A gravitationally induced decoherence model using Ashtekar variables

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joint work with Max Fahn and Michael Kobler
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work in progress with: Max Fahn, Roman Kemper, Michael Kobler

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Open Quantum Systems

We consider gravitationally induced decoherence in the context of open quantum systems

Isolated quantum system: $\mathcal{H}_S$ system Hamiltonian H_S

Dynamics: density matrix $\partial_t \rho = \frac{1}{\mathrm{i}\hbar}[\mathrm{H}_S, \rho]$

System S + environment ε : $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_\varepsilon$

Dynamics: $\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_S + \mathrm{H}_\varepsilon + \mathrm{H}_{\mathrm{int}}$ $\mathrm{H}_{\mathrm{int}}(\mathrm{t}) = \sum_\alpha \mathrm{S}_\alpha(\mathrm{t}) \otimes \mathrm{E}_\alpha(\mathrm{t})$

Aim: Effective dynamics for S: master equation $\partial_t \rho = \frac{1}{\mathrm{i}\hbar}[\mathrm{H}_{\mathrm{tot}}, \rho]$

$$\partial_t \rho_S = \frac{\mathrm{d}}{\mathrm{d}t} \mathrm{tr}_\varepsilon \left(\mathrm{U}_{\mathrm{tot}}(\mathrm{t}) \rho(0) \mathrm{U}^\dagger_{\mathrm{tot}}(\mathrm{t}) \right) = \frac{1}{\mathrm{i}\hbar} \mathrm{tr}_\varepsilon \left([\mathrm{H}_{\mathrm{tot}}, \rho] \right)$$

Lindblad equation

Lindblad equation (completely positive master equation)

$$\frac{\partial}{\partial t}\rho_S(t) = \frac{1}{i\hbar}\left[H_S + H_{\rm LS}, \rho_S(t)\right] + \sum_k \gamma_k \left(L_k \rho_S(t) L_k^\dagger - \frac{1}{2}\left\{ L_k^\dagger L_k, \rho_S(t) \right\} \right)$$

Lindblad operators L_k LS correction $H_{\rm LS}$ γ_k time-independ.

For given $H_{\rm S}$ model characterised by choice of L_k, γ_k ← QG

Simple example: dephasing

Choices: $k = 1, L = \sqrt{\gamma} H, \quad H = H_S + H_{\rm LS}$

$$\frac{\partial}{\partial t}\rho_S(t) = \frac{1}{\mathrm{i}\hbar}\left[\mathrm{H}, \rho_S(t)\right] + \gamma \left(\mathrm{H}\rho_t \mathrm{H} - \frac{1}{2}\mathrm{H}^2 \rho_S(t) - \frac{1}{2}\rho_S(t)\mathrm{H}^2 \right)$$

decay ↓

energy eigenbasis $\rho_{mn}(t) \equiv \langle m | \rho_t | n \rangle = \rho_{mn}(0) \exp\left(-\frac{\mathrm{i}}{\hbar}(E_m - E_n)\, t - \frac{\gamma}{2}(E_m - E_n)^2\, t \right)$

Gravitationally induced decoherence I

We choose gravity as the environment

Phenomenologial models often take Linblad eq. as starting point

[Ellis, Lopez, Mavromatos, Nanopoulos 1996]; [Benatti, Floreanini 1999]; [Lisi, Marrone, Montanino 2000]; [Guzzo, de Holanda, Oliveira 2016]; [Gomes, Forero, Guzzo, de Holanda, Oliveira 2019]]

However: microscopic derivation shows Lindblad needs several assumption

-Born approximation: initial separation $\rho(0) = \rho_S(0) \otimes \rho_\varepsilon(0)$

for t > 0 $\rho(t) \simeq \rho_S(t) \otimes \rho_\varepsilon(0)$ stationary state $[H_\varepsilon, \rho_\varepsilon] = 0$

-(i). Markov approx: $\int_0^t \mathrm{d}s \mathcal{K}(t,s)\tilde{\rho}(s) \cong \int_0^t \mathrm{d}s \mathcal{K}(t,s)\tilde{\rho}(t)$ tilde refers to interaction picture

-(ii). Markov approx: $t \to \infty$ $C_{\alpha\beta}(\xi)$ peaked

Born-Redfield equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_S(t) = -i\left[H, \rho_S(t)\right] - \sum_\alpha \left(\left[S_\alpha, B_\alpha\left(t, t_0\right)\rho_S(t)\right] + \left[\rho_S(t) C_\alpha\left(t, t_0\right), S_\alpha\right]\right)$$

$$B_\alpha\left(t, t_0\right) := \alpha^2 \int_0^{t-t_0} d\xi \sum_\beta C_{\alpha\beta}(\xi) S_\beta(-\xi) \quad C_\alpha\left(t, t_0\right) := \alpha^2 \int_0^{t-t_0} d\xi \sum_\beta C_{\alpha\beta}(-\xi) S_\beta(-\xi)$$

