Emerging Smooth Gravitational Wave from Spin Foam Model

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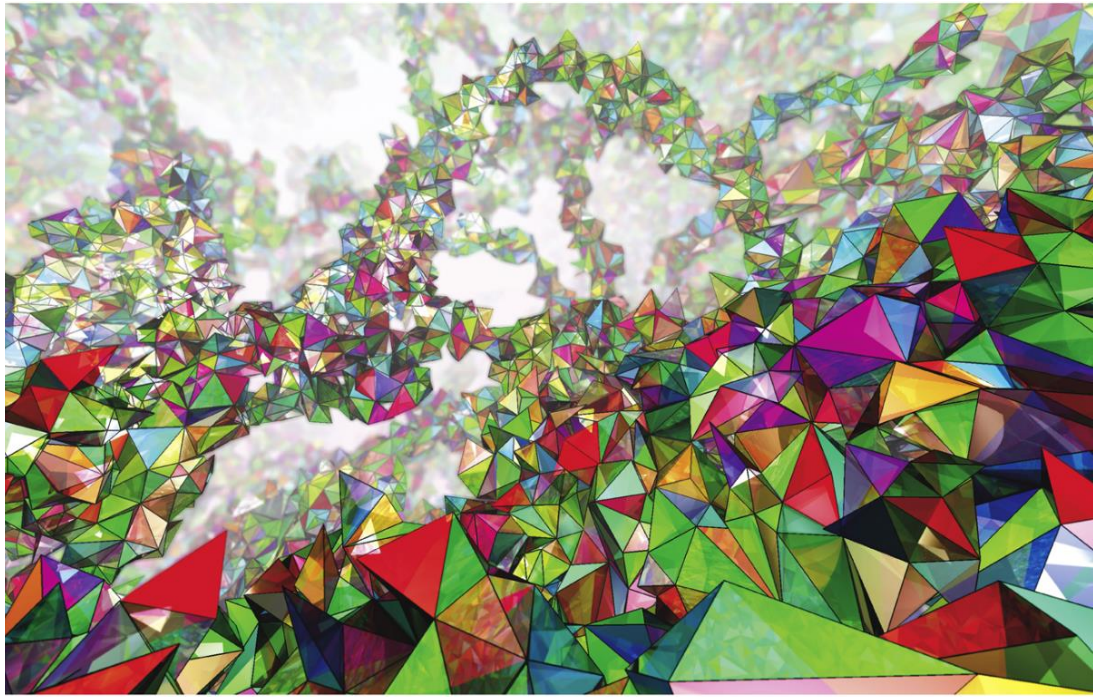
Outline

- Motivation
- Flatness and Resolution
- Non-flat Semiclassical Limit
- Semi-classical Continuum Limit of Spin-foam Model
- Emerging Gravitational Wave
- Revisiting the “Flatness”
- Conclusion

Motivation



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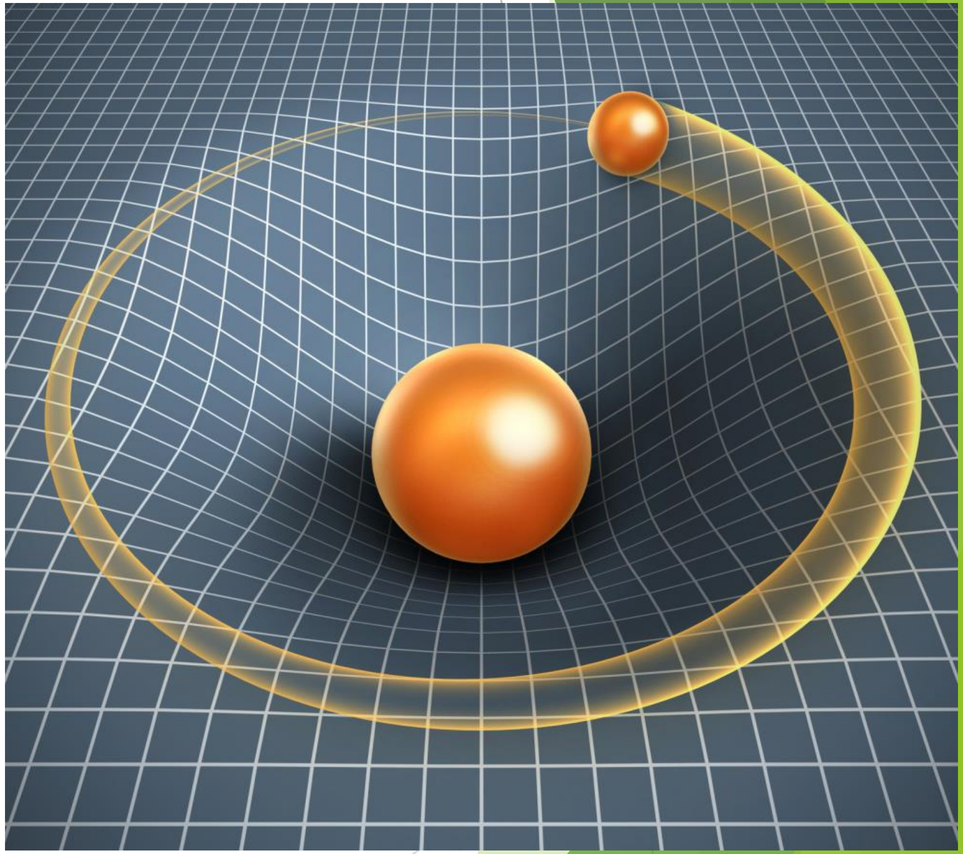


Discrete Spin-foam model

Continues Gravity theory

Motivation

- Relate discrete Spin-foam model to continuum General relativity

  i.e. reproduce smooth curved spacetime satisfying Einstein equation etc.

- Deeper discussion about semi-classical limit of covariant loop quantum gravity

- Emergent gravity program (gravity is emergent from fundamental entanglement) spin foam as a working example

Summary of results

- Propose Semi-classical continuum limit(SCL) as an IR limit

  We defined the RG-like flow of a few spin foam parameters $(J(\mu), \delta(\mu), a(\mu))$, $\mu \to 0$

  such that we can take the large limit $J \to \infty$ and continuum limit simultaneously.

