S}^R_{A'} = \frac{3g^{\frac{1}{2}}}{2\lambda_m}|v|^{\frac{1}{4}}\left((\mathcal{N}_- - \mathcal{N}_+) - \frac{\kappa\lambda_m}{6g|v|}\widehat{\Theta}\right)|v|^{\frac{1}{4}}\widehat{\phi}_{A'}$$

- λ_m: quantum area gap (full theory)
- $\mathcal{N}_\pm$: connection approx. by holonomies [Martin-Benito+Marugan+Olmedo '09, W.-E. '15, KE+HS '20]
- **Quantum algebra** between left and right SUSY constraint closes and **reproduces exactly** classical Poisson relations

Quantum algebra

$$[\widehat{S}^{LA}, \widehat{S}^R_B] = \left(i\hbar\kappa\widehat{H} - \frac{\hbar\kappa}{6g^{\frac{1}{2}}|v|^{\frac{1}{2}}}\widehat{\pi}^C_\phi\widehat{S}^R_C\right)\delta^A_B + \frac{\hbar\kappa}{6g^{\frac{1}{2}}|v|^{\frac{1}{2}}}\widehat{\pi}^A_\phi\widehat{S}^R_B$$

- $\rightarrow$ fixes some of the quantization ambiguities (requires symmetric ordering)

LQC: Dynamics: Semi-classical limit

Ansatz

$$\Psi = \sum_v \psi(v)\,|v\rangle + \left(\sum_v \psi'(v)\,|v\rangle\right) \otimes \phi^A \phi_A$$

$\psi, \psi' \in C^1(\mathbb{R}_{>0})$ (supported in sector of positive volume)

- semi-classical limit: $\lambda_m \to 0$ i.e. corrections from quantum geometry negligible
- look for state Ψ s.t. $\widehat{S}^L \Psi = 0$ and $\widehat{S}^R \Psi = 0$ ($\Rightarrow \widehat{H}\Psi = 0$)
- $\to$ **Hartle-Hawking state** as solution of $\widehat{S}^R$ $\Leftrightarrow$[D'Eath '98]

Hartle-Hawking state

$$\Psi(a) = \exp\left(\frac{3a^2}{\hbar}\right)$$

Open problems and future investigations (WIP)

Full Dynamics:

- due to factor ordering in $\hat{S}$: singularity is resolved in the quantum dynamics
- for full treatment need to add matter fields as relational clock
- **But**: prominent role of SUSY constraints $\rightarrow$ need **locally supersymmetric matter** for consistent dynamics!
- simplification as $\widehat{H}$ generically positive definite and $\widehat{S} \sim \sqrt{\widehat{H}}$

Miscellaneous:

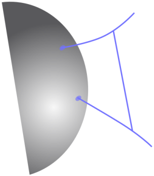
- better understanding of parity in this model

Section 7

Outlook: Boundary theory in LQG (WIP)

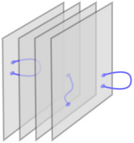
Boundary theory in LQG (WIP)

Holst-MacDowell-Mansouri in the presence of inner boundary:



- geometric theory induces super-CS theory on inner boundary
- for $\mathcal{N} = 2$: $\quad G = \mathrm{OSp}(2|2)$
- state counting feasible?

Open strings on coincident D-brane system: interesting similarities [Mikhaylov + Witten '14]



- super-CS theory in the location of the branes in the low energy limit
- for $G = \mathrm{OSp}(m|n)$

Section 8

To take home

To take home

"Classical" theory:

- SUSY: Symmetry involving bosons/fermions, internal/spacetime symmetry
- (Cartan) geometric description of $\mathcal{N} = 1$, $D = 4$ SUGRA: SUSY as gauge symmetry
- super-Ashtekar connection: Chiral SUGRA the has structure of a **super YM**
- mathematical foundation: parametrized supermanifolds

To take home

Quantum theory:

- SUSY constraint operator for real β
- rigorous construction, strucutre & properties of **graded holonomies**
- application to symmetry reduced model (cosmology)

Difficulties:

- proper definition of inner product in self-dual theory
- solution reality conditions?

$\rightarrow$ Resolvable in symmetry reduced model.

To take home

Outlook:

- include local supersymmetric matter as relational clock (simplification due to $S \sim \sqrt{H}$)
- (charged) supersymmetric BHs $\leftrightarrow$ entropy calculations in string theory [Strominger+Vafa '96, Cardoso et al. '96]
- generalization to extended SUSY ($\leftrightarrow$ compare enriched Cartan approach with other approaches [Castellani+Grassi et al '14])
- etc. etc.