Quantum gravity at the corner

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ILQGS, October 27th 2020

Based on:
Freidel, MG, Pranzetti, 2006.12527, JHEP
Freidel, MG, Pranzetti, 2007.03563, JHEP
Freidel, MG, Pranzetti, 2007.12635

Motivations

Quantum gravity

- Central role played by symmetries, but which ones?
- What are the fundamental degrees of freedom?
- Where do they live?
- What does quantum gravity assign to lower-dimensional objects (corners, points)?
- What is the role of matter?
- Many pieces of answers from different approaches

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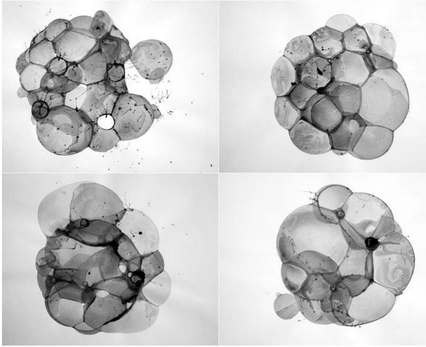
A new proposal

- Quantum gravity based on **local holography**
- Focus on local symmetry content for arbitrary subregions and their corners
- Associate Hilbert space, states, quantum numbers to these corners
- Degrees of freedom organized by representation of these **corner symmetry algebras**
- Space as entangling and fusion of these corner degrees of freedom
- Dynamics and constraints as charge conservation

Motivations



Push the logic of LQG further

- Focus on symmetries → diffeos and SU(2) so far, but why not more?
  [Ashtekar, Isham, Lewandowski, Rovelli, Smolin, Thiemann, Varadarajan, . . .]
- Space as a network of quantum geometry building blocks → generalized twisted geometries
  [Bianchi, Freidel, Haggard, Livine, Speziale, Tambornino, Weiland, . . .]
- Think in terms of coarse-graining, truncation, defects → enlarge theory space
  [Bahr, Delcamp, Dittrich, Goeller, Livine, MG, Steinhaus, Riello, . . .]

Motivations

Reconcile different approaches

- AdS/CFT, holography: focus on the boundary
- LQG-type approaches: focus on the (discrete) bulk

Resolve persistent tensions in LQG

- Interplay between discretization and quantization
- Role of the Barbero–Immirzi parameter
- Discreteness of area vs internal Lorentz invariance
- Imposition of the simplicity constraints
- Non-commutativity of the fluxes
- Access to the frame
- Construction of the dynamics, inclusion of matter, . . .

Roadmap

Setup and tools

- Boundaries turn gauge into physical symmetries: non-trivial charges and algebra at co-dimension 2 corners S
- Best studied in covariant phase space formalism: $\delta L = \text{EOMs} + d\theta$
  [Anderson, Ashtekar, Barnich, Brandt, Crnkovic, Kijowski, Lee, Wald, Witten, . . .]
- Find classifying criterion for different formulations of gravity: corner symmetry algebra $\mathfrak{g}^S$
- Focus on: entangling spheres S, no boundary conditions, no time evolution, tangent diffeos

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Simple, yet deep result

- For any formulation F of gravity, the symplectic potential is the sum of
  - a universal bulk piece, canonical GR → gives diff$(S) \subset \mathfrak{g}^S$
  - a corner piece → adds extra charges and components to $\mathfrak{g}^S$

$$\boxed{\theta_{\text{F}} = \theta_{\text{GR}} + d\theta_{\text{F/GR}} + \delta L_{\text{F/GR}}}$$

- Different formulations have different corner algebras → potentially inequivalent quantizations

Metric gravity

Potentials

- Einstein–Hilbert Lagrangian $L_{\text{EH}} = \epsilon R$
- Potential

$$\begin{aligned}
\theta_{\text{EH}} &= \tilde{\epsilon} n^{\mu} \nabla^{\nu} (\delta g_{\mu\nu} - g_{\mu\nu} g^{\alpha\beta} \delta g_{\alpha\beta}) \\
&= \tilde{\epsilon}(K g^{\mu\nu} - K^{\mu\nu}) \delta g_{\mu\nu} + \mathrm{d}(\sqrt{q}\, s_{\mu} \delta n^{\mu}) - 2\delta(\tilde{\epsilon} K) \\
&= \theta_{\text{GR}} + \mathrm{d}\theta_{\text{EH/GR}} - 2\delta(\tilde{\epsilon} K)
\end{aligned}$$