- Under SCL of the Euclidean EPRL model, we recover a linearized Einstein theory of gravity over a flat space time

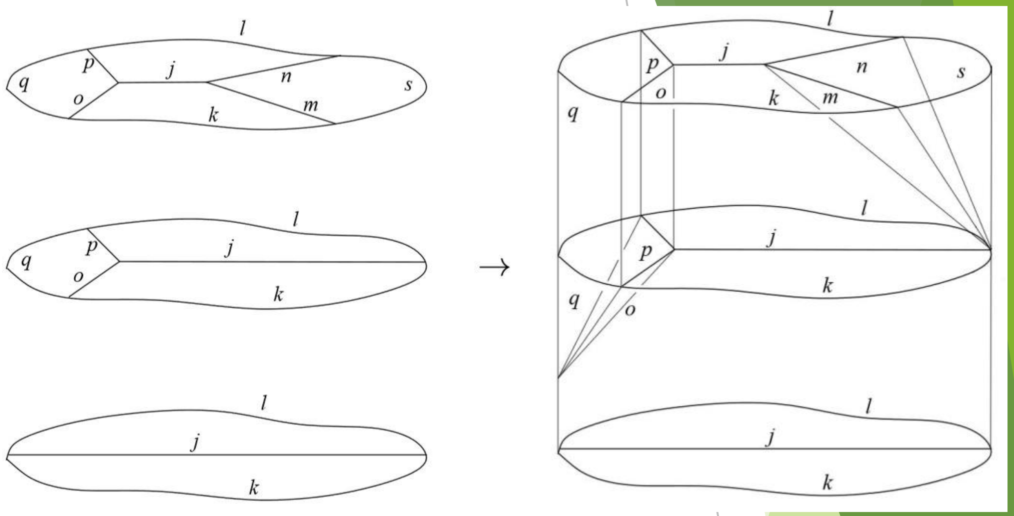
- Under SCL, we found the dominant contribution to the amplitude(all critical points) converges to smooth gravitational waves

- All low energy excitations are smooth gravitational waves (spin-2 graviton)

Spin-foam model

- Spin-foam is a state sum lattice model inspired by BF-theory
- The amplitude:

$$Z = \sum_{\vec{j},\vec{i}} \prod_{f} A_f(J_f) \prod_{\sigma} A_\sigma(J_f, i_\tau)$$



Face amplitude: $A_f(J_f)$,

4-simplex amplitude: $A_\sigma(J_f, i_\tau)$

Spin-variable(assigned on each triangle): $J_f \in \mathbb{Z}_+/2$

Intertwiner(assign to each tetrahedron): $i_\tau \in INV_{\mathrm{SU(2)}}(V_{J_1} \otimes V_{J_2} \otimes V_{J_3} \otimes V_{J_4})$

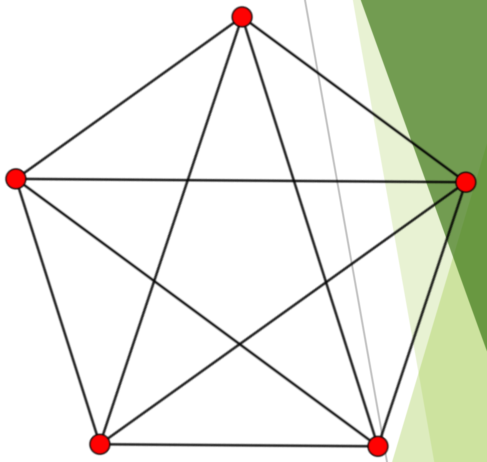
Spin-foam is a LQG model in analogy with the Faynman's path-integral formulation. It describes the histories of a evolving quantum 3D geometries (spin-networks).
(Picture from A.Perez's paper Arxiv:1205:2019)

Spin-foam model

- Semi-classically, (J_f, i_τ) relates to area of the triangle and the shape of the tetrahedron

  1) area spectrum: $A_f = \gamma l_p^2 \sqrt{J_f(J_f+1)}$

  2) intertwiner i_τ relates to the closure condition: $\sum_{f\in\tau} \vec{A}_f = 0$



Each 4-simplex is assigned with 10 spin-variables and 5 intertwiners

- Euclidean EPRL-FK amplitude:

  The 4-simplex amplitude takes the form as contracting five $spin(4)$ intertwiners together:

$$A_\sigma = trace(I_1 \otimes I_2 \otimes I_3 \otimes I_4 \otimes I_5)$$

$spin(4)$ Intertwiner

$$I_\tau^{m_1^\pm \cdots m_4^\pm} = \int \mathrm{d}h^+ \mathrm{d}h^- \prod_{f=1}^{4} \left[\underbrace{D^{(J_f^+)}_{m_f^+ n_f^+}(h^+) D^{(J_f^-)}_{m_f^- n_f^-}(h^-)}_{\text{SU(2) wigner matrix}} \underbrace{C^{n_f^+ n_f^-}_{n_f}}_{\text{clebsch gordon coefficients}} \right] \underbrace{i_\tau^{n_1 \cdots n_4}}_{\text{SU(2) intertwiner}}$$

Simplicity constraint: $j_f^\pm = \frac{1}{2}|1 \pm \gamma| j_f$

Spin-foam model

- Amplitude in exponential form:

By introducing the coherent state $|j,\vec{n}\rangle = g(\vec{n}) \triangleright |j,j\rangle$, one can have:

$$Z = \sum_J \prod_{f\in\Delta} A_f(J_f) \int \mathrm{d}g^{\pm}_{\sigma\tau}\mathrm{d}\xi_{\tau f}\; e^{\sum_f J_f F_f[g^{\pm}_{\sigma\tau},\xi_{\tau f}]}$$

and when $(\gamma < 1)$

$$F_f\left[g^{\pm}_{\sigma\tau},\, \xi_{\tau f}\right] = \sum_{\sigma, f\subset\sigma} \left[(1-\gamma)\ln < \xi_{\tau f}\middle|(g^{-}_{\sigma\tau})^{-1} g^{-}_{\sigma\tau'}\middle|\xi_{\tau' f} > +(1+\gamma)\ln < \xi_{\tau f}\middle|(g^{+}_{\sigma\tau})^{-1} g^{+}_{\sigma\tau'}\middle|\xi_{\tau' f} > \right]$$

[J.Barrett, M. Han and M. Zhang, L. Freidel, F. Conrady]

- Rescaling J_f:

$$J_f \to \lambda j_f \implies S \propto \lambda \sum_f j_f F_f$$

- We consider perturbations on a background configuration with large spins ($\lambda \gg 1$ is the typical spin)

$$\sum \text{Perturbations} \iff \text{A sum in the large spin regime}$$

Lattice with simplicial triangulation

- Hypercube lattice triangulation $\mathcal{K}$:

  1.The triangulation adapts to hypercube lattice in $\mathbb{R}^4$

  2.The vertices of each hypercube are labeled by (0-15)

  3.The triangulation is made by parallel duplicate the face-diagonal, body-diagonal and hyper-diagonal from 0 vertex to other vertices.

