Continuum limit and renormalisation in LQG

Laurent Freidel PI.

ILQGS 14

Review Talk

Review of the issue about the continuum limit in spin foam models

RG methods

summation methods

Clear some myth and confusions

Non technical check for •Bullet points

Spin foam models

•In this talk I will not talk about the continuum limit in general I will only present the issue of continuum limit from the point of view of Spin foam models

•What are spin foam models ?

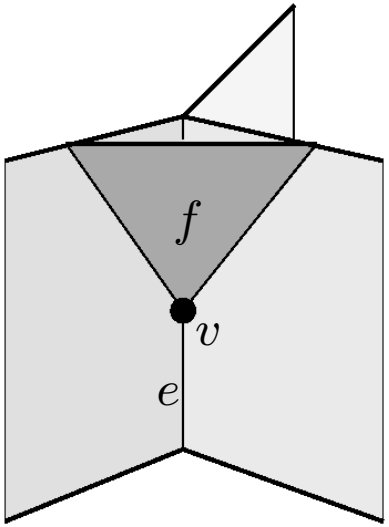
Boundary states: Spin networks Γ

Graph whose edges are labelled by representation and nodes are labelled by intertwiners

Bulk: Colored 2 dimensional complex

faces: J_f

edges: I_e



Local spin foam models

- When we talk about spin foam models we have in mind local spin foam models

In the spin representation they are characterized by a vertex amplitude an edge amplitude and a face weight.

vertex: $A_v(J_f, I_e)$ face: $\mu(J_f)$

spin foam amplitude

$$\boxed{Z(\Gamma, S) = \sum_{J_f, I_e} \prod_f \mu(J_f) \prod_v A_v(J_f, I_e)}$$

Bulk and Boundary

A spin foam model depends on two different type of discrete data: boundary Γ bulk S

- There are therefore two types of continuum limit to consider

A bulk continuum limit

refinement, RG
GFT,pert expansion

A boundary continuum limit

Different but not independent limits

Boundary states

A spin foam model fundamentally differ in principle from a usual lattice model or a dynamical triangulation model !!!

even if we can in practice borrow some techniques or ideas.

In a nutshell in lattice model we cannot take the bulk continuum limit without taking the boundary one

•In spin foam a priori we can

We can have a continuum theory label by a discrete basis of states eg rotation and spin.

Bulk continuum limit

- Lets assume that we finally have a bulk continuum limit that gives us a 2-complex independent model

$$Z(\Gamma, S) \to Z(\Gamma)$$

for any SU(2) boundary spin network states and such that

$$Z(\Gamma|_{j_e=0}) = Z(\Gamma/e)$$

This would be a continuum definition of the theory.

The reason lies the the Ashtekar-Lewandowski theorem:

The continuum Hilbert for gravity can be defined as the projective limit of the spin network Hilbert space

Embedded spin networks are a complete basis.

Bulk continuum limit

- Would that solve all the issue like the graviton computation ?

No

We would still have to understand what is the overlap between the graviton basis and the spin network basis

$$|n\rangle_{grav} = \sum_{\Gamma} A_n(\Gamma)|\Gamma\rangle$$

Since one state is local and the other is not
This overlap would involve an infinite superposition

Which can be sharply picked in the semi-classical limit.

Truncation versus discretisation

•QCD in a finite lattice is a discretisation

•Massive Quantum field theory truncated to a finite number of external particle is a truncation

The first one is an approximation, the second one is not, it is a presentation of the full amplitude in a convenient finite basis.

In practice a truncation is useful if it is related to an approximation usually a perturbative expansion.

eg massive vs massless

Γ is a truncation S is a discretization

The LQG hope is that the spin network truncation in gravity is akin to a part number truncation in a massive FT.

