Stellar collapse and shell-crossing singularities in effective loop quantum gravity

Part of the *Non-singular Black Hole* symposium, ILQGS

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In collaboration with *Luca Cafaro, Lorenzo Cipriani, Viqar Husain, Carlo Rovelli, Farshid Soltani & Edward Wilson-Ewing.*

Also based on the works of

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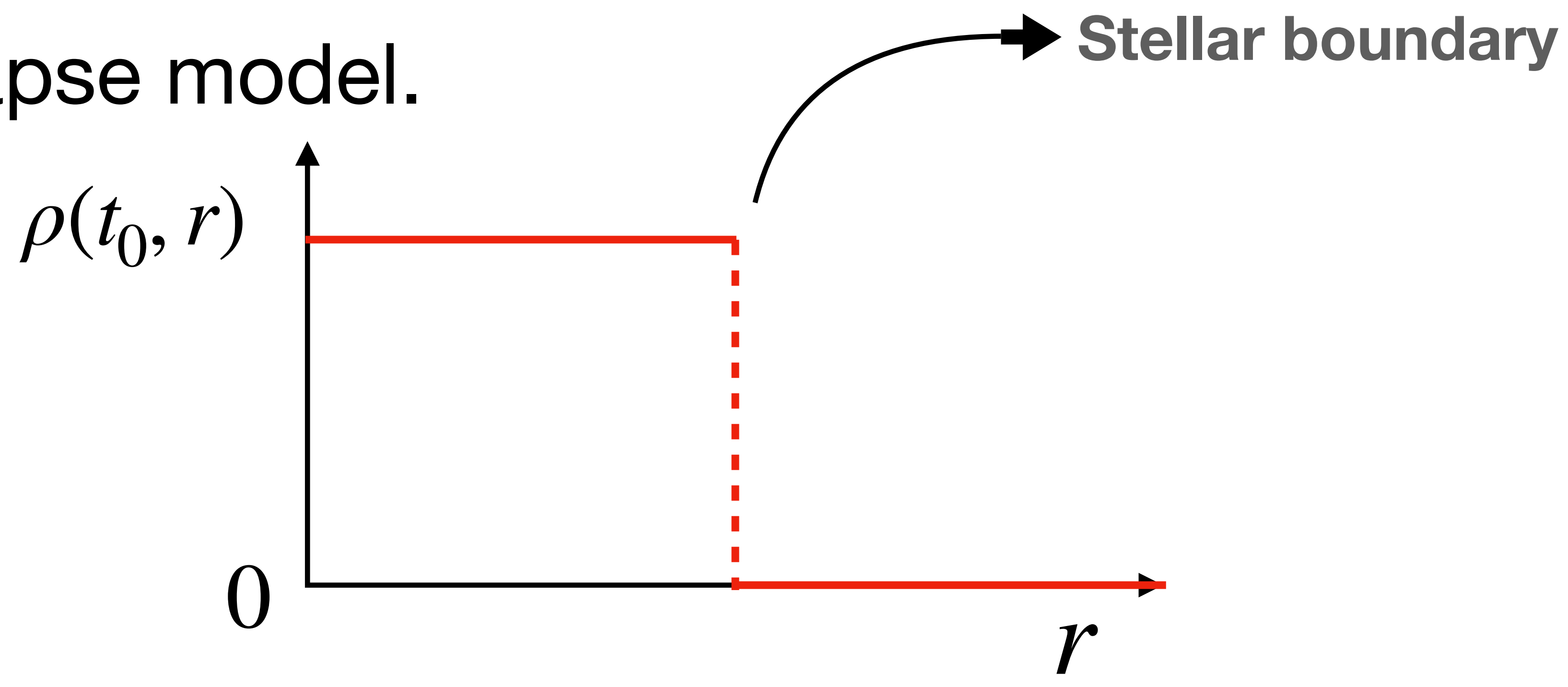
[Alonso-Bardaji, Ashtekar, Bambi, Belfaqih, Bianchi, Bobula, Bojowald, Brahma, Brizuela, Corichi, Christodoulou, D'Ambrosio, Duque, Giesel, Haggard, Han, Hossenfelder, Kelly, Lewandowski, Liu, Ma, Malafrina, Modesto, Munch, Olmedo, Pawlowski, Prémont-Schwarz, Santacruz, Singh, Speziale, Vidotto, Weigl, Yang, Zhang]

The effective Oppenheimer-Snyder collapse

The Oppenheimer-Snyder (OS) collapse is the simplest star collapse model.

$$\rho(t_0, r) = \begin{cases} \rho_0, & \text{for} \quad r \leq R \\ 0, & \text{for} \quad r > R \end{cases}$$



The problem at the effective level has been faced by employing different techniques and different polymerization schemes:

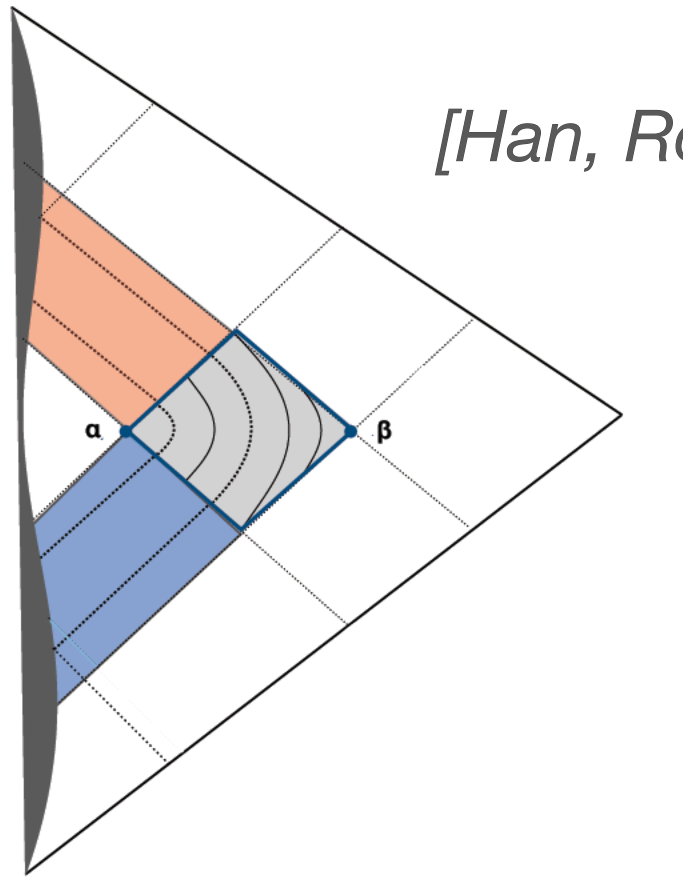
- **Equations of motion in integral form** *[Husain, Kelly, Santacruz, Wilson-Ewing, 2021]*
- **Effective equations of motion + junction conditions** *[Munch, 2021; Giesel, Liu, Singh, Weigl, 2023]*
- **Israel junction conditions** *[Bobula, Pawlowski, 2023; Lewandowski, Ma, Yang, Zhang, 2023; FF, Rovelli, Soltani, 2023; Cafaro, Lewandowski, 2024; Bobula, 2024]*
- **Israel junction conditions + gluing techniques** *[Han, Rovelli, Soltani 2023]*
- **Effective equations of motion** *[Alonso-Bardaji, Brizuela, 2023; Duque, 2024]*

The effective Oppenheimer-Snyder collapse

In this talk I will focus on results based on:

$\bar{\mu}$-scheme + **K-loop quantization**

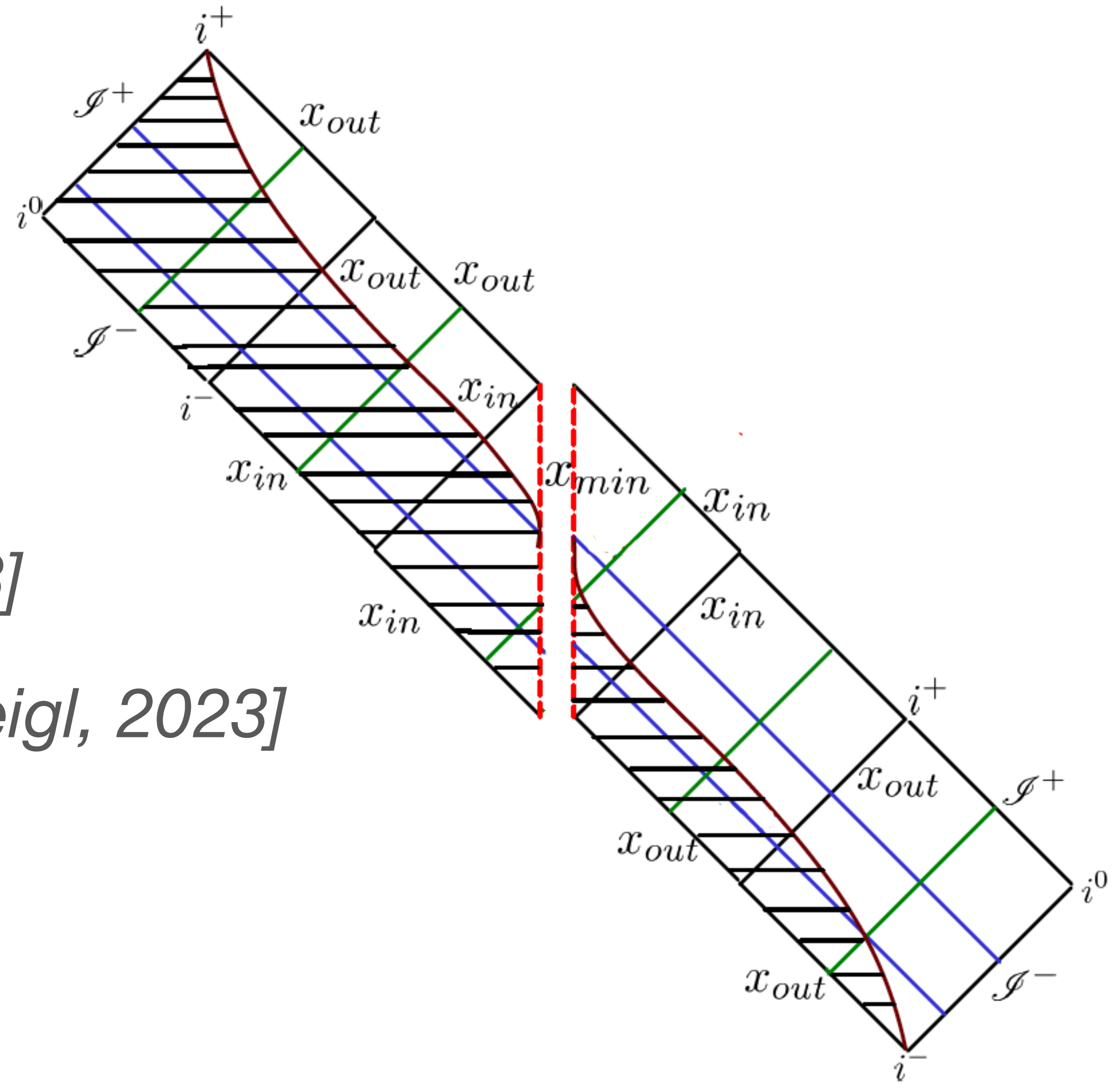
Within this scheme, all the approaches (except the first one) agree on the interior OS effective dynamics:

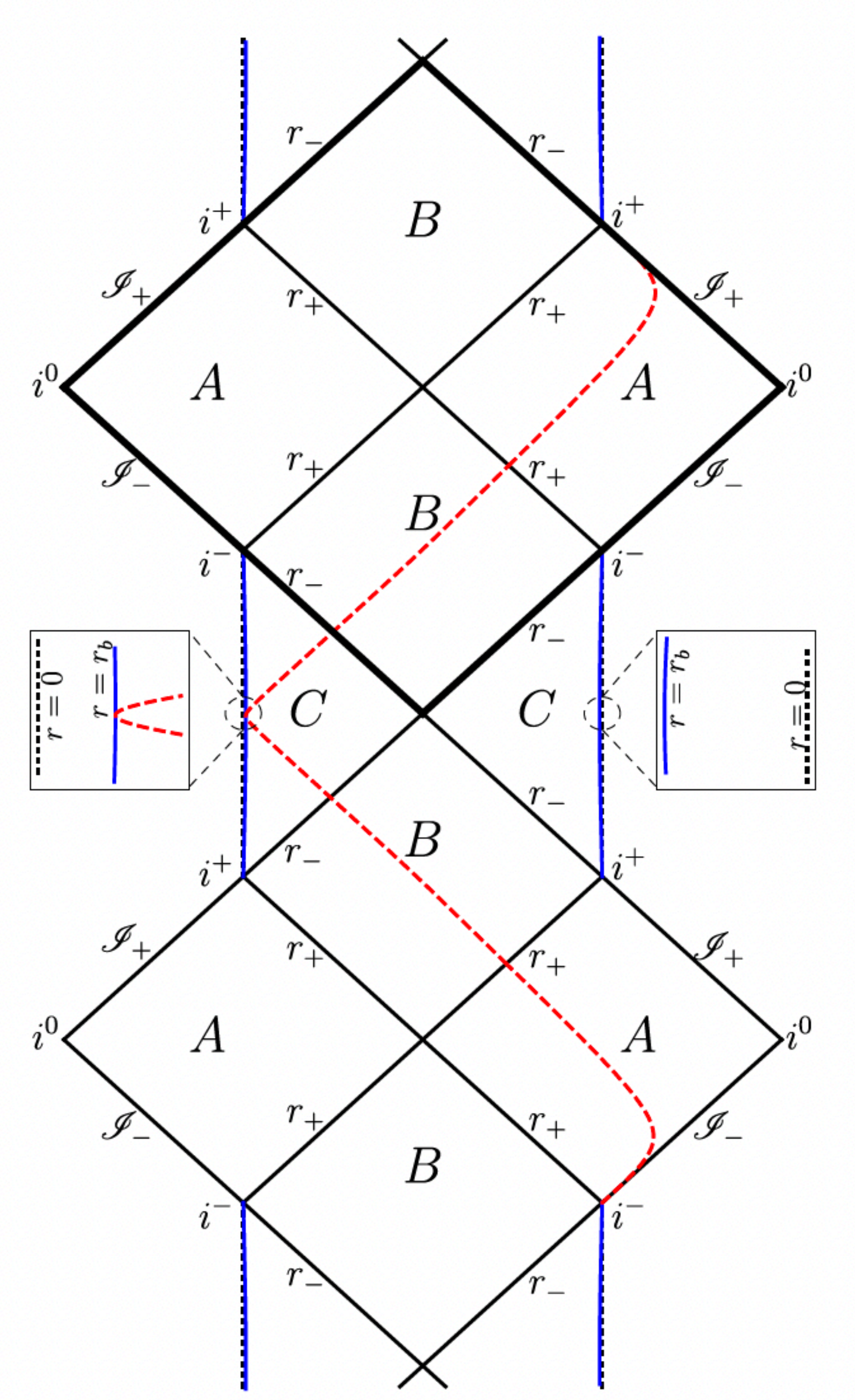


[Han, Rovelli, Soltani, 2023]

[Münch, 2023]

[Giesel, Liu, Singh, Weigl, 2023]





[Lewandowski, Ma, Yang, Zhang, 2023]

[Bobula, Pawlowski, 2023]

Is the OS a realistic description of effective stellar collapse?