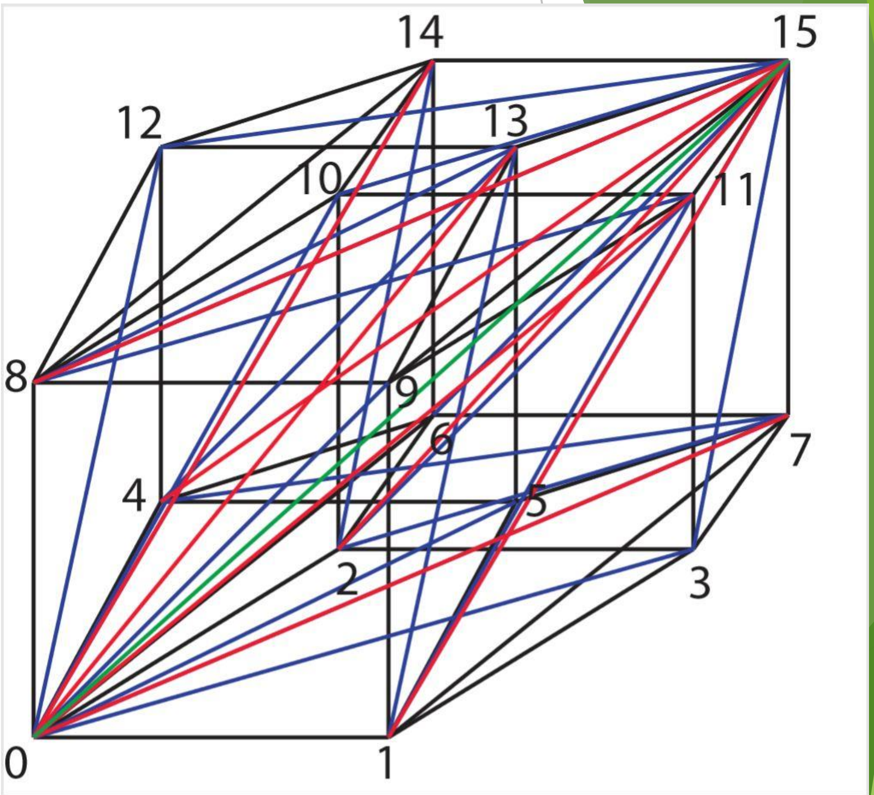
- Useful conclusions:

  1. In bulk, $N_f > N_l$

  2. In bulk, each triangle is shared by even number of 4-simplices

- Lattice refinement:

  subdividing each hypercube into 16 identical hypercubes, then triangulate by simplices in the same manner as above (Not Pachner moves for 4-simpices)



Triangulated typical Hypercube cell

[J.Barrett and R. Williams]

Asymptotic analysis of the spin foam model

- Large spin limit:

  1.From the area spectrum $A_f = \gamma l_p^2 \sqrt{J_f(J_f+1)}$, $l_p^2 \equiv 8\pi G\hbar$, one finds:

  Semi-classical limit of SFM($\hbar \to 0$) $\leftrightarrow$ Large spin limit ($\lambda \gg 1$)

  2.Large spin limit $\leftrightarrow$ Asymptotic expansion of Z $\leftrightarrow$ sum over critical points

$$\int \mathrm{d}^n x\, e^{\lambda S(x)} \approx \sum_{x_c} \left(\frac{2\pi}{\lambda}\right)^{\frac{n}{2}} \frac{e^{\lambda S(x_c)}}{\sqrt{\det H(x_c)}}$$

- A classic of SFM critical points $\leftrightarrow$ 4d simplicial geometry on $\mathcal{K}$ (without sum over spins)

  1.Asymptotically, F becomes $F = i\gamma\epsilon$ and the action reduces to $\lambda S_f \sim i\gamma J_f \epsilon_f$

  2.At one critical point large spin asymptotic of the integral:

$$\int [dX] e^{\sum_f J_f F_f[X]} \sim e^{\frac{i}{l_p^2}\sum_f a_f \epsilon_f}$$ where X stands for the variables $(g_{\sigma\tau}^{\pm}, \xi_{\tau f})$

[L. Freidel, F. Conrady, J.W.Barrett et al, M. Han, M. Zhang, S. Speziale, etc]

Perturbation and parity

- Generally, for a single 4-simplex, there are 4 types of results of the asymptotic expansion based one the boundary states:

  (Euclidean 4-simplex, BF point, vector geometry and degenerate point)

- We choose the background of the perturbation as the critical point corresponds to the Euclidean 4-simplx geometry.

- Different 4-simplices may have different orientations, however we only consider a perturbation over a background such that every single 4-simplex has a boundary state that have unified parity.

- As a discrete degrees of freedom, orientations will not changed under small perturbation.

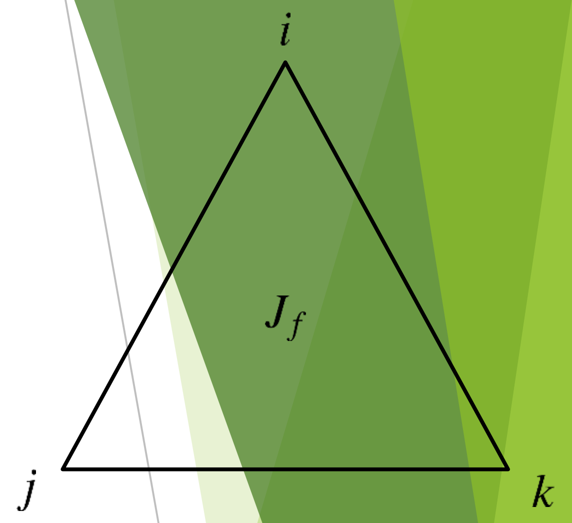
Regge-like spins

- Asymptotic results we mentioned previously works only when spins are Euclidean Regge-like.

(In the following of the presentation we use Regge like stands for Euclidean Regge-like)

- Regge-like spins $\vec{J} \in \mathbb{R}^{N_f}$ must satisfy area-length relation

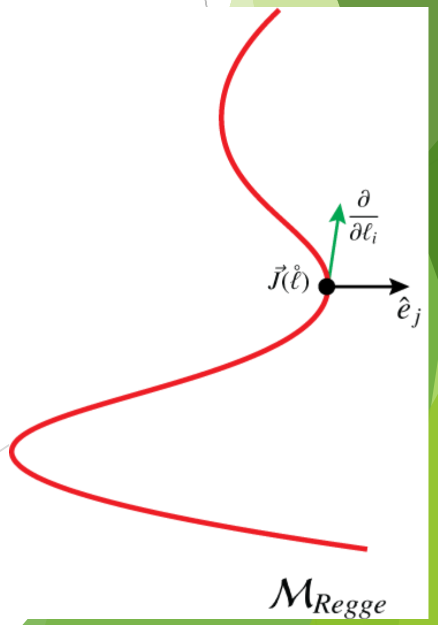
$$\gamma J_f(\ell) = \frac{1}{4}\sqrt{2(\ell_{ij}^2\ell_{jk}^2 + \ell_{ik}^2\ell_{jk}^2 + \ell_{ij}^2\ell_{ik}^2) - \ell_{ij}^4 - \ell_{ik}^4 - \ell_{jk}^4}$$