Bulk continuum limit

- The main issue is wether we can find a prescription to define the continuum limit of $Z(\Gamma, S)$

By "Continuum limit" we mean a procedure to get rid of the dependence of the amplitude on the 2-complex and a way to define a 2-complex independent sets of amplitudes

$$Z(\Gamma, S) \to Z(\Gamma)$$

And which also agree with the semi-classical limit

$$Z_\hbar(\Gamma) \to e^{\frac{i}{\hbar} S_{HJ}(\Gamma)}$$

A procedure which leads to the possibility to define in principle (if not in practice) all the spin network correlators

I will also consider as a continuum limit any perturbative expansion where the couplings are identified with physical correlators

Spin Foam Factory

Unlike 3D for 4d gravity there are a variety of model proposed

FK, EPRL, Dupuis-Livine, Oriti-Baratin, Alexandrov, Wieland, …

- It is possible to propose an infinite number of such models

All the models proposed implement in some way the simplicity constraints and can be constructed in the following way:

$$\mathrm{Tet} \to \mathrm{Inv}(SU(2)^4) \to \mathrm{Inv}(SO(4)^4)$$

$$T \to \quad |\phi(T)\rangle \quad \to |\hat{\phi}(T)\rangle$$

Amplitude $\quad A_v = \mathrm{Tr}\left(\otimes_n |\hat{\phi}(T_n)\rangle\right)$

measure $\quad \mu(T)$

Some are mathematically preferable but that doesn't mean that they are physically distinguished.

Non renormalisability

All these model differs by discretization ambiguities

Having an infinite number of models means that we have the same status as perturbative quantum gravity:
Non renormalisability

Need a continuum limit erasing these differences.

Bulk continuum limit

- The main issue is wether we can find a prescription to define the continuum limit of spin foam models

There are essentially two main philosophies

- RG or refinement approach

S is a spacetime lattice

- QFT or summation approach

S is a generalised Feynman diagram

These differ in how we interpret the 2-complex S

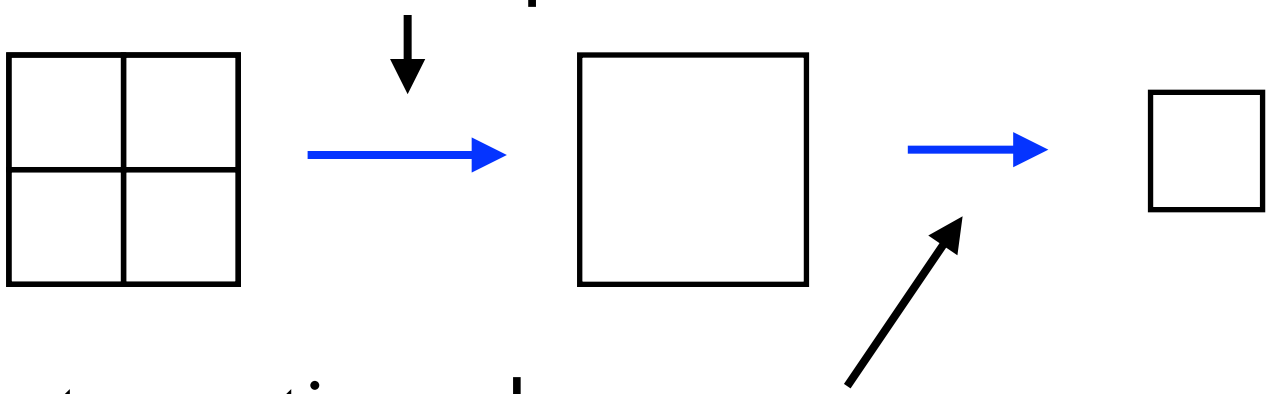
Each leads to different approximation scheme

RG continuum limit

What is the RG flow for spin or tensor models?

- Block spin transformation: one chose a family of lattices label by one parameter and such that in some sense Γ_n is coarser than Γ_{n+1}