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Relative Lagrangians

- This is due to the **well-known** fact that

$$L_{\mathsf{EH}} = L_{\mathsf{GR}} + dL_{\mathsf{EH/GR}} = \epsilon \big(R^{(3)} - K^2 + K^{\mu\nu} K_{\mu\nu} \big) + 2d\big(\tilde{\epsilon} n_{\mu} (n^{\mu} K - a^{\mu}) \big)$$

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- But really follows from the **less-known** fact that

$$\delta L_{EH/GR} + d\theta_{EH/GR} = \theta_{EH} - \theta_{GR}$$

- Boundary Lagrangians (may) have symplectic potentials, which contribute as corner terms [Freidel, Perez, Pranzetti, 2016] [MG, Jai-akson, 2019] [Harlow, Wu, 2019] [Wieland, 2017]
- Corner terms are not ambiguities, but features → formulation-dependent charges and algebra

Metric gravity

Bulk generators

- Consider a diffeomorphism δ_ξ
- Bulk piece: spatial diffeo constraint $\leftrightarrow$ momentum conservation

$$\mathcal{H}^{\Sigma}_{\mathsf{GR}} = \mathcal{H}^{\Sigma}_{\mathsf{EH}} = -\int_{\Sigma} \xi_\mu \nabla_\nu P^{\mu\nu} \approx 0$$

- Algebra $\{\mathcal{H}[\xi], \mathcal{H}[\zeta]\} = \mathcal{H}[\xi, \zeta]$
- How exactly is it represented?

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Corner charges and algebra

- GR: Brown–York charge $\mathcal{H}^S_{\mathsf{GR}} = \int_S s_\mu \xi_\nu P^{\mu\nu}$
  - vanishing if $\xi|_S = 0$
  - leads to universal component $\mathfrak{g}^S_{\mathsf{GR}} = \mathrm{diff}(S)$
- EH: Komar charge $\mathcal{H}^S_{\mathsf{EH}} = \int_S \epsilon_{\mu\nu} \nabla^\mu \xi^\nu$
  - non-vanishing if $\xi|_S = 0 \rightarrow$ leads to an extra $\mathfrak{sl}(2, \mathbb{R})_\perp$
  - $2 + 2$ decomposition reveals $\mathfrak{g}^S_{\mathsf{EH}} = \mathrm{diff}(S) \ltimes \mathfrak{sl}(2, \mathbb{R})_\perp$ [Donnelly, Freidel, 2016]

Metric gravity

	Corner symmetries $\mathfrak{g}^S$				
Formulation of gravity	$\mathrm{diff}(S)$	$\mathfrak{sl}(2,\mathbb{R})_\perp$	$\mathfrak{sl}(2,\mathbb{R})_\parallel$	$\mathfrak{su}(2)$	boosts
Canonical general relativity (GR)	✓				
Einstein–Hilbert (EH)	✓	✓			
Einstein–Cartan (EC)	✓				✓
Einstein–Cartan–Holst time gauge	✓		✓	✓	
Einstein–Cartan–Holst (ECH)	✓		✓	✓	✓

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Back to our motivations

- Different formulations of gravity reveal different components of $\mathfrak{g}^S$
- If $\mathfrak{g}^S$ plays a role in quantizing gravity, what is the **full symmetry algebra**?
- Bigger algebra: more quantum numbers (handles) to reconstruct bulk dof and dynamics
- Let us continue justifying why
  - this **has** to do with quantum gravity
  - looking at extra components of $\mathfrak{g}^S$ has physical implications