- $N_f > N_l$ indicates that all the Regge-like spins consist a subspace $M_{Regge} \subset \mathbb{R}^{N_f}$
- We consider perturbations on background geometry $\mathring{\ell} \to \mathring{\ell} + \delta\ell$
- In the base point $\vec{J}(\mathring{\ell})$, we can define vector basis: $\left(\frac{\partial}{\partial \ell_i}, \hat{e}_j\right)$

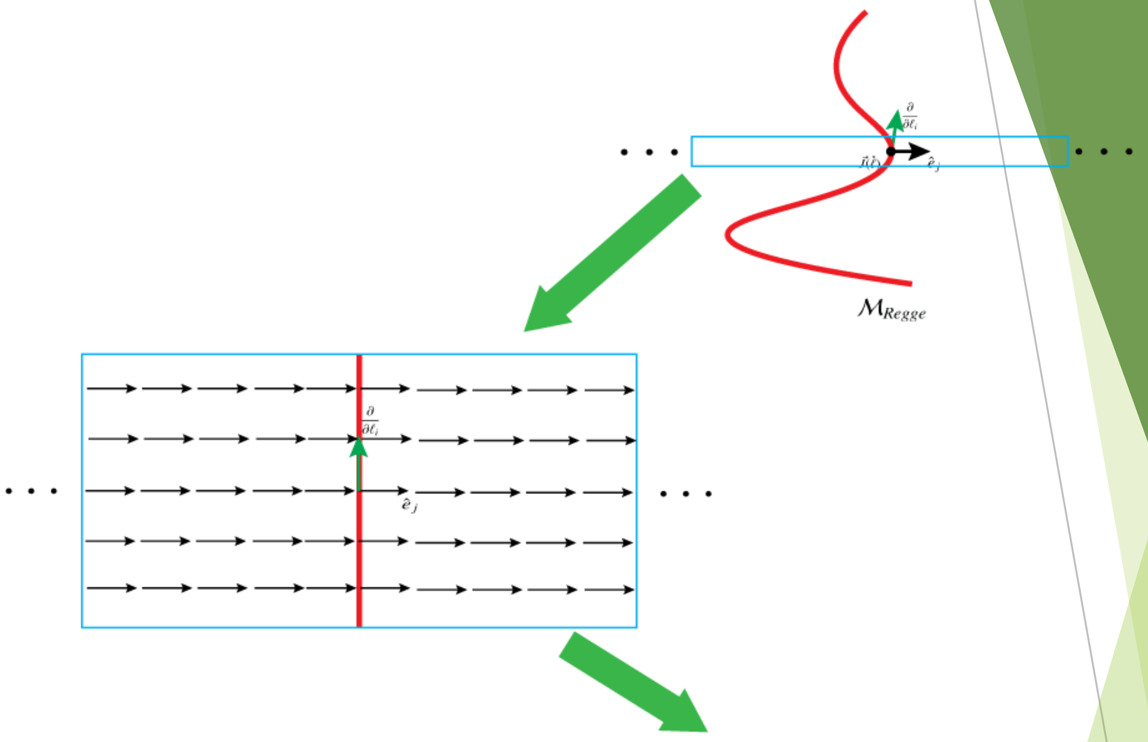
$\frac{\partial}{\partial \ell_i}$ are the vectors locally tangent to M_{Regge}

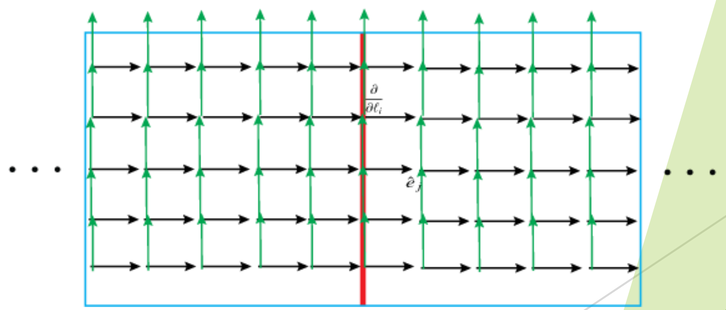
$\hat{e}_j$ are the constant vectors locally transverse to M_{Regge}



Infinite band

- $\vec{\ell}$ can parametrize the Regge-like subspace
- The neighborhood of $\vec{J}(\mathring{\ell})$ on M_{Regge} can be transversely extent to a infinite band on $\mathbb{R}^{N_f}$
- $\hat{e}_j(\mathring{\ell})$ can be extent to the infinite band as a constant vector, since the space $\mathbb{R}^{N_f}$ is flat.
- The extension of $\frac{\partial}{\partial \ell_i}(\vec{\ell})$ can be considered as its parallel transport along $\hat{e}_j$
- We can have a coordinate $(\vec{\ell}, \vec{t})$ based on the frame field $\left(\frac{\partial}{\partial \ell}, \hat{e}\right)$
- One can proof that $\partial_t \left(\frac{\partial}{\partial \ell}\right) = 0$ for each constant $\vec{t}$ line.





All the spin variables can be expressed as:

$$\vec{J} = \underbrace{[\vec{J}(\mathring{\ell}) + \frac{\partial \vec{J}}{\partial l}(\ell)\delta\ell]}_{\vec{J}(\ell)} + \sum_{i=1}^{M} t_i \hat{e}^i(\mathring{\ell}), \quad (M = N_f - N_l)$$

Spin sum and Flatness

- Ideas for Flatness

  1. Semi-classical limit of the action is proportional to the spin variable:

$$S \propto \sum_f j_f F_f$$

  2. Spin-sum in the amplitude calculation:

$$Z = \sum_j e^{\sum_f j_f F_f}$$

EOM with respect to spin gives $F = 0 \mod 4\pi i k$. (Can be seen after Poisson Resummation)

EOM indicates flatness at discrete level $\gamma\epsilon = 0 \mod 4\pi k$ (Reminds that $F = i\gamma\epsilon$)

In semi-classical limit, spin-foam model always gives flat geometry!

[V. Bonzom F. Hellmann W. Kaminski C. Perini M. Han]

An idea

1. Continuous gravity theory $\Longleftrightarrow$ Large-j limit $+$ Continuum limit(Lattice refinement)

   (Similar to the hydrodynamical limit in Freidel ILQGS(2014))

2. Since the continuum limit also confines the deficit angles to be 0, the flatness indicates that large-j limit and the refinement limit cannot be treated separately.

