Gravitational waves in the de Sitter universe

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to appear on arXiv soon (joint work with Jerzy Lewandowski)

Diamentowy Grant

FACULTY OF PHYSICS
UNIVERSITY OF WARSAW

Outline

Gravitational waves are real

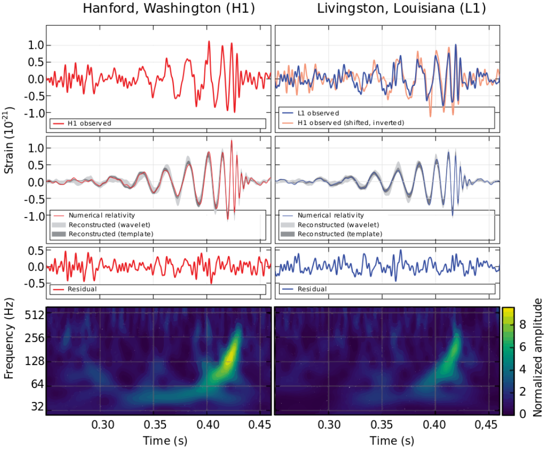

Figure: LIGO

Cosmological constant is positive

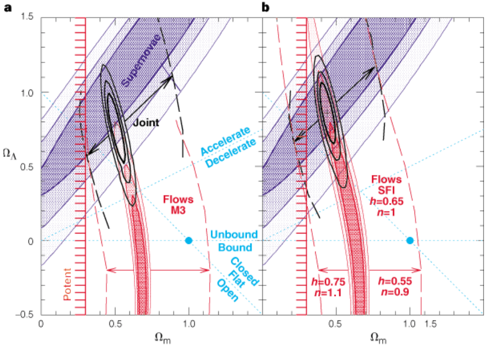

Figure: Zehavi and Dekel (1999)

So what's the problem?

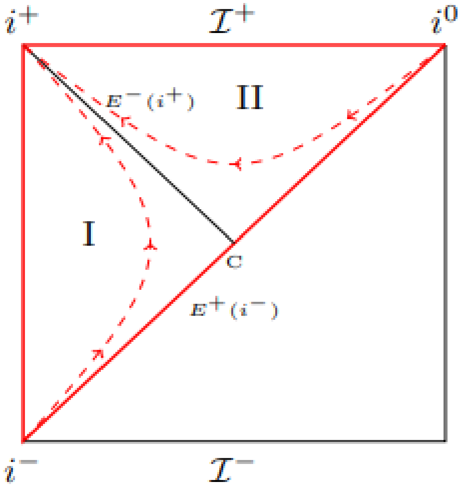

Figure: Ashtekar, Bonga and Kesavan (2015)

Recent progress

- Different energies [Kastor and Traschen 2002, Penrose 2011, Ashtekar, Bonga and Kesavan 2015, Szabados and Tod 2015, Chruściel, Hoque and Smołka 2020...]

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- Different boundary conditions and gauges [Ashtekar, Bonga and Kesavan 2015, He and Cao 2015, Poole, Skenderis and Taylor 2019, Comperé, Fiorucci and Ruzziconi 2019...]
- Physical solutions [Ashtekar, Bonga and Kesavan 2015, Bishop 2016, Date and Hoque 2016...]

Good news: we are not going to propose a new definition!

Our strategy

We consider gravity linearized around the de Sitter universe

$$g = -\left(1 - \frac{\Lambda r^2}{3}\right) du^2 - 2dudr + r^2 \mathring{\gamma}_{AB} dx^A dx^B. \tag{1}$$

Those coordinates are well-suited both for *scri* (with a topology $\mathbb{R} \times \mathbb{S}^2$) and for the $\Lambda \to 0$ limit.
We use symplectic definitions of charges which are known to be well-defined up to boundary terms – those we need to find independently.

Charges

Let X be a Killing vector of the de Sitter universe. We define a charge of a light cone C_u as

$$E(u) = \frac{1}{2}\int_{C_u} \omega^a(h, \mathcal{L}_X h)\epsilon_{abcd}\frac{1}{3!}dx^b dx^c dx^d \tag{2}$$

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(plus some boundary terms to to be established later on)
We will focus mainly on the de Sitter energy given by $X = T = \partial_u$.

Gauge choices

One can consider a family of solutions with the asymptotics as $r \to 0$:

$$\frac{h_{AB}(r,u,x)}{r^2} = \frac{h^{(-1)}_{AB}(u,x)}{r} + \frac{h^{(-3)}_{AB}(u,x)}{r^3} + \dots$$
$$\frac{h_{Au}(r,u,x)}{r^2} = h^{(0)}_{Au}(u,x) + \frac{\mathring{D}^B h^{(-1)}_{AB}(u,x)}{2r^2} + \frac{h^{(-3)}_{Au}(u,x)}{r^3} + \dots \tag{3}$$

Moreover,

$$h_{ra} = 0, \quad \text{and} \quad h_{AB}\mathring{\gamma}^{AB} = 0. \tag{4}$$

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It's the analog of the Bondi gauge in AF spacetimes. The main differences are [Friedrich '86]

- $h^{(0)} \neq 0$
- $h^{(-3)}$ are free data up to 'momentum' constraints

Charges calculated

Straightforward but tedious calculation gives the following difference of energy between cones:

$$E(u_1) - E(u_2) = \frac{1}{32\pi}\int_{u_1}^{u_2} du \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi$$
$$\mathring{\gamma}^{BE}\left(\mathring{\gamma}^{FC} h^{(-1)}_{BC,u} h^{(-1)}_{EF,u} - 6 h^{(-3)}_{B} h^{(0)}_{E,u}\right) -$$
$$\frac{1}{2}\mathring{\gamma}^{BE}\partial_u\left(\mathring{\gamma}^{FC} h^{(-1)}_{BC} h^{(-1)}_{EF,u} + 6 h^{(-3)}_{B} h^{(0)}_{E}\right) \quad (5)$$

