LQC of black holes (recent progress)

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Background

- Black holes: Promising observational windows to quantum gravity phenomena, in the dawn of GW astronomy.

- Long history of works for the quantization of black holes in LQG & LQC. Satisfactory yet simple description?

[Ashtekar, Bojowald, Modesto, Cartin, Khanna, Boehmer, Vandersloot, Chiou, Campiglia, Gambini, Pullin, Sabharwal, Brannlund, Kloster, De Benedictis, Olmedo, Dadhich, Joe, Singh, Haggard, Rovelli, Corichi, Saini, Cortez, Cuervo, Morales-Técotl, Ruelas, Yonika, Bianchi, Christodoulo, D'Ambrosio, Alesci, Bahrami, Pranzetti, Kelly, Santacruz, Wilson-Ewing, Zhang, Ma, Song, Bodendorfer, Mele, Münch, Mena Marugán, García-Quismondo, Perez, Speziale, Viollet, Han, Liu, Alonso-Bardaji, Brizuela, Vera, …]

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  - ★ Spherical symmetry as starting point.

  - ★ Use LQC techniques to describe the interior.

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- Highlight: Extension of Kruskal spacetime by AOS.

[Ashtekar, Olmedo, Singh 2019]

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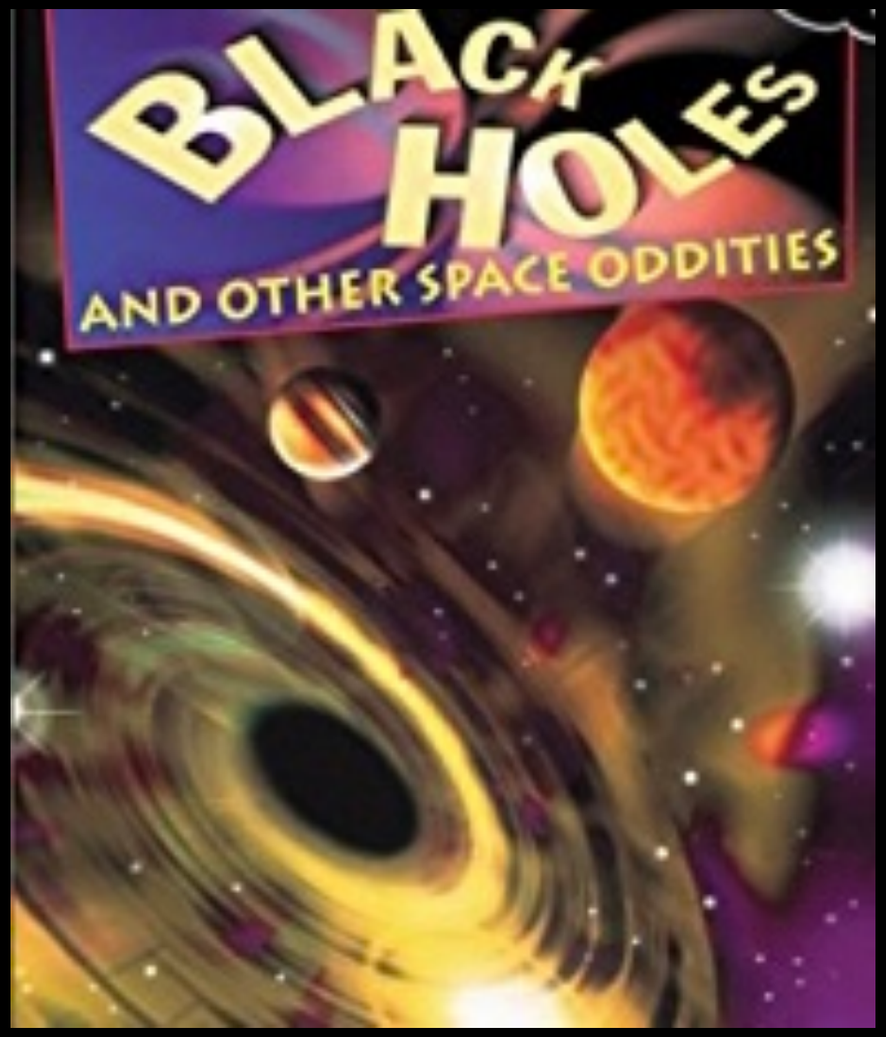
  - ⋆ Quantization? (General properties, eff. regimes, etc.)