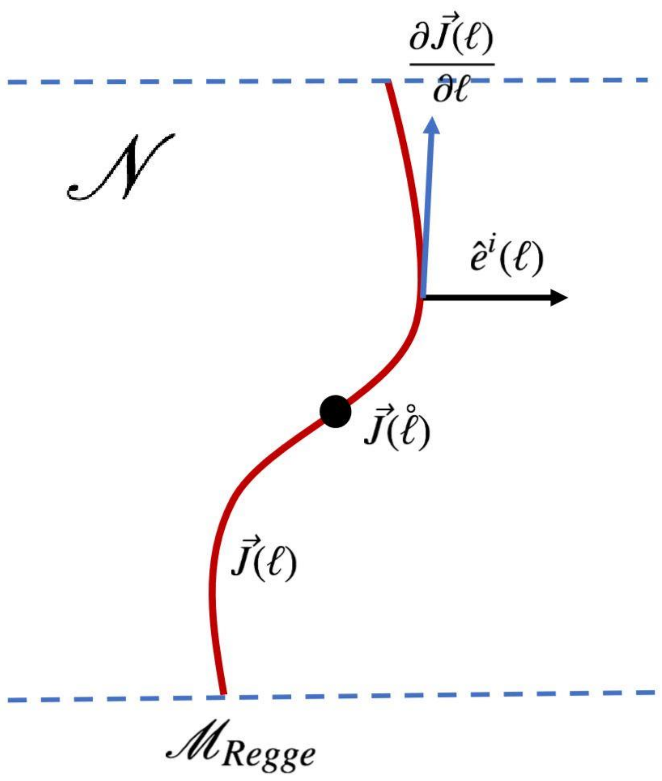
3. Our proposal is to tweak the large-j limit and leave a window for creating the small deficit angle and close that window at the same time we take the continuum limit.

An origin of flatness: non-Regge-like spins

- If the sum was only along Regge-like spins, we would have quantum Regge calculus without flatness

$$Z = \sum_l \int [dX] e^{\sum_f j_f(l) F_f[X]} \sim \sum_l e^{i \sum_f j_f(l) \epsilon_f(l)}$$

- Flatness comes from summing the non-Regge-like spins



Resolution: Deforming non-Regge-like spin sum

- Poisson Resummation: $\sum_{n\in\mathbb{Z}} f(n) = \sum_{k\in\mathbb{Z}} \int \mathrm{d}n\, e^{2\pi i k n} f(n)$

The amplitude becomes:

$$Z = \sum_{\vec{k}\in\mathbb{Z}^{N_f}} (2\lambda)^{N_f} \int \left[\mathrm{d}\vec{j}\mathrm{d}X\right] e^{\lambda \sum j(F[X]+4\pi i k)} = \sum_{\vec{k}\in\mathbb{Z}^{N_f}} (2\lambda)^{N_f} \int \left[\mathrm{d}\vec{j}\mathrm{d}X\right] e^{\lambda\left\langle \vec{j}(l),\, \vec{F}[X]+4\pi i\vec{k}\right\rangle + t_i \overbrace{\left\langle \hat{e}^i(l),\, \vec{F}[X]+4\pi i\vec{k}\right\rangle}^{\Phi^i_{(k)}}}$$

where X stands for the variables $(g^{\pm}_{\sigma\tau}, \xi_{\tau f})$.

($<,>$ is the standard Euclidean inner product in $\mathbb{R}^{N_f}$)

In order to simplify the discussion, we keep $k = 0$. For the case $k \neq 0$, the result can be considered as F shifts by $4\pi k$

- t-deformation integral:

Along the non-Regge-like direction, one can introduce a Gaussian regulator (We will push $\delta \to 0$ at the very end)

$$\int \mathrm{d}\vec{J} = \int [\mathrm{d}\ell \mathrm{d}t]\, \mathcal{J}(\mathring{\ell}) \to \int [\mathrm{d}\ell]\, \mathcal{J}(\mathring{\ell}) \int [\mathrm{d}t]\, e^{-\frac{\delta}{4}\sum_i t_i^2} \qquad \mathcal{J} = \det\left(\frac{\partial J}{\partial l}(l),\ \hat{e}\right)$$

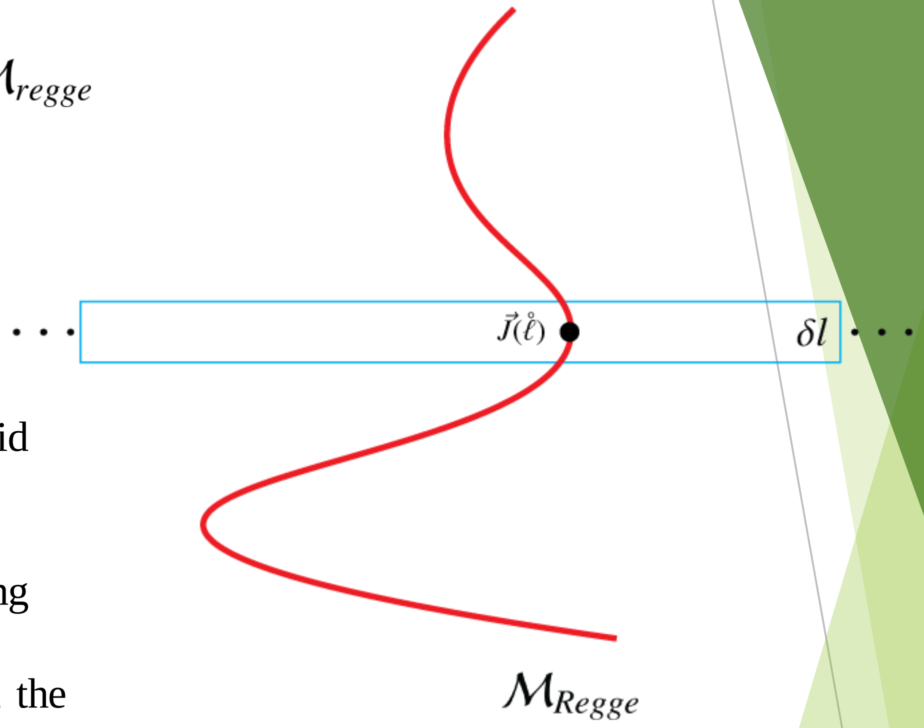
The integral over t integrates out the non-Regge-like degree of freedom

$$Z = \int [\mathrm{d}\ell \mathrm{d}X]\, e^{\lambda\left\langle \vec{j}(\ell), \vec{F}[X]\right\rangle} D^{(k)}_{\delta}(\ell, X), \quad D^{(k)}_{\delta}(\ell, X) \propto e^{\sum_{i=1}^{M} \frac{1}{\delta} \Phi^i_{(k)} \Phi^i_{(k)}}$$