Tetrad gravity

Strategy

- Start from BF theory $L_{BF} = B_{IJ} \wedge F^{IJ}[\omega] \quad \rightarrow \quad \theta_{BF} = B_{IJ} \wedge \delta\omega^{IJ}$
- Impose simplicity $B_{IJ} = E_{IJ}[e] = (* + \beta)(e \wedge e)_{IJ}$
- Work without time gauge to access boosts [Alexandrov, MG, Livine, Noui, . . .]
- Use internal normal n^I [Peldan, Alexandrov, Bodendorfer, Thurn, Thiemann, Wieland]

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Decomposition

- Introduce boost and spin 2-forms and 1-forms (frames)

$$B_{IJ} \stackrel{\Sigma}{=} B_{[I} n_{J]} + \varepsilon_{IJ}{}^{K} S_K \qquad B_I = \frac{1}{2}(b \times b)_I \qquad S_I = \frac{1}{2}(s \times s)_I$$

- Decompose connection as

$$\omega^{IJ} = K^{[I} n^{J]} + \Gamma^{IJ} \qquad K^I = d_\omega n^I$$

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- Bulk + corner decomposition (up to total δ)

$$\theta_{BF} = B_I \wedge \delta K^I - d_\Gamma s_I \wedge s^I + d\left(B_I \delta n^I - \frac{1}{2} s_I \wedge \delta s^I\right)$$

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- Simplicity is now $B_I = E_I$ and $s_I = \sqrt{\beta}\, e_I$

$$\theta_{\mathsf{ECH}} \approx E_I \wedge \delta K^I + d\left(E_I \delta n^I - \frac{\beta}{2} e_I \wedge \delta e^I\right)$$

Tetrad gravity

From bulk to corner

- Introducing 4-momentum aspect $P^I \equiv (K \times e)^I$, we have $\theta_{\mathsf{ECH}} \approx \theta_{\mathsf{GR}} + \theta_{\mathsf{ECH/GR}}$ since

$$P^{\mu\nu}\delta g_{\mu\nu} = P_I \wedge \delta e^I = E_I \wedge \delta K^I + \delta(\dots)$$

- Shift emphasis from bulk to corner using $\beta(e \wedge e)_{IJ} \wedge \delta\omega^{IJ}[e] \approx -\beta \mathrm{d}(e_I \wedge \delta e^I)$

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Generators and charges

- Diffeomorphisms
  - constraint $\nabla_\nu P^{\mu\nu} \approx 0$ as a conservation $d_\Gamma P^I \approx 0$
  - charge gives usual $\mathcal{H}^S_{\text{ECH}}[\xi] = \int_S \xi \lrcorner E_{IJ}\omega^{IJ}$ (contains topological Komar charge) [De Paoli, Speziale, Oliveri, 2018, 2020] [Perry, Godazgar, Godazgar, 2020]
- Lorentz transformations
  - Gauss constraint as charge conservation $d_\omega E_{IJ} \approx 0$
  - Lorentz charges $\mathcal{H}^S_{\text{ECH}}[\alpha] = \int_S \alpha^{IJ}E_{IJ}$ with boosts and rotations since

$$E_{IJ} = E_{[I}n_{J]} + \beta\,\varepsilon_{IJ}{}^K (e \times e)_K$$

- Algebra $\mathfrak{g}^S_{\text{ECH}}$ contains diff(S) and an **ultra-local** $\mathfrak{sl}(2,\mathbb{C})$

Zooming on the corner

Hierarchy of phase spaces, geometrical, and algebraic structures

- Go back and focus on BF corner phase space $\theta_{\mathsf{BF}} = B_I \delta n^I - \frac{\beta}{2} e_I \wedge \delta e^I$

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- Comes from precursor Poincaré–Heisenberg phase space
- Massive particle analogy
  - momentum $p^I = \gamma n^I = \beta^{-1} n^I$
  - Pauli–Lubanski vector $W^I = \gamma S^I$
  - Casimirs $p^2 = -m^2 = -\gamma^2$ and $W^2 = m^2 s(s+1)$, and $\gamma = 0$ gravity as massless limit
  - Total angular momentum $J^{IJ} = B^{[I} n^{J]} + \varepsilon^{IJ}{}_K S^K$
- Simplicity breaks Poincaré to Lorentz: area element $\propto$ Lorentz-invariant Poincaré spin Casimir
- Space-like nature of surface S selects the discrete $\mathfrak{sl}(2,\mathbb{R})$ representations