BH interior & its "effective" LQC models

Interior region in GR

- Metric: Cosmology of Kantowski-Sachs type.

$$ds^2 = -N(t)^2 dt^2 + \frac{p_b^2(t)}{L_o^2 |p_c(t)|} dx^2 + |p_c(t)| d\Omega_S^2, \quad x \in (0, L_o)$$

- Symmetry-reduced Ashtekar-Barbero variables:

$$E_i^\alpha \to (p_b, p_c) \qquad A_\alpha^i \to (\dot{b}, c)$$

$$\{b, p_b\} = \gamma, \qquad \{c, p_c\} = 2\gamma, \qquad \gamma \to \text{Immirzi}$$

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$\propto$ ADM mass

Regularization of Hamiltonian

$$H_{KS}[N] = NL_o \frac{b}{\gamma\sqrt{|p_c|}}\left(O_b^{KS} - O_c^{KS}\right), \quad \text{in GR}: O_b^{KS} = O_c^{KS} = m$$

- Anticipating lack of continuity in LQC: $\hat{b}, \hat{c}$ not well-def.
- Use symmetries and rewrite the curvature of A_α^i in terms of an holonomy circuit enclosing a minimum area Δ.

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- Regularized Hamiltonian [ignoring $\mathcal{O}(\delta_b)$]:

$$H_{KS}^{\text{reg}}[N_T] = L_o O_b^{KS}\left[b \to \sin(\delta_b b)/\delta_b)\right] - L_o O_c^{KS}\left[c \to \sin(\delta_c c)/\delta_c\right],$$

$$N_T = \frac{\gamma\delta_b\sqrt{|p_c|}}{\sin(\delta_b b)}, \quad |\delta_b|, |\delta_c| \text{ coord. lengths of min. area plaquettes}$$

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“Effective” models

$$H_{KS}^{\mathrm{reg}}[N] = NL_o \frac{\sin(\delta_b b)}{\gamma \delta_b \sqrt{|p_c|}} \left(O_b - O_c\right), \quad \mathrm{effLQC} : O_b = O_c = m$$

- Several studies of the evolution generated by $H_{KS}^{\mathrm{reg}}[N]$.
- Freedom in the choice of parameters δ_b, δ_c.

[Ashtekar, Bojowald, Modesto, Campiglia, Gambini, Pullin, Boehmer, Vandersloot, Chiou, Joe, Singh, Corichi, Olmedo, Saini, Bodendorfer, Mele, Münch, Mena Marugán, García-Quismondo, Han, Liu, …]

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[Ashtekar, Olmedo, Singh]