Then we implement a block spin transformation

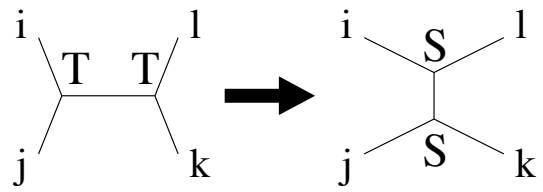


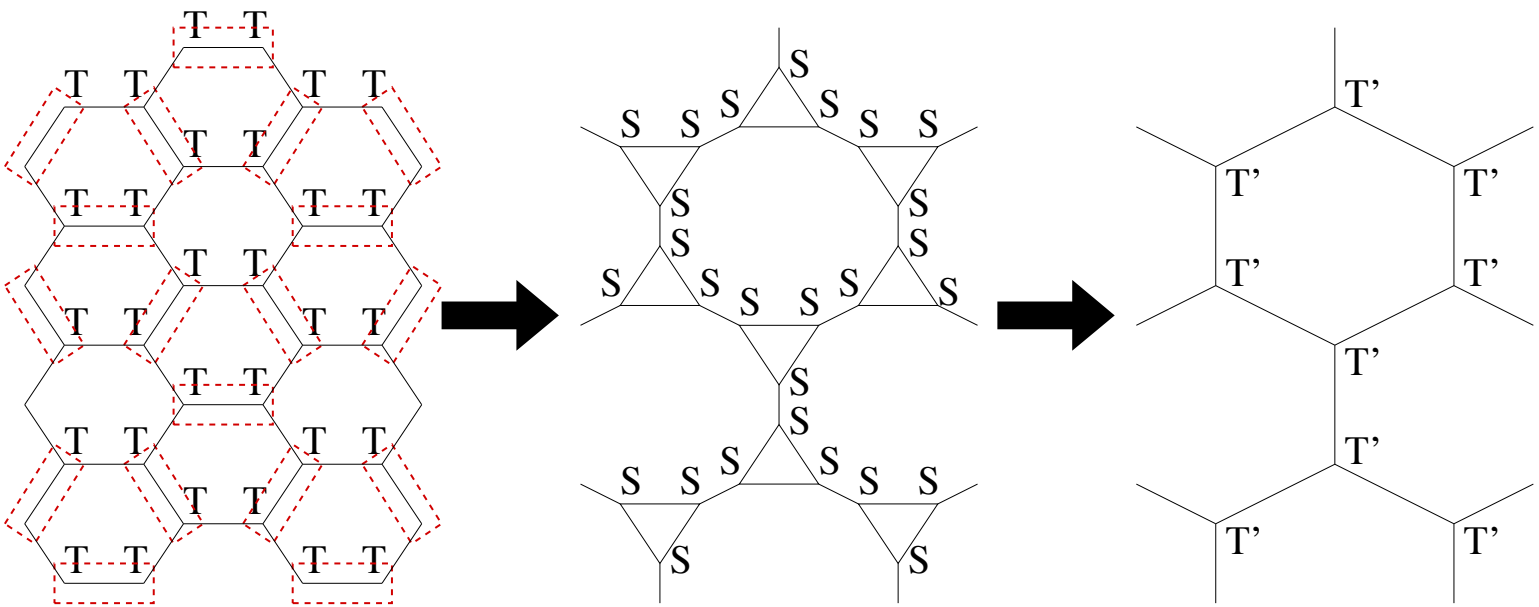
Then we take a truncation scheme

RG continuum limit

Examples

Block spin transformation:





(a) (b) (c)

RG continuum limit

Usually a truncation scheme exists in specific case

In 2d if the system is gapped then the procedure converges

•The success of the RG truncation procedure relies on locality: The ability to have a coarse gaining procedure that distill out the massive modes.
It really worked well for gapped systems

The spin foam RG flow is fundamentally different from the usual lattice theory RG flow

Spin nets RG continuum limit

- The spin foam RG flow is fundamentally different from the usual lattice theory RG flow

In usual spin foam models the number of allowed spin is infinite or very large, In tensor model we have to restrict the number of spins to have a finite small value (cut off).

In usual lattice theory we have a set of lattices equipped with a total order

This linear order can then be interpreted as a change of scale, we can think of individual building block as having a fixed scale so the RG is controlled by one scale parameter

In spin foam we have only a partial order which leads to an infinite number of possible coarse graining.

Spin nets RG continuum limit

In spin foam we have only a partial order which leads

The set of two complexes contains a priori all duals to triangulations or cellular decomposition of space-time

since there is no natural background structure.

Given two arbitrary graph we cannot say which one is the finer or the coarser!

- This means that we do not have an a priori sense of scale and an RG direction.

One naive attempt is to arbitrarily chose a subset of cellular decomposition which have a partial order and study the spin foam RG on this

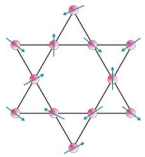
Spin nets RG continuum limit

Chose a subset of cellular decomposition which have a total order and study the spin foam RG on it

However there is an infinite set of possible such choices!

In this approach we have to hope or prove that there is some level of universality.