The spin-foam amplitude reduced to the integral along Regge-like spins up to a measure factor $D^{(k)}_{\delta}(\ell, X)$

T-deformation

1. The perturbation δl is only done on Regge-like submanifold $\mathcal{M}_{regge}$ among a critical point $\vec{J}(\mathring{\ell})$
2. We do the full integral (from $-\infty$ to ∞) in t direction with Gaussian deformation
$$\int [\mathrm{d}t]\, e^{-\frac{\delta}{4}\sum_i t_i^2}$$
3. The Gaussian deformation plays a role as the regulator to avoid the divergent result from integral of t.
4. Apparently, there maybe non-Regge like critical points existing in the spin sum region. But δ here act as a adjustable cut-off introduced by the Gaussian deformation. (i.e. when δ is large, the support of the spin sum will stay closely around $\mathcal{M}_{regge}$, when δ is 0, then deformation will go back to the trivial case.)
5. The shape of the spin sum region does depend on the choice of the transverse direction, but the result(convergence and EOM) doesn't depend on this choice



The spin sum is done inside the blue rectangular(infinite band)

Resolution: Deforming non-Regge-like spin sum

- The term D_δ is also an exponential function:

$$D_\delta^{(k)}(\ell, X) \propto e^{\sum_{i=1}^{M} \frac{1}{\delta} \Phi^i_{(k)} \Phi^i_{(k)}} \quad \text{v.s.} \quad e^{\lambda \left\langle \vec{j}(\ell), \vec{F}[X] \right\rangle} \sim e^{i\lambda S_{regge}}$$

- The exponent in D_δ is scaled by:

$$\frac{1}{\delta} \gg 1$$

How is the competition between $J \to \infty$ and ${}^{1}/_{\delta} \to \infty$?

Resolution: Deforming non-Regge-like spin sum

- The term D_δ is also an exponential function:

$$D_\delta^{(k)}(\ell, X) \propto e^{\sum_{i=1}^{M} \frac{1}{\delta} \Phi^i_{(k)} \Phi^i_{(k)}} \quad \text{v.s.} \quad e^{\lambda \left\langle \vec{j}(\ell), \vec{F}[X] \right\rangle} \sim e^{i\lambda S_{regge}}$$

- The exponent in D_δ is scaled by:

$$\frac{1}{\delta} \gg 1$$

How is the competition between $J \to \infty$ and ${}^{1}/_{\delta} \to \infty$?

$$\lambda \gg \delta^{-1} \gg 1$$

Keep the D_δ subleading at the semi-classical limit ($J \to 0$)

Equation of Motions

Remarks:

$$S = \lambda \sum_f j_f F_f(X)$$
$$F = i\gamma\epsilon$$

- In the regime $\lambda \gg \delta^{-1} \gg 1$, the EOMs are given by

$$ReS = \partial_X S = \partial_l S = 0$$

$$\mathrm{Re}(S) = \partial_X S = 0 \Longleftrightarrow \text{SFM critical points} \leftrightarrow \text{4d simplicial geometry}$$

$$\partial_\ell S = 0 \Longleftrightarrow \left\langle \frac{\partial \vec{j}(\ell)}{\partial \ell}, \vec{F}[X] \right\rangle = 0 \quad \text{or} \quad \left(\sum_f \frac{\partial a_f(\ell)}{\partial \ell} \epsilon_f(l) = 0 \right)$$

- Formally one can have asymptotic formula:

$$\int [dldX] e^{\lambda S[l,X]} D_\delta[l,X] \sim e^{\lambda S[l_c,X_c]} D_\delta[l_c,X_c]$$

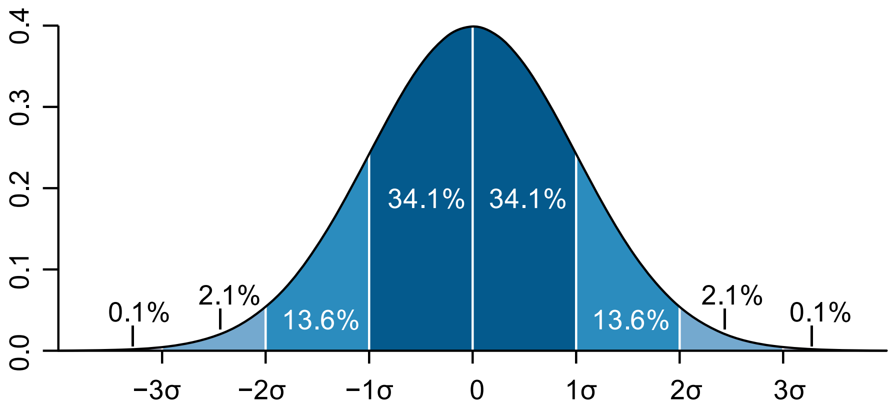
Constraints from measure data D_δ

- $\mathrm{Re}(S) = \partial_X S = 0$ indicates that $F = i\gamma\epsilon$
- Plug $F = i\gamma\epsilon$ into D_δ :

$$D_\delta \propto e^{\frac{1}{\delta}\sum_i <e^i(l),\vec{F}>^2} = e^{-\frac{1}{\delta}\sum_i <e^i(l),\gamma\vec{\epsilon}>^2}$$

- When $\delta \ll 1$, the measure D_δ is supported on:

$$|< e^i, \gamma\vec{\epsilon} >| \leq \delta^{1/2}$$



Constraints from measure data D_δ

- Reminds that $\gamma\epsilon = 0$ and $F = i\gamma\epsilon$

$$\left\langle \frac{\partial \vec{j}(\ell)}{\partial \ell}, \vec{F}[X] \right\rangle = 0$$

$$\left| \left\langle \hat{e}^i, \gamma\vec{\epsilon} \right\rangle \right| \leq \delta^{\frac{1}{2}}$$

$$|\gamma\epsilon_f(\ell)| \leq \delta^{\frac{1}{2}}$$

- If $k \neq 0$ then the bound of deficit angle becomes

$$|\gamma\epsilon_f(\ell) + 4\pi k_f| \leq \delta^{\frac{1}{2}}$$

However, when we focus on the perturbation over flat background k must be 0

- At discrete level $\delta \to 0$ recovers the flatness

The role of δ

- The parameter δ opens a window for non-zero deficit angles.