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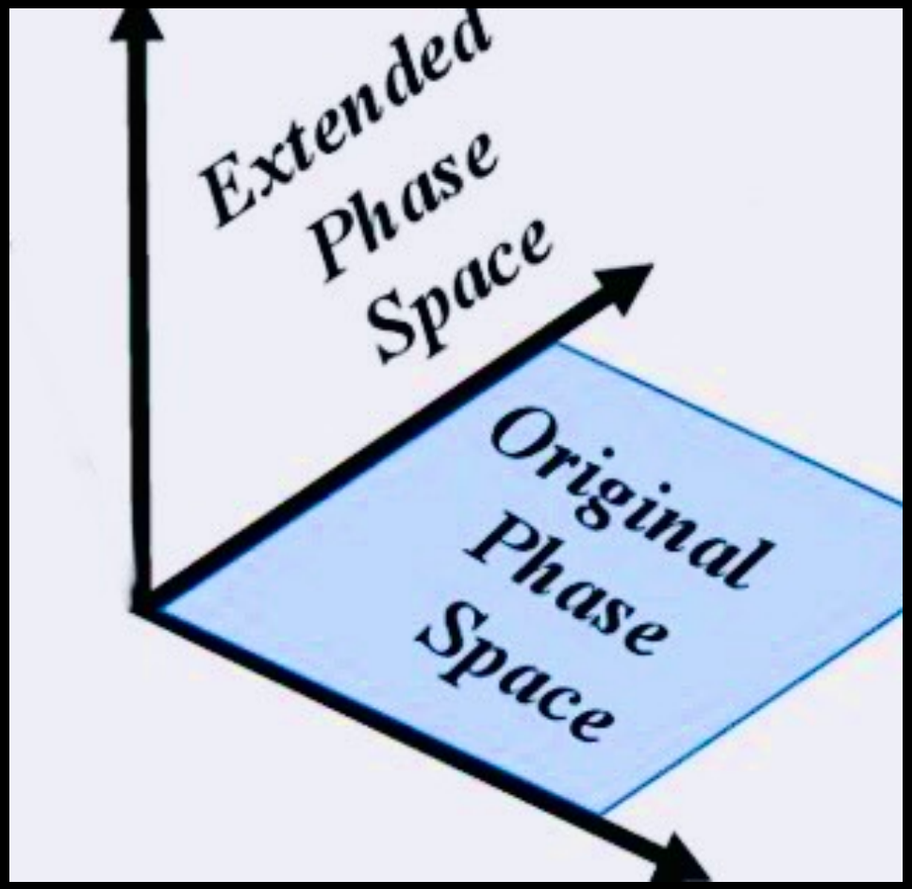
- 2019: “Quantum Transfiguration of Kruskal BHs” (**AOS**),

$$\delta_b \sim \left(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m}\right)^{1/3}, \quad L_o\delta_c \sim \frac{1}{2}\left(\frac{\gamma\Delta^2}{4\pi^2 m}\right)^{1/3},$$

<u>on solutions</u>



AOS type of models: Hamiltonian formulation

Caveats on derivation of e.o.m

- The AOS "effective" dynamics is generated by:

$$H_{\mathrm{AOS}}^{\mathrm{eff}} = L_o \left(O_b - O_c \right) (= H_{KS}^{\mathrm{reg}}[N_T])$$

treating the parameters δ_b, δ_c as pure constants.

- On each solution, constraint implies $O_b = O_c = m$.

- On each sol., declare $\delta_b = \delta_b(m), \delta_c = \delta_c(m)$ for large BHs.

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- However, m is a function on phase space (const. motion)!

[Bodendorfer, Mele, Münch, Mena Marugán, García-Quismondo]

Extended phase space

- The AOS model can be derived from a Hamiltonian

$$\underline{N}H_{\mathrm{AOS}}^{\mathrm{eff}} + \lambda_b \Psi_b + \lambda_c \Psi_c,$$

on a extended phase space that includes δ_b, δ_c (& momenta).

- Ψ_b, Ψ_c : Relation between δ_b, δ_c and BH mass on solutions:

$$\Psi_b = K_b\left(O_b, O_c\right) - \delta_b, \quad \Psi_c = K_c\left(O_b, O_c\right) - \delta_c,$$

$$K_b(m,m) \sim \left(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m}\right)^{1/3}, \quad K_c(m,m) \sim \frac{1}{2L_o}\left(\frac{\gamma\Delta^2}{4\pi^2 m}\right)^{1/3}$$

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- <u>Idea</u>: fix the gauge so that $\lambda_b = \lambda_c = 0$ to remove δ_b, δ_c.

- <u>Result</u>: The reduced phase space is ***<u>not</u>*** symplectomorphic to that of Kantowski-Sachs cosmologies ($\neq$ GR algebra).