Is that plausible? : In usual system it is sometime true sometimes not.

For instance an antiferromagnet on a Kagomé lattice  has a macrocopical number of dof, it is a liquid: frustration. It is fundamentally different from the same magnet on a square lattice.

Universality?

•The ability to prove universality relies on the possibility to split the tensors excitations into massless and massive modes the massive modes are eliminated by the RG flow, the massless modes survives and their effective behaviour is controlled in the limit by a fundamental scale.

In frustrated system the splitting is not universal.

Challenges for spin foam and tensor models: Give some evidence for a RG truncation scheme: a splitting between massless and massive modes in a theory with a background H

Extra challenges for spin foam: The spin is a scale and a dynamical variable

Scale for spin Foam?

A fundamental difference between spin and tensor models versus spin foam models is that in the former the number of spin variables D is fixed. While in spin foam it varies, with the maximum value being the infrared cutoff of space (that is extremely large in planck unit).

In tensor models the number of building block can be identified with the scale of the system.
In spin foam it is impossible, one building block with large j can be of macroscopical size. So the number of block and the scale are different objects.

In other work in a spin foam S we have two parameters:
N # of building blocks J average spin per building blocks

A Proper RG for Spin foam

In a spin foam S we have two parameters:
N # of building blocks J average spin per building blocks

•The definition of RG for spin foam should be formulated in term of this two scales

$N \to \infty, \qquad J \to a^d$ Usual lattice continuum limit

$N = cst, \qquad J \to \infty$ The semi-classical limit

$N \to \infty, \qquad J \to \infty$

$J^{\frac{d}{2}}/N \to cste$

The spin foam continuum limit

Hydrodynamics limits

$$N \to \infty, \qquad J \to \infty$$
$$J^{\frac{d}{2}}/N \to cste$$

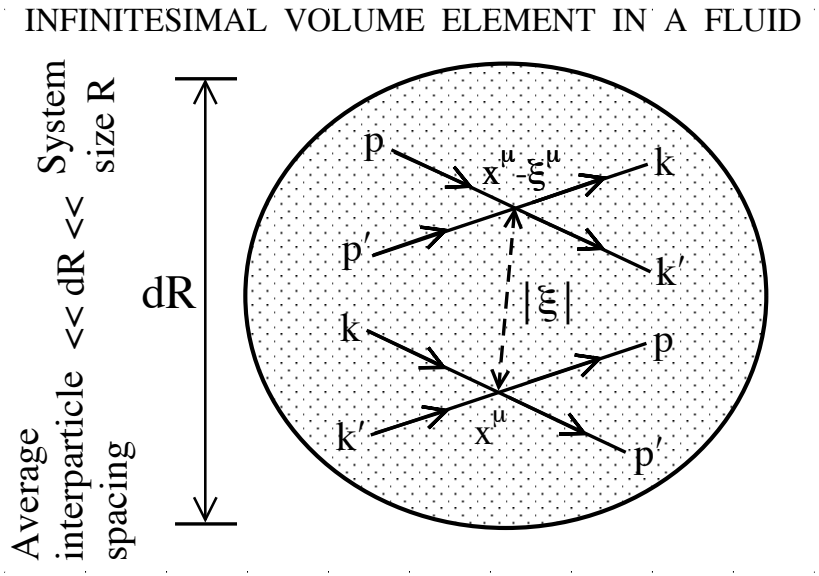
The spin foam continuum limit

is similar to an hydrodynamical limit

Landau-Lifshitz, Fluid Mechanics, page 1, §1, para 1:

"Any small volume element in the fluid is always supposed so large that it still contains a very great number of molecules. Accordingly, when we speak of infinitely small elements of volume, we shall always mean those which are 'physically' infinitely small, i.e. very small compared with the volume of the body under consideration, but large compared with the distances between the molecules"

An "infinitesimal" fluid element is a macroscopical object from the molecular point of view with size: the mean free path



Hydrodynamics limits

$$N \to \infty, \qquad J \to \infty$$
$$J^{\frac{d}{2}}/N \to cste$$

The spin foam double scaling limit

Interestingly there are evidences that this limit is also exactly the regime in which the flatness feature of spin foam models discovered by Kaminski and Hellman disappear.