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Explicit description of symmetry breaking towards gravity

16 dof	(p^I, X^I, z^I)	
		⤵ kinematical constraints ($p \perp$ corner)
12 dof	(n^I, B^I, e^I)	
Poincaré	$(n^I, B^I, \overbrace{S^I, q_{ab}, \vartheta})$	$S^2 = \beta^2 q$
		⤵ simplicity constraints $S^I = \beta B^I$
8 Dirac observables	$(J^{IJ}, q_{ab}, \vartheta)$	$\rightarrow \mathfrak{sl}(2,\mathbb{C}) \oplus \mathfrak{sl}(2,\mathbb{R})_{\parallel} \oplus \mathfrak{u}(1)$

Discreteness at the corner

Look at tangential metric

- Decompose tangential frame e^I_a at the corner in terms of
  - spin operator $S^I = \frac{\beta}{2}(e \times e)^I$
  - tangential metric $q_{ab} = e^I_a e^J_b \eta_{IJ}$
  - twist angle ϑ
- $\mathfrak{sl}(2,\mathbb{R})_\parallel$ algebra [Freidel, Perez, 2015]

$$\{q_{ab}(x), q_{cd}(y)\} = -\frac{1}{\beta}(q_{ac}\varepsilon_{bd} + q_{bc}\varepsilon_{ad} + q_{ad}\varepsilon_{bc} + q_{bd}\varepsilon_{ac})(x)\delta^2(x-y)$$

- Geometrical balance relation $S^2 = \beta^2 q$ relating $\mathfrak{su}(2)$ and $\mathfrak{sl}(2,\mathbb{R})$ Casimirs
- Space-like surface: **internal Lorentz-invariant quantization of area in the continuum** (see also [Wieland, 2017, 2018] with spinors and null surfaces)

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Corner frame provides a complex structure

- Ambiguity $e^I_a \mapsto e^I_a(\vartheta) = e^I_a \cos\vartheta + \star e^I_a \sin\vartheta$ with $\star e^I_a = \sqrt{q}^{\,-1} q_{ab}\varepsilon^{bc} e^I_c$
- Complex structure as $\star^2 = -1$
- Area conjugated to angle $\beta\{\sqrt{q}, \vartheta\} = 1$ and $\beta\{\sqrt{q}, \cdot\} = \star$
- Jacobi implies that $\star$ structure is Poisson-compatible

Simplicity at the corner

Shifting emphasis from bulk to corner lifts ambiguities

- Start from BF potential

$$\theta_{\mathsf{BF}} = B_I \delta n^I - \frac{\beta}{2} e_I \wedge \delta e^I$$

- Simplicity is relation $C^I \equiv B^I - \beta^{-1} S^I = 0$ between boost B^I and spin $S^I[e]$

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- 2nd class with themselves (no spin foam quantization map, no secondary constraints)

$$\{C^I, C^J\} = C^{[I} n^{J]} - (1+\beta^{-2}) \varepsilon^{IJ}{}_K S^K$$

- 3 constraints split into
  - 1 1st class $\mathcal{C} \equiv C^2 + (\beta + \beta^{-1}) C^I S_I = 0$

    continuum version of diagonal simplicity [Livine, Oriti, 2002] [Rovelli, Speziale, 2011]
  - 2 2nd class $C_a \equiv C_I e^I_a = 0$

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Gupta–Bleuler

- Use compatible complex structure to build $C^\pm_a = (1 \pm i \star) C_a$ with $\{C^\pm_a, C^\pm_b\} = 0$
- Replace 2nd class C_a by quantum holomorphic 1st class $C^-_a |\Psi\rangle = 0$
- Alternatively, use master constraint $\mathcal{M} = C_a q^{ab} C_b$
  - Classically ok since $\mathcal{M} = 2 C^+_a q^{ab} C^-_b$
  - Quantum level not immediate since (commutator) anomaly $\mathcal{M} = 2 C^+_a q^{ab} C^-_b + \mathcal{A}$