[BEN, Mena Marugán, García-Quismondo]

Road to quantization

- The AOS model can be derived from a Hamiltonian

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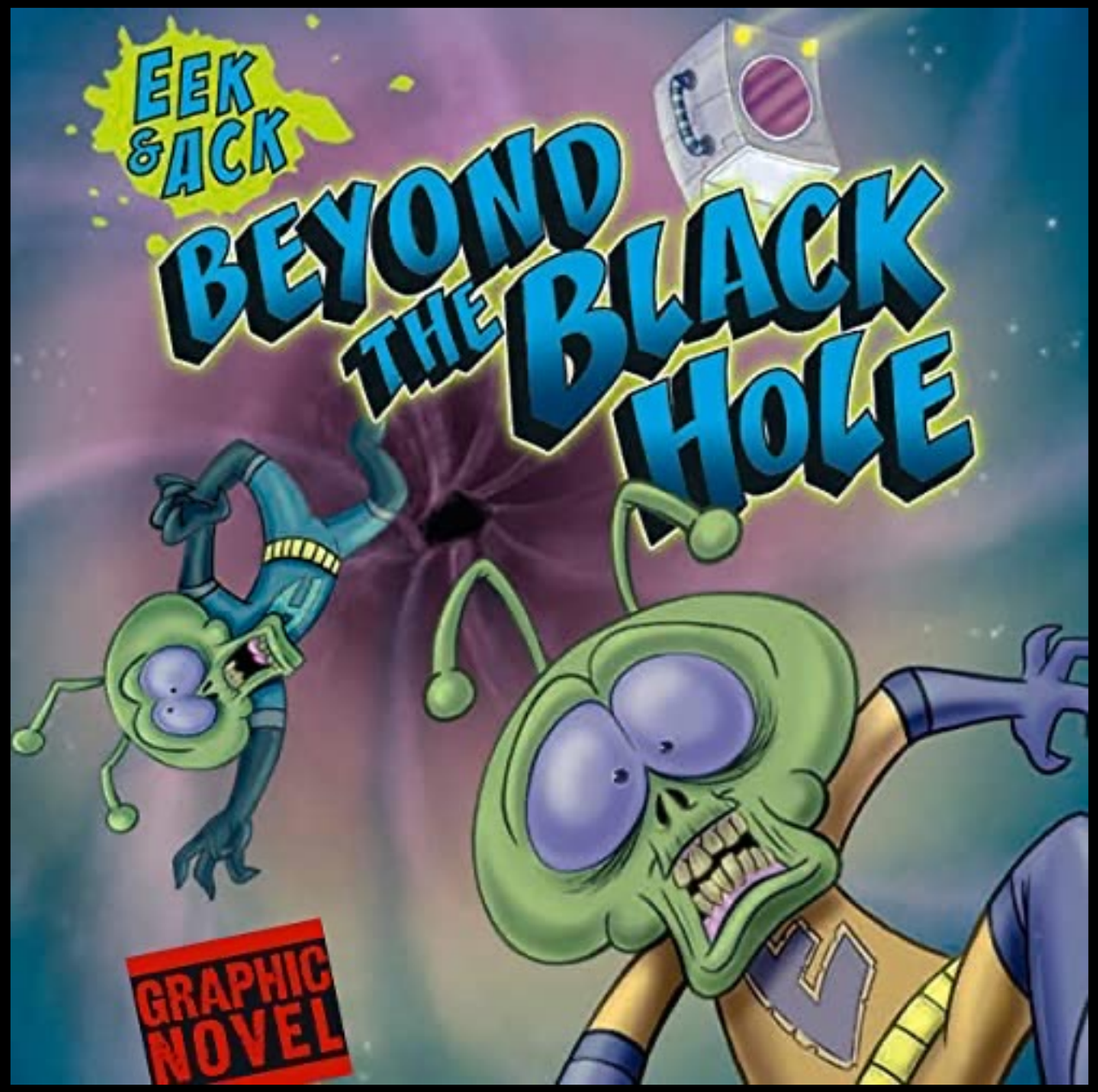
on a extended phase space that includes δ_b, δ_c (& momenta).

- LQC under control for triad/connection algebra in GR.

- Quantization strategy: Use LQC tools to find a quantum representation of constraints in the extended framework.



Quantization: Kinematics

Holonomy-flux algebra

- Matrix elements of holonomies of A^i_α along θ and x:

$$\mathcal{N}_{\mu_b} = e^{ib\mu_b/2}, \qquad \mathcal{N}_{\mu_c} = e^{ic\mu_c/2}, \qquad \mu_b, \mu_c \in \mathbb{R}\,.$$

- Fluxes of E^α_i across surfaces determined by p_b and p_c.