M. Han has studied this hydrodynamics approximation on a fixed lattice.

In this limit we demand building block to have very small total curvature not small volume.

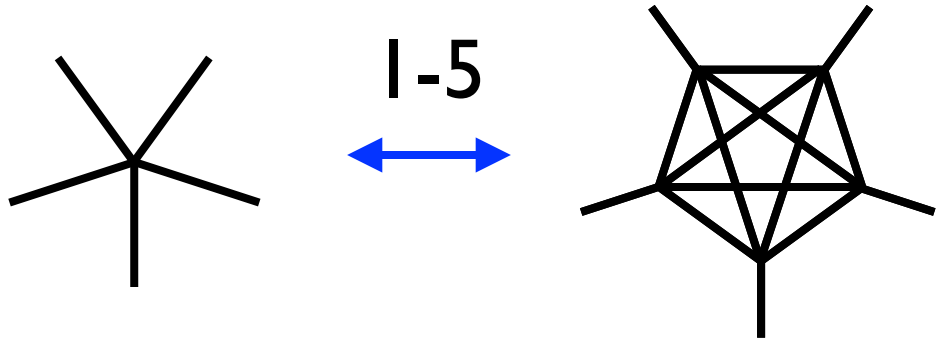
Partial RG

- Putting the two main feature of spin foam:
  the dynamical spin (scale)
  the fluctuating nature of the lattice
  suggests to study a modification of the RG flow

Coarse graining moves

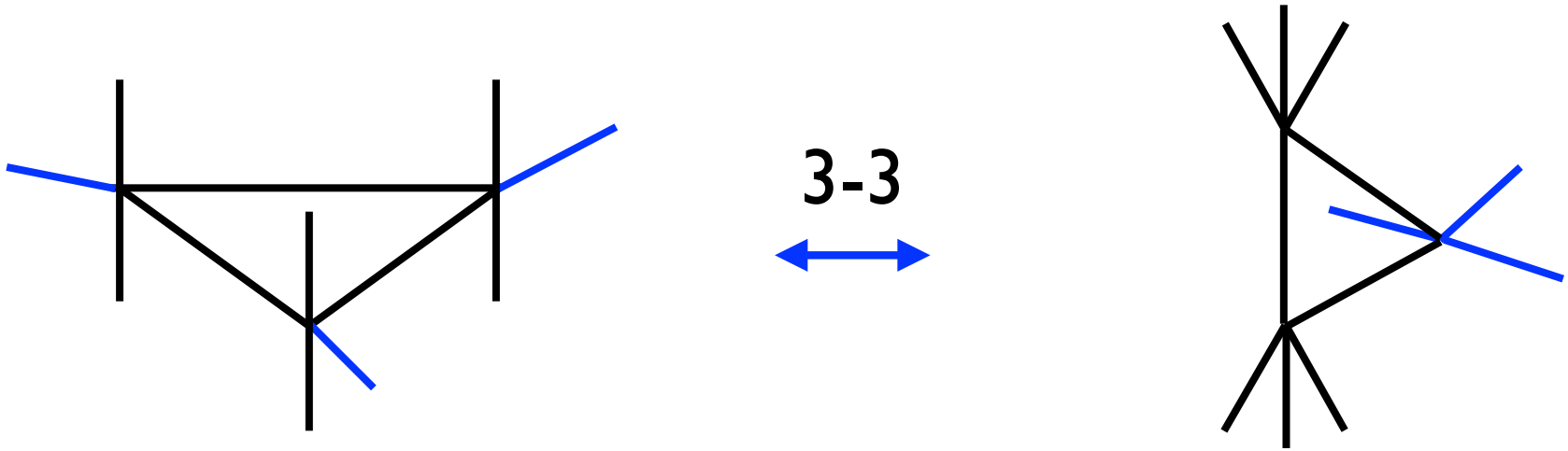
•In 4d triangulated spin foam we have two types of moves:

Coarse graining or refinement moves 1-5 or 2-4



changes the number of spin foam vertices

Mixing moves 3-3



Partial RG

The idea is that one should treat the refining moves in terms of an RG implementation.
While the coarse mixing moves in terms of a summation

If successful this will lead to a vertex weight invariant under 1-5 and 2-4 but not under 3-3

A very nice result of Dittrich and Steinhaus shows that this is true at one loop for the Regge action.
Is it true at all loop for spin foam vertex?