Simplicity at the corner

Classical solutions

- Rewrite $\mathcal{C} = 0$ and $\mathcal{M} = 0$ in terms of Lorentz and Poincaré spin Casimirs ($\mathbf{Q}, \widetilde{\mathbf{Q}}, S^2$)
- Selects Lorentz weights determined by Poincaré spin as ($\mathsf{k} = \mathsf{s}$, $\rho = \beta^{-1}\mathsf{s}$)
- $|S| \propto$ corner area, and LQG kinematical spins $L_I = (*J)_{IJ} t^J$ are boosted areas $|L| = j \geqslant s$

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8 Dirac observables

- Total angular momentum J^{IJ} (Lorentz charges)
- Tangential metric q_{ab}
- Angle ϑ
- Algebra $\mathfrak{g}^S_{\text{ECH}} = \text{diff}(S) \ltimes \big(\mathfrak{sl}(2,\mathbb{C}) \oplus \mathfrak{sl}(2,\mathbb{R})_{\parallel} \oplus \mathfrak{u}(1)\big)$
- Note that q_{ab} and ϑ are not charges of gauge transformations

Symmetries at the corner

	Corner symmetries $\mathfrak{g}^S$				
Formulation of gravity	$\text{diff}(S)$	$\mathfrak{sl}(2,\mathbb{R})_\perp$	$\mathfrak{sl}(2,\mathbb{R})_\parallel$	$\mathfrak{su}(2)$	boosts
Canonical general relativity (GR)	✓				
Einstein–Hilbert (EH)	✓	✓			
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Einstein–Cartan–Holst time gauge	✓		✓	✓	
Einstein–Cartan–Holst (ECH)	✓		✓	✓	✓

??? $\longrightarrow$

$\big\downarrow$

Corner quantization

Continuous representations

- Study quantum anomaly $\mathcal{A}$ and states $C^-_a|\Psi\rangle = 0 = \mathcal{C}|\Psi\rangle$
- Quantization of the frame e^I_a (needs tensor operators → intertwiners → bulk information?)
- Represent $\mathfrak{g}^S_{\text{ECH}} = \text{diff}(S) \ltimes \mathfrak{h}^S$ to get QG building blocks, knowing that

$$\mathbf{C}_{\text{SL}(2,\mathbb{R})} = -\beta^2 \mathbf{q} \qquad \mathbf{C}_{\text{SU}(2)} = \beta^2 \mathbf{q} \qquad \mathbf{C}^{(1)}_{\text{SL}(2,\mathbb{C})} = (\beta^2 - 1)\mathbf{q} \qquad \mathbf{C}^{(2)}_{\text{SL}(2,\mathbb{C})} = -2\beta\,\mathbf{q}$$

- Discreteness of area element $\sqrt{\mathbf{q}}$ gives discrete measure on $\mathfrak{h}^S$
- $\text{diff}(S^2)$ not unreasonable [Penna, 2018] [Donnelly, Freidel, Moosavian, Speranza]

Discrete subalgebras

- LQG-type truncations from piecewise-constant smearings on partition of S
- Defect-like picture [Dittrich, MG] from smearing on circles around punctures [Freidel, Perez, Pranzetti, 2016] [Freidel, Livine, Pranzetti, 2019]

Edge modes

Why

- What brought us here in the first place: boundaries break gauge-invariance
- Possibility to restore gauge-invariance by suppressing the charges [De Paoli, Speziale, 2018]
  - goes against the systematic treatment of relative potentials $\theta_{F/GR}$
  - loose observables and quantum numbers
- **Edge modes** allow to have **gauge-invariance as well as non-trivial symmetry charges**
- Contains information for gluing and coarse-graining

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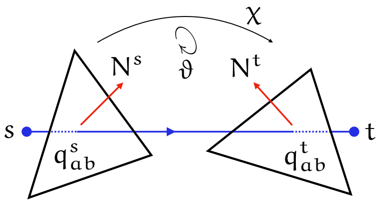
- What brought us here in the first place: boundaries break gauge-invariance
- Possibility to restore gauge-invariance by suppressing the charges [De Paoli, Speziale, 2018]
  - goes against the systematic treatment of relative potentials $\theta_{F/GR}$
  - loose observables and quantum numbers
- **Edge modes** allow to have **gauge-invariance as well as non-trivial symmetry charges**
- Contains information for gluing and coarse-graining