- Holonomy-flux algebra:

$$\{\mathcal{N}_{\mu_b}, p_b\} = \frac{i}{2}\mu_b \gamma \mathcal{N}_{\mu_b}, \qquad \{\mathcal{N}_{\mu_c}, p_c\} = i\mu_c \gamma \mathcal{N}_{\mu_c}$$

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Two copies of homogeneous & isotropic LQC

Kinematic Hilbert space

- $\mathscr{H}^{\text{kin}}_{\text{LQC}}$: space of square summable functions on $\mathbb{R}^2$.

$$\text{Basis } |\mu_b, \mu_c\rangle \quad \langle \mu_b, \mu_c | \mu_b', \mu_c', \rangle = \delta_{\mu_b, \mu_b'} \delta_{\mu_c, \mu_c'}$$

- $\hat{\mathcal{N}}_{\mu_{b,c}} \to$ translation, $\hat{p}_{b,c} \to$ multiplication operators.

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- Schrödinger representation for δ_b, δ_c and momenta.

- Total (kinematic) Hilbert space:

$$\mathscr{H}^{\text{kin}}_T = \mathscr{H}^{\text{kin}}_{\text{LQC}} \otimes L^2(\mathbb{R}, d\delta_b) \otimes L^2(\mathbb{R}, d\delta_c)$$

Hamiltonian constraint

$$H_{\mathrm{AOS}}^{\mathrm{eff}} = L_o \left(O_b - O_c \right)$$

- The "angular" part reads

$$O_c = \frac{\sin(\delta_c c)}{\gamma L_o \delta_c} p_c, \qquad \sin(\delta_c c) = \frac{1}{2i} \left(\mathcal{N}_{2\delta_c} - \mathcal{N}_{-2\delta_c} \right)$$

- Quantization: $\sin(\delta_c c) p_c / \delta_c \to \hat{\Omega}_c$ on (a subset of) $\mathcal{H}_T^{\mathrm{kin}}$.

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- For each fixed pair δ_b, δ_c change the LQC basis:

$$|\mu_b, \mu_c\rangle \to |\tilde{\mu}_b = \mu_b \delta_b^{-1}, \tilde{\mu}_c = \mu_c \delta_c^{-1}\rangle$$

$\rightsquigarrow$ **$\hat{\Omega}_c^2$ same as in FLRW cosmology**

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- Operator $\hat{O}_c \propto \hat{\Omega}_c$ essentially self-adjoint on

$$\mathrm{Cyl}_{\varepsilon_c}^{\pm} = \mathrm{span}\{ \, | \tilde{\mu}_b, \tilde{\mu}_c \rangle : \tilde{\mu}_c = \pm \left(\varepsilon_c + 2n \right), n \in \mathbb{N} \}, \quad \varepsilon_c \in (0,2]$$

- Abs. continuous spectrum equal to $\mathbb{R}$, independent of δ_c.

[Lewandowski, Kaminski, Mena Marugán, Martín-Benito, Olmedo, Pawlowski]

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$$\overline{\mathrm{Cyl}_{\varepsilon_c}^{\pm}} \leftrightarrow \overline{\mathrm{span}\{ \,|\tilde{\mu}_b \in \mathbb{R}\rangle} \otimes L^2(\mathbb{R}, dm)$$

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$$O_b = -\frac{1}{2\gamma L_o} \left[\Omega_b + \gamma^2 \delta_b^2 \tilde{p}_b^2 \Omega_b^{-1} \right], \quad \Omega_b = \frac{\sin(\delta_b b)}{\delta_b} p_b, \quad \tilde{p}_b = \frac{p_b}{\delta_b}$$

- Quantization: "Minimal" requirements on $\hat{O}_b$.

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- Quantization: "Minimal" requirements on $\hat{O}_b$:
  - ⋆ Built from $\hat{\Omega}_b, \hat{\tilde{p}}_b$: Only depends on δ_b^2 for each pair δ_b, δ_c.
  - ⋆ Preserves super-selection sectors of $\hat{\Omega}_b$ (similar to $\hat{\Omega}_c$).
  - ⋆ Ess. self-adjoint, no singular sp., point sp. = discrete.

