Jerzy (Jurek) Lewandowski 1959-2024

Scientific work

Jurek is an important figure in LQG research:

1. **Foundations of LQG** Spin networks, Ashtekar-Lewandowski measure on the holonomy-flux algebra, LOST uniqueness result, volume operators...
2. **LQC** Foundations, analysis of APS type models, quantum field on quantum backgrounds...
3. **Spin foams** Extensions to all LQG Hilbert space (KKL)...
4. **Deparametrization** Using world-line of an observer or scalar fields to define canonical evolution in GR...

He contributed also significantly to classical GR:

1. **Shearfree geodesic null congruences, CR manifolds, twistors, Cartan connection and Einstein's equations** Study of relations between CR structures, algebraically special solutions and Fefferman conformal classes of metrics.
2. **Isolated horizons** Mechanics and geometry of isolated horizons, extremal horizons.

Some of his results are famous in our community. Some other are less known although, I think they deserve attention. I will focus in my talk on joint work. Not all are directly related to quantum gravity at first sight.

Conformal normal Cartan connection

This is a topic of Jurek's talk on Loops' 24, ILQGS seminar and planned talk in September in Cambridge.

[A. Bac, W. Kaminski, J. Lewandowski, M. Broda]

Conformal normal Cartan connection

Conformal normal Cartan connection provides a useful way to deal with conformal geometry. Among other things it defines a derivatives in some bundles associated with the Cartan bundle.

There is a reason why conformal Cartan connection is interesting for relativists.

1. Well adapted to conformal boundary,
2. The metric is conformal to a solution of Einstein's equations iff there exists a covariantly constant section in an associated bundle (tractor bundle with fiber $\mathbb{R}^{d+2}$), $\nabla^T_\mu I^I = 0$
3. Hard to check (one needs to find a solution to some overdetermined system), but there exists a necessary condition: Bach tensor needs to vanish.

For a while it was not obvious if this condition is not by accident also sufficient, but Nurowski and Plebanski provided a counterexample in the class of Fefferman metrics.

One of the slide from Jurek's talk at Loops '24

Examples of reduced holonomy:

The spinor representation of the NCCC is the local twistor connection of Penrose. Iff the NCCC can be gauge transformed to the following non-generic form:

$$A = \begin{pmatrix} \psi & \theta^4 & \mathrm{Re}\omega_1 & \mathrm{Im}\omega_1 & -\omega_0 & 0 \\ -\theta^4 & \psi & -\mathrm{Im}\omega_1 & \mathrm{Re}\omega_1 & 0 & -\omega_0 \\ \mathrm{Re}\omega_3 & \mathrm{Im}\omega_3 & 0 & -2\theta^4 & \mathrm{Im}\omega_1 & \mathrm{Re}\omega_1 \\ -\mathrm{Im}\omega_3 & \mathrm{Re}\omega_3 & 2\theta^4 & 0 & -\mathrm{Re}\omega_1 & \mathrm{Im}\omega_1 \\ \omega_4 & 0 & -\mathrm{Im}\omega_3 & -\mathrm{Re}\omega_3 & -\psi & \theta^4 \\ 0 & \omega_4 & \mathrm{Re}\omega_3 & -\mathrm{Im}\omega_3 & -\theta^4 & -\psi \end{pmatrix}$$

then the spacetime conformal geometry admits solutions to the twistor equation:

$$\nabla^{(A}{}_{A'}\omega^{B)} = 0$$

This is the Fefferman family of spacetime metric tensors. Among them are known examples of the Bach flat metric tensors that are not conformal to Einstein.

Two important results of Jurek from '80:

1. Characterization of all spacetimes allowing nontrivial solutions to twistor equation.
2. Fefferman metrics are never conformally Einstein. Thus if there exists a Bach flat Fefferman metric then it is a counterexample.