- In sufficiently refined triangulation, a near smooth simplicial geometry satisfies

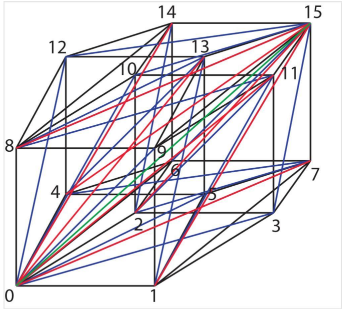
$$|\epsilon_f| \simeq a^2/\rho^2 << 1$$

compatible with small non-zero δ

- One can always triangulate an arbitrary non-singular curvature with small deficit angles, provided by a refined lattice. So we won't loss any geometry by using this way of description.

- We close the window ($\delta \to 0$) together with continuum limit, and we can approach arbitrary curved space-time

Semi-classical continuum limit (SCL)



Triangulated typical Hypercube cell

$\mathcal{K}_1 \longrightarrow Z(\mathcal{K}_1) \longrightarrow$ **discrete geometries on** $\mathcal{K}_1$

$\mathcal{K}_{1/2} \longrightarrow Z(\mathcal{K}_{1/2}) \longrightarrow$ **discrete geometries on** $\mathcal{K}_{1/2}$

$\cdots$

$\mathcal{K}_\mu \longrightarrow Z(\mathcal{K}_\mu) \longrightarrow$ **discrete geometries on** $\mathcal{K}_\mu$

$\cdots$

continuum limit: $\mu \to 0$

$$J = J(\mu)$$
$$\delta = \delta(\mu)$$
$$\text{Geometry} = \text{Geometry}(\mu)$$
$$\vdots$$

Lattice Refinement:

Subdividing hypercubes followed by triangulation (Not Pachner moves for 4-simpices)

Spin foam continuum limit ($\mu \to 0$) V.S. Regge geometry continuum limit (lattice spacing $a \to 0$)

What's the relation?

Large-j v.s. small lattice spacing

- $\mu \to 0 \implies \delta(\mu) \to 0$ $\qquad$ $J \gg \delta^{-1} \gg 1 \implies J(\mu) \to \infty$

  Combining large-j and continuum limit

- Area spectrum $A_f \simeq \gamma l_p^2 J_f$ indicates lattice spacing $\ell(\mu) \simeq l_p \sqrt{\gamma J(\mu)}$

  Apparently there is a contradiction between large spin limit and small lattice spacing limit!

- Resolution:

The lattice spacing $l(\mu)$ is dimensionful, its numerical value depends on the choice of unit.

UV unit: l_p

IR unit: μ^{-1}

Numerical value of edge length: $\begin{cases} \sqrt{\gamma J(\mu)} & \text{In } l_p \text{ unit} \\ a(\mu) & \text{In } \mu^{-1} \text{unit} \end{cases}$

We set the flow parameter μ to be energy scale(of the flat background)

$$\ell(\mu) = l_p \sqrt{\gamma J(\mu)} = a(\mu)\mu^{-1}$$

No contradiction between large spin limit and small lattice spacing limit($a \to 0$), if the μ^{-1}blows up faster than $\sqrt{J(\mu)}$

$$-\frac{2}{\mu} < \frac{1}{J}\frac{dJ}{d\mu} < 0$$

Bound the quantum correction

- Asymptotic expansion on the refining sequence:

$$\left| Z(\mathcal{K}_\mu) - (\text{large-j approximation}) \right| \leq \left(\frac{2\pi}{\lambda(\mu)}\right)^{\frac{\mathcal{N}}{2}} \frac{C(\mu)}{\lambda(\mu)}.$$ $\mathcal{N}$ is the number of degree of freedom

Reminds: (large-j approximation)$\propto \left(\frac{2\pi}{\lambda(\mu)}\right)^{\frac{\mathcal{N}}{2}}$

Positive factor $C(\mu)$ bounds $\frac{1}{\lambda}$ quantum corrections

Semi-classically converge to Regge geometries for all μ if and only if quantum corrections $C/_\lambda$ are always small

$C(\mu)$ likely grows when the triangulation refines (the worst case)

$$\frac{C(\mu)}{\lambda(\mu)} \leq \frac{C(1)}{\lambda(1)}$$ For all $\mu \to 0$ OR $$\frac{1}{\lambda}\frac{d\lambda}{d\mu} < \frac{1}{C}\frac{dC}{d\mu}$$

This constraints the running behavior of $\lambda(\mu)$

Semi-classical continuum limit (SCL) as a RG-like flow

- Definition:

We define the semiclassical continuum limit (SCL) as the flow of the 3 parameters $\lambda(\mu), a(\mu), \delta(\mu)$, where $a(\mu), \delta(\mu) \to 0$ and $\lambda(\mu) \to \infty$ for $\mu \to 0$.
The flow should satisfy:

$$\frac{1}{\lambda}\frac{d\lambda}{d\mu} < \frac{1}{C}\frac{dC}{d\mu} \qquad -\frac{2}{\mu} < \frac{1}{\lambda}\frac{d\lambda}{d\mu} < 0 \qquad \frac{\delta(\mu)^{1/2}}{a(\mu)^2} \leq L \ \text{ bounded from above}$$

- Theorem:

SCL is well defined because the flows satisfying the requirements always exist.

- Example:

$$\lambda(\mu) = \lambda(1)\mu^{-2+u}, \qquad 0 < u < \frac{2}{5}$$
$$a(\mu) = \mu^{u/2}\sqrt{\gamma\lambda(1)l_p^2}$$
$$\lambda(\mu)^{-1/2}\mu^{1-u/2} \ll \delta(\mu) \leq L^2\mu^{2u}$$

$\ell(\mu) = l_p\sqrt{\gamma J(\mu)} = a(\mu)\mu^{-1}$

$\delta(\mu)$: deformation parameter

$\lambda(\mu)$: typical background spin

$a(\mu)$: background lattice spacing

- SCL is a low energy limit of spin foam models

Gravitational Wave in Spin Foam Model

$$
\begin{array}{ccccccc}
\mathcal{K}_1 & \mathcal{K}_{1/2} & \cdots & \mathcal{K}_\mu & \cdots \\
\downarrow & \downarrow & & \downarrow & \\
Z(\mathcal{K}_1) & Z(\mathcal{K}_{1/2}) & \cdots & Z(\mathcal{K}_\mu) & \cdots \\
\downarrow & \downarrow & & \downarrow &
\end{array}
$$

continuum limit: $\mu \to 0$

SCL gives $a(\mu) \to 0$

discrete geometries on $\mathcal{K}_1$ **discrete geometries on** $\mathcal{K}_{1/2}$ **discrete geometries on** $\mathcal{K}_\mu$ $\cdots \longrightarrow$ Smooth geometries (solutions of Einstein equation)

[J.W. Barrett R. Williams 1988]

- Convergence in SCL:

  Linearised Regge geometries from spin-foam critical points $\longrightarrow$ Smooth gravitational wave geometries (solutions of Einstein equation)

- Smooth gravitational waves are all low energy excitations of spin-foams under SCL

Conclusions and Discussions

- Propose Semi-classical continuum limit(SCL) as an IR limit

  We defined the RG-like flow of a few spin foam parameters $(J(\mu),\ \delta(\mu),\ a(\mu)),\ \ \mu \to 0$

  such that we can take the large limit $J \to \infty$ and continuum limit simultaneously.