Physical states

(in collab. with A. García-Quismondo, G.A. Mena Marugán & A. Mínguez)

Constraint equations

- Total Hamiltonian: $\underline{N} H^{\text{eff}}_{\text{AOS}} + \lambda_b \Psi_b + \lambda_c \Psi_c \,.$

- Quantum Hamiltonian constraint: $(\psi_p | \hat{O}_b = (\psi_p | \hat{O}_c \,.$

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$$\hat{O}_b \, \psi_p(\tilde{\mu}_b, m, \delta_b, \delta_c) = m \, \psi_p(\tilde{\mu}_b, m, \delta_b, \delta_c), \quad m \in \mathbb{R}$$

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Looking for physical states in the algebraic dual of:

$$C_0^\infty(\text{Sp}^c_{\hat{O}_b}) \cup \text{span}\{ \, | \, \omega_n \in \text{Sp}^d_{\hat{O}_b} \rangle \}$$

Physical Hilbert space

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- Additional constraints: $\delta_b = K_b(O_b, O_c)$, $\delta_c = K_c(O_b, O_c)$.

- Quantum imposition of all constraints on $(\psi_p|$:

$$(\psi_p | \leftrightarrow \xi(m), \quad m \in \text{Sp}_{\hat{O}_b}[\delta_b = K_b(m, m)]$$

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- Inner product on $\mathscr{H}_p$ from spectral measure of $\hat{O}_b$.

Possibilities: Discrete mass

$$O_b = -\frac{1}{2\gamma L_o}\left[\Omega_b + \gamma^2\delta_b^2\Omega_b^{-1}\tilde{p}_b^2\right], \quad \Omega_b = \frac{\sin(\delta_b b)}{\delta_b}p_b, \quad \tilde{p}_b = \frac{p_b}{\delta_b}$$

- Imagine that it is possible to define $\hat{O}_b$ such that

$$\hat{O}_b^2 \propto \hat{\Omega}_b^2 + 2\gamma^2\delta_b^2\hat{\tilde{p}}_b^2 + \gamma^4\delta_b^4\hat{P}, \qquad \hat{P} \text{ positive}$$

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$$\rightsquigarrow \langle\hat{O}_b^2\rangle \geq \propto 2\gamma^2\delta_b^2\langle\hat{\tilde{p}}_b^2\rangle \longrightarrow \mathrm{Sp}_{\hat{O}_b^2}[\delta_b \neq 0] \ \underline{\textbf{discrete}}$$

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- Then $\mathrm{Sp}_{\hat{O}_b}[\delta_b \neq 0]$ discrete: Gaps in the physical BH mass.

[Zhang, Ma, Song, Zhang, 2020-2022]

- Consistent with recent works quantizing $H^{\mathrm{eff}}_{\mathrm{AOS}}\sin(\delta_b b)p_b/\delta_b$.

- Satisfactory $\delta_b, \delta_c \to 0$ limit? Spectrum becomes continuous.

Possibilities: Cont. mass

$$O_b = -\frac{1}{2\gamma L_o}\left[\Omega_b + \gamma^2 \delta_b^2 \Omega_b^{-1} \tilde{p}_b^2\right], \quad \Omega_b = \frac{\sin(\delta_b b)}{\delta_b} p_b, \quad \tilde{p}_b = \frac{p_b}{\delta_b}$$

- The standard habitat for physical states may be too small.

- Imagine (e.g.) a "direct" quantization of O_b.

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- Experience in LQC indicates that this eq. is solvable for continuous values of the BH mass. Good $\delta_b \to 0$ limit.

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Now we search on the algebraic dual of $D(\hat{O}_b)$ ("much" bigger!!)

Conclusions & outlook

- Aim: LQC of a BH interior leading to AOS-type models.
- Extended phase space: motivation from dynamics of effective model & convenient for quantization.
- Operators in Hamiltonian from FLRW LQC. ✓
- <u>Important</u>: Choice of dual space to construct physical Hilbert space → Discrete/continuous BH mass. ⚠
- WdW limit? Relational dynamics? Effective AOS?