Bach tensor

Jurek was interested in this counter-example:

1. He wrote a paper (with P. Nurowski and another with M. Korzynski) explaining the origin of this construction: Bach tensor for a Fefferman metric is governed by a single scalar.
2. Important property was a formula for Bach tensor as a part of Yang-Mills current for conformal normal Cartan connection.
3. Bach tensor is an Euler-Lagrange equation for Weyl square Lagrangean. Relation to Yang-Mills equations allows to derive conformally invariant presymplectic current (basically YM pre-symplectic current).
4. This current differs from Λ times GR current by a trivial corrections (corrections that does not alter symplectic form)
5. Such conformally invariant currents are important for Noether charges (Wald-Zoupas construction).

Schroedinger picture versus group averaging

This is application of foundations of LQG to LQC and deparametrization. Technically demanding, we have almost given up writing down the paper. Then Jurek rewrote alone the whole manuscript to the form (more) digestable by a reader.

[WK, J. Lewandowski, T. Pawlowski]

Schroedinger picture

1. Loop Quantum cosmology with a massless scalar field
$\mathcal{H}_{kin} = L^2(\mathbb{R}) \otimes L^2(\mathbb{Z}_+)$

$$\hat{C} = -\frac{1}{2}B(v)\partial_\phi^2 - \hat{C}_{gr}$$

2. Deparametrization

$$i\partial_\phi \Psi = \pm \hat{H}\Psi, \quad \hat{\Theta} := 2B^{-1/2}\hat{C}_{gr}B^{-1/2}$$

with physical hamiltonian $\hat{H} = \sqrt{\hat{\Theta}}$ and physical Hilbert space

$$\mathcal{H}_{phys} = P_{[0,\infty)}(\Theta)L^2(\mathbb{Z}_+).$$

Actually two copies but one argue that only one copy is physical future evolving.

3. It works well for $\Lambda \geq 0$, hamiltonian is self-adjoint, thus dynamics uniquely defined.

Negative cosmological constant

1. However, for negative cosmological constant $\hat{\Theta}$ has many self-adjoint extensions (non-unique dynamics).
2. Semiclassically, the wave packet achieve v infinity in a finite ϕ time (visible already on semiclassical equation of motion). The extension label way the packet bounce from infinity.
3. Apparently, the semiclassical trajectory is indepedent of the choice of extension. The only difference in the evolution is a total phase of wave packet (given by choice of extension).
4. Even, so for large volume universe should behave classically, but such bouncing are not classical. Classically, ϕ has finite range (in standard geodesic time) and no dynamics exists outside this range.

Here, the other work of Jurek comes in rescue.

Group averaging

1. To solve the puzzle we should look at the original constraint

$$\hat{C} = -\frac{1}{2}B(v)\partial_\phi^2 - \hat{C}_{gr}$$

   and find the physical Hilbert space $\ker \hat{C}$.

2. This operator is self-adjoint and it has continuous spectrum. The definition of kernel needs care.
3. Group averaging provides such a definition.

$$\langle[\psi],[\psi']\rangle_{ph} := \frac{1}{2\pi}\int_{-\infty}^{\infty} d\lambda \ \langle\psi, e^{i\lambda\hat{C}}\psi'\rangle$$

   Operators commuting with $\hat{C}$ are observables.

4. The observable O_F obtained by group averaged $\sqrt{B|\Pi|}\delta(\phi-\phi_0)\hat{F}(v,b)\sqrt{B|\Pi|}$ is quantum relational observable.

Comparison

We have quite control over generalized eigenfunctions of $\hat{C}$ and H we can perform construction and at least describe results.

1. For $\Lambda > 0$ the result coincides with deparametrization excepts that O_F is F mixing both future and past evolving sectors.
2. For $\Lambda < 0$ the situation is different. The physical Hilbert space is a direct intergral of all possible Hilbert spaces corresponding to different extensions.
3. The relational observables acts by mixing all the different Hilbert spaces.
4. If we compute expectation value of observable outside of classically allowed range of ϕ the phases of wave packets (should) cancel giving vanishing. At least for large ϕ it follows from Riemann-Lebesgue lemma.