The 1-5 move is under study, the hope is to be able to get a 1-5 fixed point vertex amplitude.
The challenge is to devise a RG truncation which is efficient

Hnybida, Chen, Banburski

1-5 and 2-4 represents the flat space sector so we should be able to renormalize them

RG and Symmetry

1-5 and 2-4 represents the flat space sector so we should be able to renormalize them exactly

This is suggested by the idea that there should be a link between exact fixed point amplitude and amplitude respecting the bulk symmetries

This idea is natural in lattice QCD context where the concept of improvements and perfect action has been implemented successfully. It also was suggested in the context of context of Regge models and suggested by us in spin foam model. It is exemplified in 3d gravity

In 3d the perfect amplitude is also the fixed point amplitude.

L.F,-Louapre, Dittrich- Hoen,

Summation

Going back to the original problem of how to define a amplitude independent of the 2-complex

Another even more natural and simple proposal is to sum over 2-complexes with a specific measure.

$$Z(\Gamma) = \sum_{S \in C} m(S) Z(\Gamma, S)$$

Note that if C possess a total order, then taking the infinite sum limit is the same as taking the refinement limit.

If C does not these are widely different concepts.

What measure?

Summation and GFT

•A remarkable feature of spin foam models is that the summation over all spin foam amplitudes can be recasted in terms of a Group Field Theory (GFT)

Boulatov, De Pietri, Rovelli, L.F, Reisenberger

A similar property is true for tensor models

Ambjorn, Durhus

This is a remarkable feature of these models that suggests a new way to discuss the continuum limit.

The GFT fields $\phi(g_1, \cdots, g_d)$ $g_i \in G$

Tensor fields $\phi_{a_1 \cdots a_d}$ $a_i = 1, \cdots, N$

GFT

New ways to discuss the continuum limit

GFT depends on coupling constants g

on a loop counting parameter λ

$$S_g(\phi) = \phi K^{-1} \phi + V_g(\phi)$$

To each boundary state Γ we assign a functional $O_\Gamma(\phi)$

The spin foam amplitude is simply defined to be:

loop counting

$$Z(\Gamma) = \int D\phi e^{-\frac{S_g(\phi)}{\lambda}} O_\Gamma(\phi)$$

No more discretization dependence!

This is it seems, radically simpler and more compact than RG

Summation $\neq$ Refining

$$Z(\Gamma) = \int D\phi e^{-\frac{S_g(\phi)}{\lambda}} O_\Gamma(\phi)$$

What is the relation with usual approach?
In what sense is it a continuum limit?

The refinement continuum limit aims at computing the Physical scalar product $Z(\Gamma^*, H\Gamma) = 0$

It has a kernel generated by the Hamiltonian constraints

We can prove that $Z(\Gamma^*, \Gamma) > 0$ L.F 2005 Thiemann 2014

Similar to Hadamard versus Feynman propagator $\Box G_F = \hbar 1$

Summation $\neq$ Refining

$$Z(\Gamma) = \int D\phi e^{-\frac{S_g(\phi)}{\lambda}} O_\Gamma(\phi)$$

Possess no kernel $Z(\Gamma^*, \Gamma) > 0$

Schwinger-Dyson: $Z(\hat{H}\Gamma) = \lambda(T + D)$ ← disconnection changes

loop counting

L.F

Handle addition

from degenerate geometry

Bonzom

So Unless $\lambda \to 0$ sum $Z^{GFT}(\Gamma) \neq Z^{LQG}(\Gamma)$ refine

Tensor models

Lets start by discussing the tensor models analysis:
One key development that as led to numerous progress is to promote GFT or tensor fields to coloured tensor.

R. Gurau

Allows to keep track of the face, bubble structure of the 2-complex the GFT generates.
The other idea is to propose for tensor models like in matrix model a large N limit

R. Gurau, V. Rivasseau, V. Bonzom

N size of the tensors $\to \infty$

$g \sim N^{-d} \qquad d \geq 0$ degree of the coupling

$\lambda \sim N^{-D+1}$ D: dimension semiclassical

Large N Limit

$$N \to \infty \qquad g \sim N^{-d} \qquad \lambda \sim N^{-D+1}$$

A remarkable series of results: Dominance of spherical topology, criticality expressing that the limit is a infinite sum of discrete geometry (continuum limit), Universality, non perturbative completion (Borel summability).