Dust collapse in LTB gauge

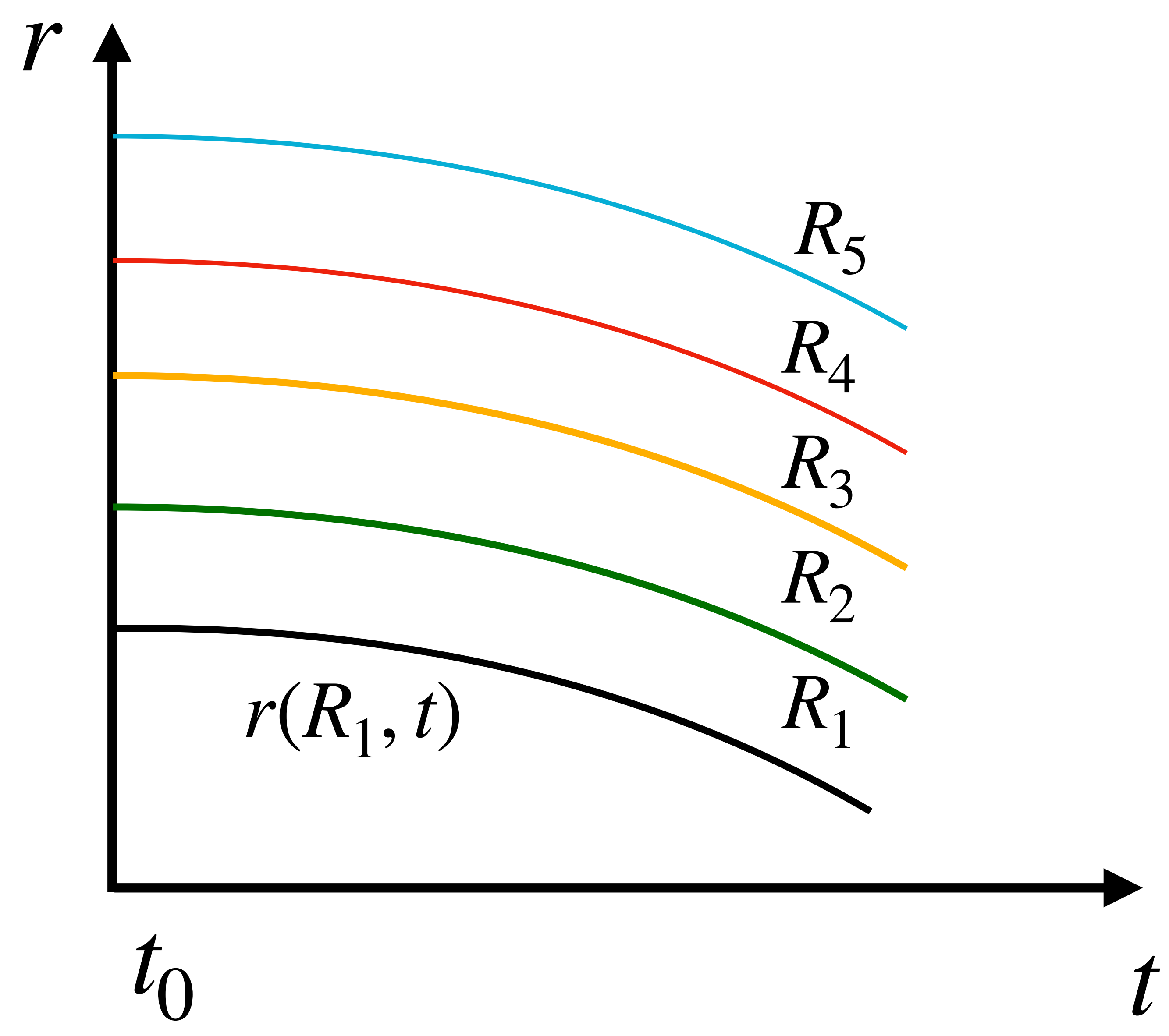
The metric describing dust evolution in Lemaître-Tolman-Bondi coordinates reads

$$ds^2 = -dt^2 + \frac{\left[\partial_R r(R,t)\right]^2}{1+\varepsilon(R)} dR^2 + r(R,t)^2 d\Omega^2$$

This metric describes **both** the matter and vacuum region (if exists) of the spacetime.

Interpretation: we can imagine to divide the spatial part of the manifold in spherical shells parametrized by the radial coordinate R.

The spacetime evolution is described through the evolution of the **areal radius r of the shell R at time t.**



Effective dynamics in LTB coordinates

The LTB effective equations for spherically symmetric generic dust collapse (marginally bound case):

$$\left(\frac{\dot{r}}{r}\right)^2 = \frac{2Gm}{r^3}\left(1 - \frac{2\Delta Gm}{r^3}\right), \qquad \textit{[Giesel, Liu, Singh, Weigl, 2023]}$$

where Δ is the minimum area gap in LQG: $\Delta \sim l_P^2$ and $m(R)$ is the mass function, fixed by the initial energy density profile.

The general solution of the EoM is:

$$r(R,t) = [2Gm(R)]^{1/3}\left[\frac{9}{4}\left(t - \alpha(R)\right)^2 + \Delta\right]^{1/3}.$$

$\Longrightarrow$ Each shell R should bounce at: $t = \alpha(R)$.

But: The solution cannot be trusted after *shell crossing singularities*' formation.

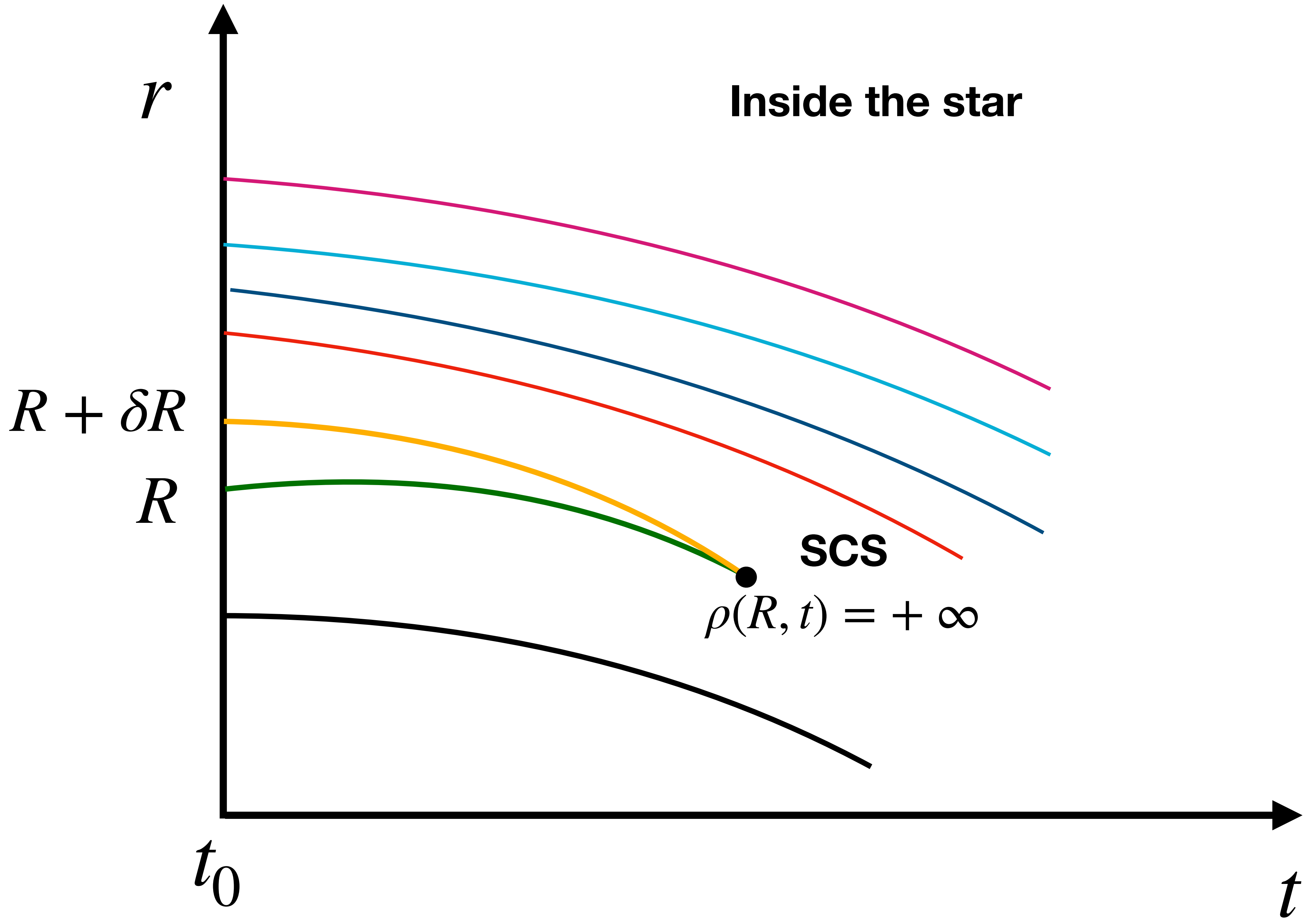
Shell-crossing singularities in generic LTB space-times