How

- [Donnelly, Freidel, MG, Balasubramanian, Parrikar, Speranza, Takayanagi, Tamaoka, . . .]
- Extended potential $\theta_{\text{ext}} = \theta^{\Sigma} + \boldsymbol{\theta}^{S}[\mathbf{B}, \mathbf{n}, \mathbf{e}]$ with **independent** fields at the corner
- Replace $\delta|_S$ by $\delta - \chi^{-1}\delta\chi$ with $\chi = (\varphi, \rho) \in SL(2, \mathbb{C})^S \times SL(2, \mathbb{R})^S$ a choice of gauge frame
- Usual gauge transformations δ_α lead to generator $\mathcal{H}[\alpha] = \mathcal{H}^{\Sigma}[\alpha] + \mathcal{H}^{S}[\alpha]$ with
  - bulk components (e.g. Gauss law) ≈ 0
  - corner equation of motion imposing (1^{st} class) continuity conditions

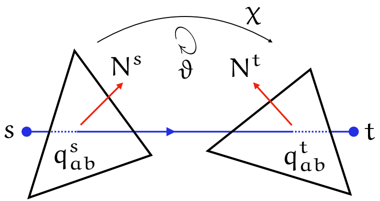
$$B^I \overset{S}{=} \varphi^I{}_J \mathbf{B}^J \qquad n^I \overset{S}{=} \varphi^I{}_J \mathbf{n}^J \qquad e^I_a \overset{S}{=} \varphi^I{}_J \rho_a{}^b \mathbf{e}^J_b$$

- Symmetries Δ_α acting only on edge modes, leading to gauge-invariant corner charges $\mathcal{Q}[\alpha]$



Initial incarnation

- Map between $T^*SU(2)$ and geometrical data (N^s, N^t, j, ϑ) [Freidel, Speziale, 2010]
- SU(2) holonomy g rotating $N^s = g \triangleright N^t$
- APD $SL(2,\mathbb{R})$ transformation needed to map metrics (i.e. frames instead of fluxes) [Haggard, Rovelli, Wieland, Vidotto, 2013] [Freidel, Livine, 2018]
- Attempts to generalize [Dupuis, Freidel, MG, Livine, Ziprick, Speziale, Tambornino, Wieland]



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Corner reconstruction

- Data from Poincaré spin operator, metric, and twiste angle $(\mathcal{S}^s, \mathcal{S}^t, s, \vartheta)$
- $SL(2,\mathbb{C})$ holonomy φ such that $\mathcal{S}^s = \varphi \triangleright \mathcal{S}^t$
- Comes from edge mode gauge frames (half-holonomies) and bulk-corner continuity $e = \chi \triangleright \mathbf{e}$

$$e^s = e^t \qquad \Leftrightarrow \qquad \mathbf{e}^s = \chi_{st} \triangleright \mathbf{e}^t \quad \text{with} \quad \chi_{st} = \chi_s^{-1}\chi_t$$

Perspectives

We have shown that

- Using canonical bulk + corner split and **focusing on the corner contains physics**
- When applied to tetrad gravity, naturally **leads to LQG features**
  - internal normal n^I in the phase space
  - corner metric is non-commutative when $\gamma \neq 0$
  - simplicity constraints satisfy a self-2$^{\text{nd}}$ class algebra in the continuum
  - internal Lorentz-invariance and discrete area spectrum
  - new quantum numbers, generalized twisted geometries
- Focusing on the corner paves the road for where to go next

Next

- Search for biggest symmetry algebra (include topological terms)
- Continuum quantization of $\mathfrak{g}^S$
- Space as fusion of corner states
- Dynamics and conservation of charges
- Matter as defects of geometry

Announcement

LOOPS'21

- July 19 – 23 in Lyon (France)
- Summer school in Marseille, July 12 – 16
- Hopefully in person