- Often also rotating wave approx: $\sum_{\omega\omega'} \mathrm{e}^{\mathrm{i}(\omega-\omega')t} f\left(\omega, \omega'\right) \simeq \sum_\omega f(\omega, \omega)$

Gravitationally induced decoherence II

Microscopic derivation: First steps in a given model

Existing work in ADM variables: [[Anastopoulos, Hu 2013]; [Blencowe 2013]; [Oniga, Wang 2016]; [Lagouvardos, Anastopoulos 2021]; [Asprea 2021]]

Scalar field coupled linearised gravity in Ashtekar variables

Need true Hamiltonian system: reduced quantisation, geometrical clocks, classical dynamical reference frame

$$\text{system} : \phi^{GI}, \pi^{GI} \qquad \text{environment} \; : \delta\mathcal{A}_a^i, \delta\mathcal{E}_i^a$$

In field theory interaction given $\frac{\kappa}{2}\int_M d^4x \delta h^{\mu\nu} T_{\mu\nu}[\Phi, \eta]$

Physical Hamiltonian: Fock quantisation

$$H = \int_{\mathbb{R}^3} d^3k \left\{ \omega_k a_k^\dagger a_k + \Omega_k \left[\left(b_k^+\right)^\dagger b_k^+ + \left(b_k^-\right)^\dagger b_k^- \right] \right\} + \sqrt{\frac{\kappa}{2}} \int d^3k \frac{1}{\sqrt{\Omega_k}} \sum_{r \in \{\pm\}} \left[b_k^r J_r^\dagger(\vec{k}) + \left(b_k^r\right)^\dagger J_r(\vec{k}) \right]$$

$$+ \kappa U \otimes 1_\varepsilon$$

U self-interaction term scalar field

Gravitationally induced decoherence III

Microscopic derivation: First steps in a given model

Assumptions of the model [Max Fahn, K.G., Michael Kobler '22]

[[Nakajima 1958]; [Zwanzig 1960]; [Shibata, Takahashi, Hashitsume 1977]; [Chaturvedi, Shibata 1979]]

Starting point: Time-ConvolutionLess (TCL) equation truncated at second oder

Can be derived from Nakajima-Zwanzig equation, time-local, can involve non-Markovian processes

Assume thermal state for the gravitational environment

Resulting master equation

$$\frac{\partial}{\partial t}\rho_S(t) = -i\left[H_S + \kappa U + \kappa H_{LS}, \rho_S(t)\right] + \mathcal{D}_{\text{first}}\ [\rho_S]$$

$$\mathcal{D}_{\text{first}}\ [\rho_S] := \frac{\kappa}{2}\int \frac{d^3k d^3p d^3l}{(2\pi)^{\frac{6}{2}}} \sum_{r;ab} R_{ab}(\vec{p},\vec{l};\vec{k},t)\left(j_r^b(\vec{k},\vec{l})\rho_S(t) j_r^a(\vec{k},\vec{p})^\dagger - \frac{1}{2}\left\{j_r^a(\vec{k},\vec{p})^\dagger j_r^b(\vec{k},\vec{l}), \rho_S(t)\right\}\right)^{\Box}$$

not of Lindblad type, [Partiallly confirm results of [Oniga, Wang 2016], [Anastopoulos, Hu 2013] and [Lagouvardos, Anastopoulos 2021]]

Gravitationally induced decoherence IV

Next steps: work in progress

Final master equation still very complicated

Strategy: projection on the 1-particle case

Already here renormalisation necessary, before or after rotating wave approx?

Modell of AHB: 1 particle case non-relativistic limit, 1 dim [Anastopoulos, Hu 2013]; [Blencowe 2013]

$$\frac{\partial}{\partial t}\rho = \frac{1}{i\hbar}[H,\rho] + \tau\left(H\rho H - \frac{1}{2}\left\{H^2,\rho\right\}\right) \quad H = \frac{p^2}{2m} \qquad \tau = \frac{32\pi G k_B T_{grav}}{9}$$

[first steps in confirming same result in NR limit]

Why interesting in QG in a broader context:

Allows different perspective to QG effects in open quantum systems

Interesting in the context of matter interactions where gravity is weak

Consider loop quantisation of decoherence models [Feller, Livine '16] [K.G., Michael Kobler '22]

If we construct LQG inspired models do they have characteristic properties?

Thank You!