The energy density is given by:

$$\rho(R,t) = \frac{\partial_R m(R)}{4\pi r^2 \partial_R r(R,t)}$$

If :
- $\partial_R r(R,t) = 0$ for some R at some time t (two shells cross)
- For the same R: $\partial_R m(R) \neq 0$ (they cross in the matter region)

$$\implies \rho(R,t) = +\infty, \quad R_{\mu\nu}g^{\mu\nu} = +\infty.$$



A shell-crossing singularity (SCS) forms: it is a physical *weak* singularity.

In classical GR, many initial configurations develop SCS, but one can choose suitable initial profiles that don't develop such singularities *[Hellaby, Lake, 1984]*.

Can SCS be avoided in the effective dynamics?

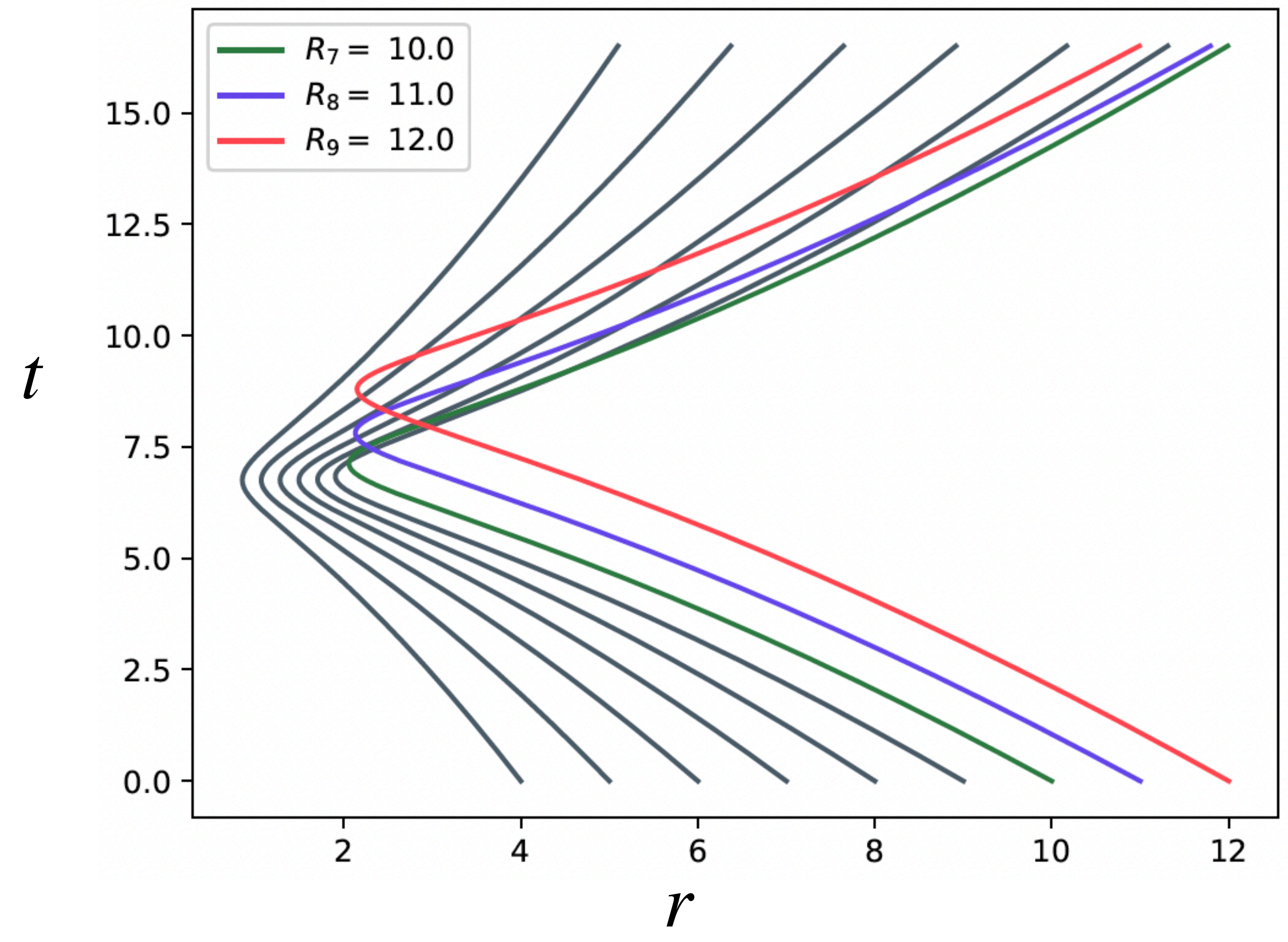
Theorem: "*for the marginally bound case, a shell-crossing singularity will necessary form at some R if the initial energy density profile is non-negative, continuous, of compact support and for which $m(R)$ is not everywhere zero*." *[FF, Husain, Wilson-Ewing, 2024]*

Similarly for non-compact profiles with enough large inhomogeneities.