- Under SCL, we found the dominant contribution to the amplitude(all critical points) converges to the smooth gravitational waves

- All low energy excitations are smooth gravitational waves (spin-2 graviton)

Conclusions and Discussions

- Flatness and Singularity Resolution

Because of the (deformed) flatness, the deficit angles are bounded by deformed factor $\delta^{\frac{1}{2}}$, in order to prevent the large spin amplitude from being suppressed.

$$|\gamma\epsilon| < \delta^{1/2} \ll 1$$

For small deficit angles,

$$|\epsilon| \sim \frac{\ell^2}{\rho^2} \ll 1$$

So

$$\underbrace{l_p^2 \ll \ell^2}_{\text{Large spin limit}} \underbrace{\ll \rho^2}_{\text{Flatness}}$$

This is valid for arbitrary finite curvature

However, near a curvature singularity (e.g. big bang or black hole) $(\rho^2 \to 0)$

$$\ell^2 \to 0 \longrightarrow \cancel{l_p^2 \ll \ell^2} \longrightarrow$$

The spin sum is not dominated by the large spins

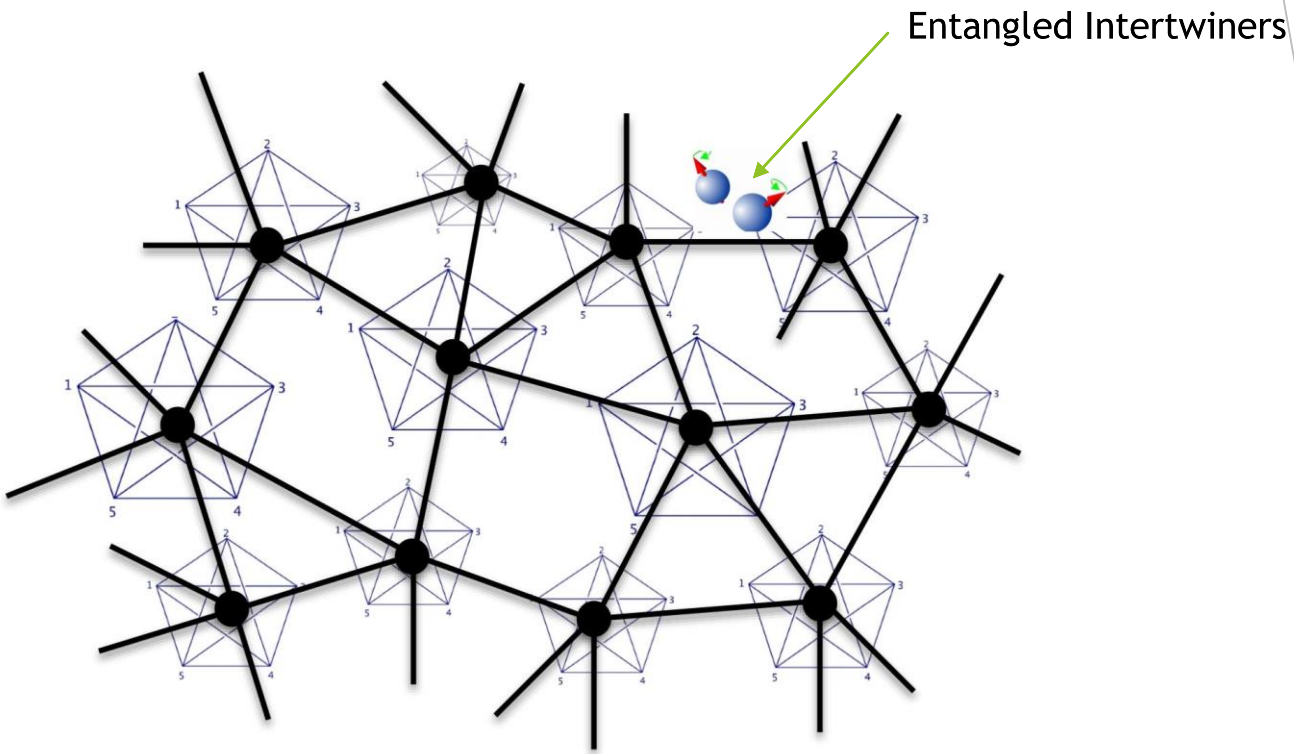
Large J amplitudes are all suppressed. The spin foam amplitude near the singularity is dominated by small spins.

Small spin amplitudes are well defined while having large quantum fluctuations.

[M. Han and M. Zhang 2016]

Conclusions and Discussions

- Tensor Network and Emergent Gravity



Relates to emergent gravity from entanglement

Details can be seen in our paper

[Xiao-GangWen, Zhenghan Wang, Zheng-Cheng Gu, Xiao-liang Qi, Zhao Yang, Yi-Zhuang You, J. Maldacena,etc]

Thanks for your attention!

Simplicity constraint(back up)

- Euclidean EPRL-FK model:

Gauge group: $spin(4) = \mathrm{SU}(2)^{+} \otimes \mathrm{SU}(2)^{-}$

Weak imposed linear simplicity constraint: $J_f^{\pm} = \frac{1}{2}|1 \pm \gamma| J_f$ where γ is the Barbero-Immirzi parameter.

Spin variables $J_f^{\pm} \in \mathbb{Z}/2$.
γ can be expressed as a ratio $^{p}/_{q}$, then $J_f \in q\mathbb{Z}$ for $p+q$ odd, $J_f \in q\mathbb{Z}/2$ for $q+p$ even.

Convergence Solution (back up)

- For a fixed triangulation, the dof of linearised distributional curvature is described by the deficit angles δ_f satisfying the Bianchi identity

- From the linearised deficit angles satisfying the Bianchi identity, one can reconstruct the linearized metric(edge-length variables) up to linearised diffeomorphisms(the translation of the lattice point)

- On the hypercube lattice triangulation, the deficit angle configurations that satisfied the Regge equation of motion and Bianchi identity can converge to a continues smooth Geometry after the coarse graining

- The solutions in the bifurcation branch following the dispersion relation $k^2 = 0$ can define a sequence of the discrete geometries converging to gravitational wave solutions

- The hyper-diagonal zero mode is still there at the discrete level. But it doesn't show up in the smooth curvature in the continuum limit, since hyper-diagonal zero modes converge to zero curvature.