Large N Limit

$$N \to \infty \qquad g \sim N^{-d} \qquad \lambda \sim N^{-D+1}$$

A remarkable series of results: Dominance of spherical topology, criticality expressing that the limit is a infinite sum of discrete geometry (continuum limit), Universality, non perturbative completion (Borel summability).

BUT

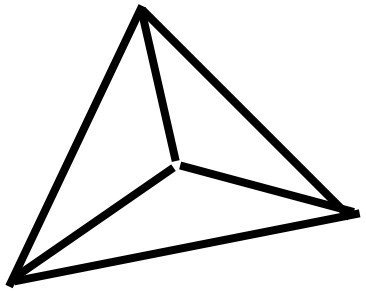
These describes totally degenerate or crumpled geometry. It is therefore not suited as a quantum gravity model :(

Large N Limit

In the large N the limit is dominated by melonic 2 complexes

In the triangulation language a melon is the most highly degenerate triangulation

Glue two tetrahedra along 3 faces



A pillow with 2 faces

Glue a large number of pillows on their faces: you get a melon

An infinitesimally small triangulation (as large as the largest triangle), with an infinite number of tetrahedra.

In Schwinger-Dyson only the degenerate contribution dominates

Not the continuum geometry we want in QG

Large N Limit

Large N limit of Tensor models leads to highly degenerate geometries

Geometries with an infinite number of cells but zero volume

Large spin limit: Geometries with a few cells but infinite volume

The idea is simple: inflate the tensor model with the spin and take a large N limit while also sending the spin per cell to infinity. similar to hydro limit.

We have to study GFT models and not tensor models

GFT

The GFT from LQG possess an ultra local term and do not allow for an RG understanding

One expect radiative correction to modify the propagator adding a kinetic term B. Geloum, Bonzom

dynamical GFT models Oriti

Remarkable series of results leading to B. Geloum, Rivasseau,Oriti,

Proof of renormalisability of abelian and non abelian 3D GFT models and asymptotic freedom.

Finiteness of the 3D dynamical Boulatov model B. Geloum

Closed equation for the two point function of 3D and 4D GFT models D. Ousmane Samary

GFT

The introduction of these dynamical GFT models introduces a scale and the possibility to organise an RG flow.

•It also introduces a notion of locality!

The GFT locality is however a locality in momentum space selecting the most local interaction terms on the group.

This GFT locality has however no a priori link to the usual notion of locality in a given space-time

In fact the GFT locality is related to the space-time largest scales

This duality between GFT locality and space-time locality is one of the most fascinating and puzzling point of the whole program

GFT perturbative expansion

Does the GFT defines a continuum limit related to the refinement limit?

Puzzle: Unlike usual field theory, GFT depends on coupling constants, that have no classical analogs !

- The spin foam vertex does not have the same status as the interaction vertex.

Therefore the interpretation of the GFT amplitudes and of its perturbative expansion is unclear.

Together with its continuum limit properties one needs to understand its classical properties

i-e sum of tree level diag = Hamilton-Jacobi functional

Oriti, L.F

Meaning of the loop counting parameter: 3rd quantization

Synthesis

- The problem of the continuum limit is the most central, urgent and important problem in loop quantum gravity

- It is not going to be solved by simply transposing old methods and RG ideas, new methods are needed. many new and exciting dev, but more are needed.

Spin foam are radically different due the presence of
- fluctuating lattices
- dynamical space-time scale

- I have argued that the solution should come from a combination of the techniques under developments: Refinement, summation and semi-classical + a numerical effort.

Synthesis

One need to devise a generalization of RG that incorporates both coarse graining moves and mixing moves to incorporate the random nature of spin foam lattices

This suggests to incorporate both RG techniques to deals with coarse graining moves together with summation techniques to deal with mixing moves.

- It is natural to consider a type of hydrodynamical limit that merges the naive continuum limit with the naive semi-classical limit in order to avoid the tensor crumpled phase and the flatness problem.

- It is crucial to understand the role of symmetries in RG and the role of coupling constants: These requires a classical analysis. diff symmetry in the spin foam language?