Numerical Example:

$$\rho(R, t_0) = C\left(1 - \tanh\frac{R - R_0}{\sigma}\right),$$

$$C \propto m_{tot} = 5, \quad R_0 = 10, \quad \sigma = 1.1$$

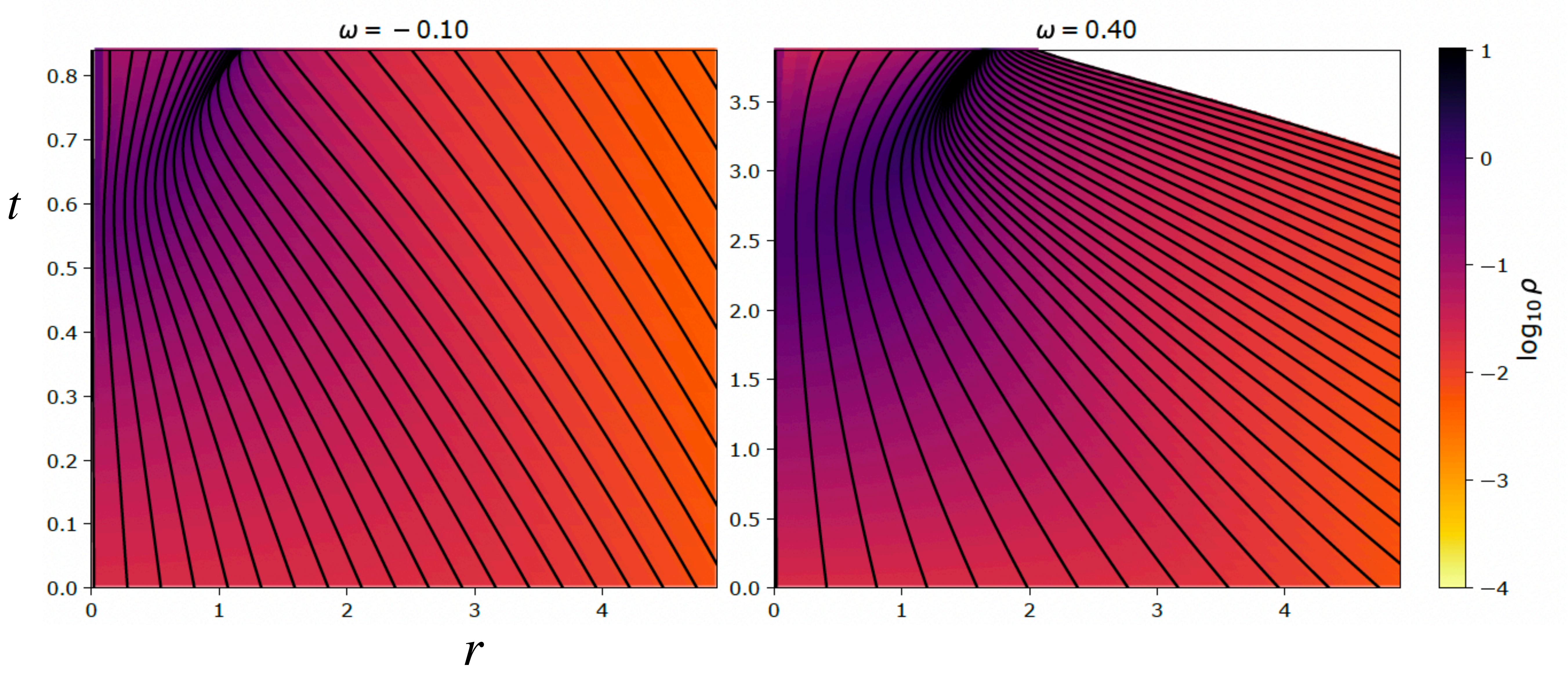


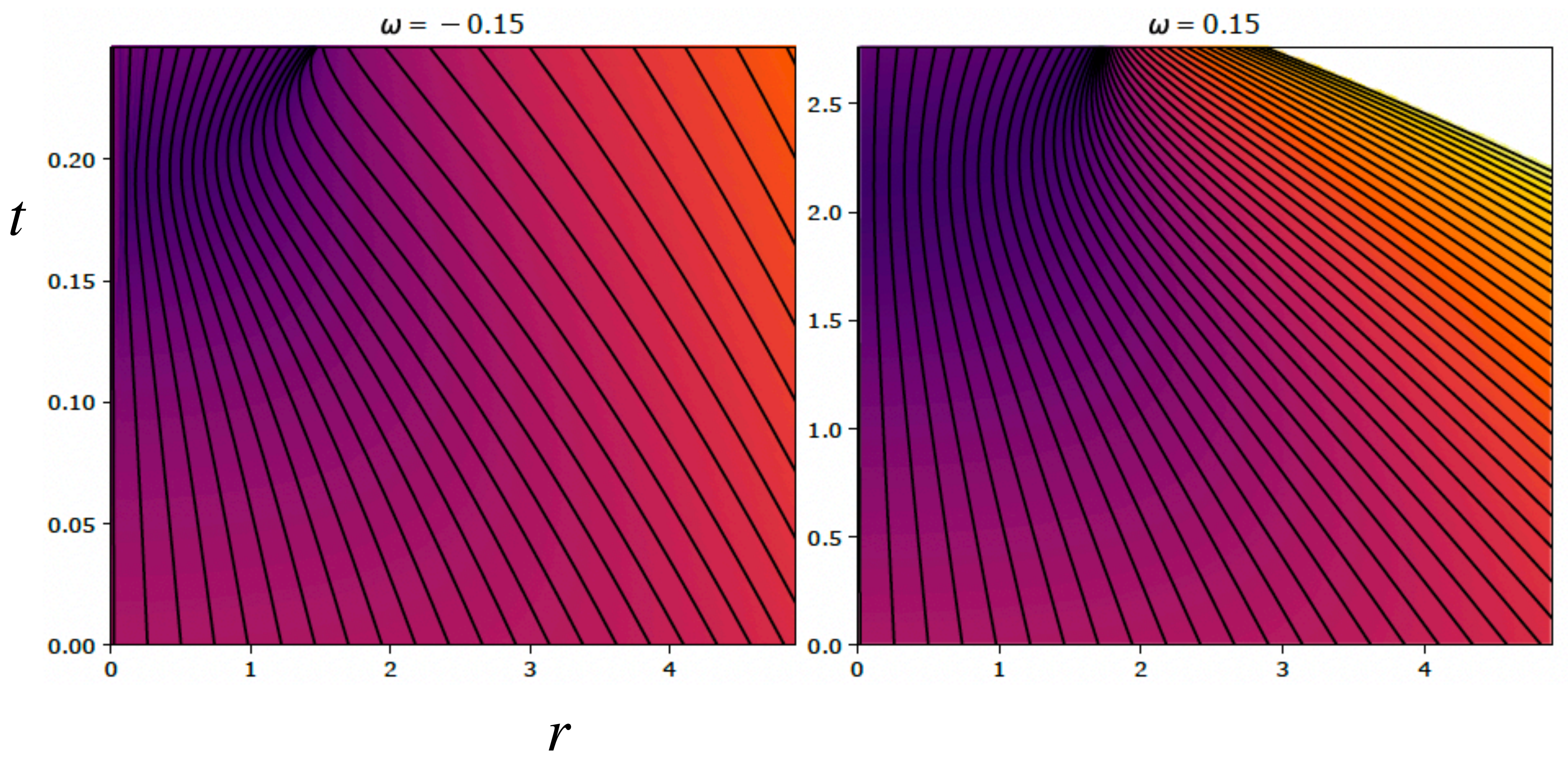
General picture: quantum gravitational bounce of the core followed by SCS formation on the tail.

Same for the non-marginally bound case *[Cipriani, FF, Wilson-Ewing, 2024].* ⟶ OS IS NOT a good prototype for dust.

Does the inclusion of pressure change this picture?

For perfect fluids with linear equation of state $p = \omega\rho$: *[Cipriani, Cafaro, FF & Soltani, 2024]*





$$\rho(t_0, R) = C\left[\frac{\pi}{2} - \arctan\frac{R - R_0}{\lambda}\right] ,$$

$$R_0 = 13, \quad \sigma = 3.5, \quad \lambda = 1 \; , \qquad \varepsilon(t_0, R) = \tilde{C}\left[\exp\left(-\frac{R^2}{2\sigma^2}\right) - 1\right]$$

$$\rho(t_0, R) = C\left[1 - \tanh\left(\frac{R - R_0}{\lambda}\right)\right],$$

$$R_0 = 11, \quad \sigma = \lambda = 3.5, \quad c = G = 1, \quad \gamma^2\Delta = 0.1$$

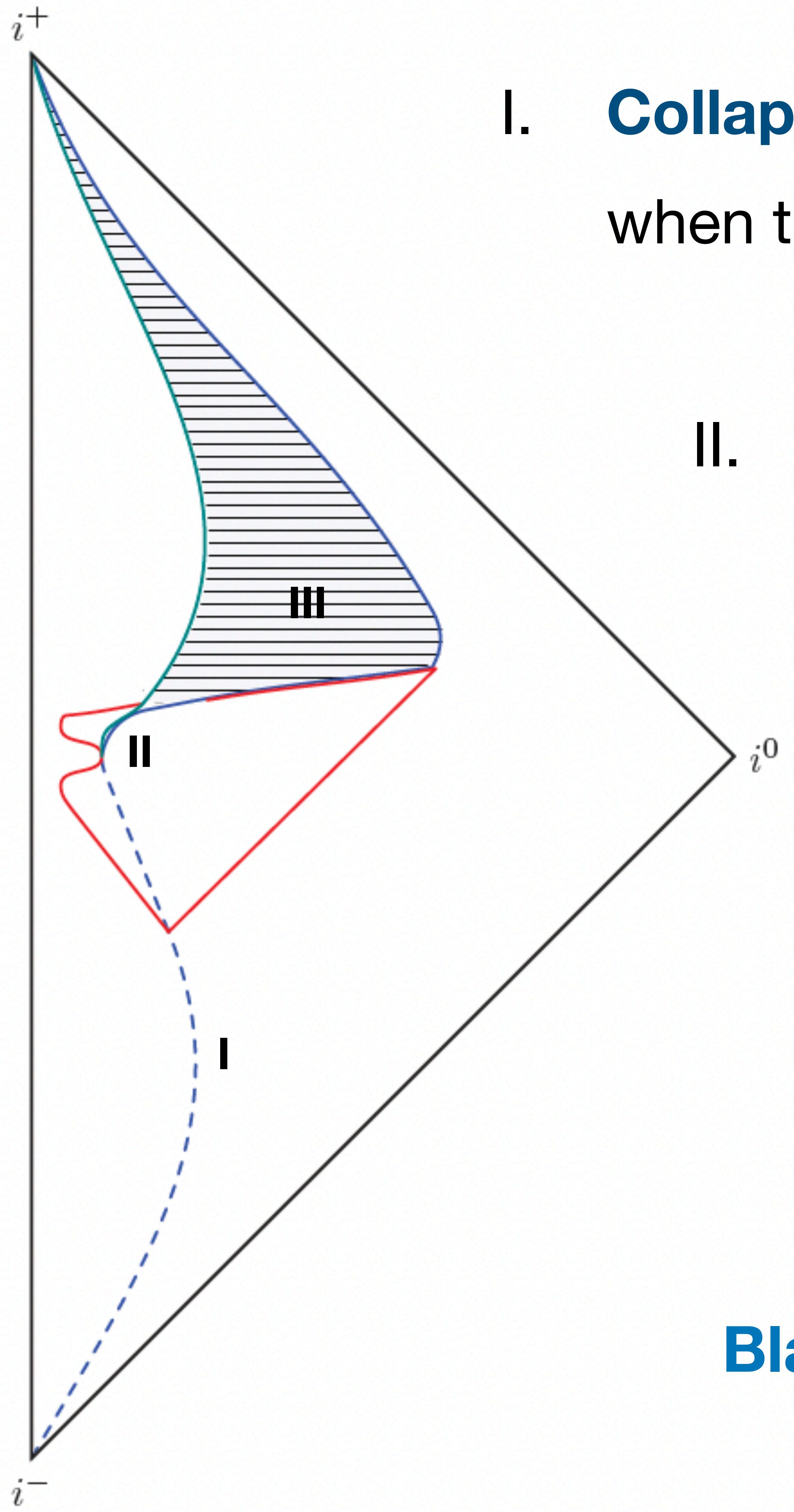
⟶ Shell-crossing singularities seem to be a **general feature** of star collapse in this effective model.

Discussion

- Shell-crossing singularities are a central feature of stellar collapse within the $\bar{\mu}$+K-loop quantization scheme, both in the dust case and fluid with pressure.
- The Oppenheimer-Snyder evolution is not a good description for stellar collapse within this scheme.
- How to extend the dynamics beyond shell-crossing singularities? One possibility is through weak solutions *[Husain, Kelly, Santacruz, Wilson-Ewing, 2022; Cipriani, FF, Wilson-Ewing, 2024].*
- Do particular equations of state can remove SCS within this model? We have the tools to investigate this.
- Interesting to investigate additional/other polymerizations (Michał's talk) in Oppenheimer-Snyder case and beyond, presence of SCS?
- What is the role of Hawking radiation here? (Jonas' talk)

Thanks for your attention!

Backup slides: shockwave dynamics for dust



I. **Collapse phase**: the energy density of the star progressively increases and its volume decreases; when the horizon forms the star becomes as a black hole.

II. **Bouncing phase**: when the energy density of the core becomes planckian, the shells of the core bounce and crush the collapsing shells of the tail.
→ A *shell-crossing singularity* forms, together with a *discontinuity* in the gravitational field.

III. **Shockwave phase**: the whole matter content of the star rapidly concentrates in a thin shell moving outward together with the discontinuity. When the shell reaches the outer horizon, the black hole disappears.

Black hole life-time:

$$T \sim \frac{8\pi M^2}{3m_p}$$

[Husain, Kelly, Santacruz, Wilson-Ewing, 2022]

Backup slides: beyond dust

The model can be generalized to matter source with non-vanishing pressure *[E.Wilson-Ewing, 2024, Cipriani, Cafaro, FF & Soltani, 2024]*, using similar tools as in effective LQC *[Singh, 2009]*.

Effective loop quantum gravity gets access to more realistic stellar collapse

In LTB gauge, the only equation of motion that takes quantum corrections:

$$\dot{r}^2 = N(R,t)^2\left(\frac{2Gm(R,t)}{r} + \varepsilon(R,t)\right)\left[1 - \frac{\gamma^2\Delta}{r^2}\left(\frac{2Gm(R,t)}{r} + \varepsilon(R,t)\right)\right]$$

Notice that $N = N(R,t)$, $\varepsilon = \varepsilon(R,t)$, $m = m(R,t)$. The other equations contain also $p(R,t)$.

Within this framework, we can consider any equation of state (linear, polytropic,..), for perfect and non-perfect fluids.

Backup slides: effective EOMs for perfect fluid

Complete set of effective equations of motion for effective stellar collapse with pressure (perfect fluid):

$$\dot{r}^2 = N(R,t)^2\left(\frac{2Gm(R,t)}{r} + \varepsilon(R,t)\right)\left[1 - \frac{\gamma^2\Delta}{r^2}\left(\frac{2Gm(R,t)}{r} + \varepsilon(R,t)\right)\right]$$

$\dot{m} = -4\pi p r^2 \dot{r}$ $\longrightarrow$ m depends on time, due to radial pressure.

$$\dot{\varepsilon} = -2(1+\varepsilon)\frac{p'}{\rho + p}\frac{\dot{r}}{r'}$$

$$\dot{\rho} = -p'\frac{\dot{r}}{r'} - (\rho + p)\left(\frac{\dot{r}'}{r'} + 2\frac{\dot{r}}{r}\right)$$

$\longrightarrow$ ε depends on time differently from the dust case. This because pressure enters in the energy balance of the shells.

$$\frac{N'}{N} = -\frac{p'}{p+\rho}$$

$\longrightarrow$ The lapse function cannot be set to 1, unless making a radial coordinate transformation, going outside the LTB gauge.

Backup slides: plots for fluid with only tangential pressure

The first class of solutions we investigated is the non-perfect fluid with only tangential pressure:

$$T^{\mu}_{\nu} = diag(-\rho, 0, \Sigma, \Sigma), \text{ with } \Sigma = \omega\rho$$

Initial profile:

$$\rho(R, t_0) = C\left[\pi - \arctan\left(\frac{R - R_0}{\sigma}\right)\right]$$