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For the future use we take

$$Q_T := E(-\infty) - E(+\infty). \tag{6}$$

Charges calculated

A similar calculation gives also an amount of radiated angular momenta:

$$
\begin{aligned}
J(-\infty) - J(\infty) =: \\
Q_S[h] = \frac{1}{64\pi} \int_{\mathbb{R}} du \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \\
\mathring{\gamma}^{BE} \mathring{\gamma}^{FC} \left(\mathcal{L}_S h^{(-1)}_{BC} h^{(-1)}_{EF,u} - h^{(-1)}_{BC} \mathcal{L}_S h^{(-1)}_{EF,u} \right) \\
- 6 \mathring{\gamma}^{AB} \left(\mathcal{L}_S h^{(-3)}_A h^{(0)}_B - \mathcal{L}_S h^{(0)}_A h^{(-3)}_B \right).
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Unfortunately, calculation of $Q_{T_{(i)}}$ is too complicated so we will present the result later on.

Weyl tensor

It is convenient to work with Weyl tensor at $\mathcal{I}^+$. Let

$$E_{ab} := C_{acbd} n^c n^c \tag{8}$$

be the electric part of (linearized) C_{abcd} at the surface $r = \mathrm{const.}$ It vanishes as $r \to \infty$ so we introduce

$$\mathcal{E}_{ab} = \lim_{r\to\infty} HrE_{ab}. \tag{9}$$

In particular we have:

$$\begin{aligned} \mathcal{E}_{uu} &= -H^3 h_{uu}^{(-3)} \\ \mathcal{E}_{uA} &= -\frac{3}{2} H^3 h_A^{(-3)} + H h_{A,u}^{(-2)} \end{aligned} \tag{10}$$

Weyl tensor cnd

We can rewrite our charges as

$$\begin{aligned} Q_T &= \frac{1}{16\pi H}\int_{\mathcal{I}^+} d^3x\sqrt{\mathring{q}}\mathcal{E}_{cd}\mathcal{L}_T\left(H^{-2}h^{(0)}_{ab}\right)\mathring{q}^{ac}\mathring{q}^{bd} \\ Q_S &= \frac{1}{16\pi H}\int_{\mathcal{I}^+} d^3x\sqrt{\mathring{q}}\mathcal{E}_{cd}\mathcal{L}_S\left(H^{-2}h^{(0)}_{ab}\right)\mathring{q}^{ac}\mathring{q}^{bd} \end{aligned} \tag{11}$$

They look surprisingly similar...

Ashtekar–Bonga–Kesavan charges

Some time ago [2015] the same charges were calculated in the Poincare patch:

$$\begin{aligned} Q_T^P &= \frac{1}{2H\kappa}\int_{\mathcal{I}^+} d^3x\sqrt{\mathring{q}^P}\mathcal{E}_{ij}^P\left(\mathcal{L}_T h_{kl}^P + 2Hh_{kl}^P\right)\mathring{q}^{ikP}\mathring{q}^{jlP} \\ Q_{T_{(i)}}^P &= \frac{1}{2H\kappa}\int_{\mathcal{I}^+} d^3x\sqrt{\mathring{q}^P}\mathcal{E}_{ij}^P\mathcal{L}_{T_{(i)}} h_{kl}^P\mathring{q}^{ikP}\mathring{q}^{jlP} \\ Q_S^P &= \frac{1}{2H\kappa}\int_{\mathcal{I}^+} d^3x\sqrt{\mathring{q}^P}\mathcal{E}_{ij}^P\mathcal{L}_S h_{kl}^P\mathring{q}^{ikP}\mathring{q}^{jlP}. \end{aligned} \tag{12}$$

Those are exactly the same if we remember to transforms both sets of initial data by a conformal transformation with $\Omega = e^{-Hu}$:

$$\begin{aligned} \mathring{q}^P &= \Omega^2\mathring{q} \\ h^P &= \Omega^2 H^{-2} h^{(0)} \\ \mathcal{E}^P &= \Omega^{-1}\mathcal{E}. \end{aligned} \tag{13}$$

Momentum in the Bondi frame

In this way we can 'derive' also momentum:

$$Q_{T_{(i)}} = \frac{1}{16\pi H}\int_{\mathcal{I}^+} d^3x\sqrt{\mathring{q}}\mathcal{E}_{cd}\left(\mathcal{L}_{T_{(i)}}h^{(0)}_{ab} - 2e^{Hu}Hg_i h^{(0)}_{ab}\right)\mathring{q}^{ac}\mathring{q}^{bd}, \qquad (14)$$

where

$$T_{(i)} = e^{Hu}\left(g_i\partial_u - g_i\left(Hr+1\right)\partial_r - \left(\frac{1}{r}+H\right)\mathring{D}^A g_i\partial_A\right). \qquad (15)$$

Of course all (not necessarily important) boundary terms are lost in the process.

Comparison

What can we learn from ABK work? Among many other things, boundary conditions. As $u \to \infty$, $|\vec{x}| = H^{-1}e^{-Hu} \to 0$ at $\mathcal{I}^{+P}$. From smoothness it follows that:

$$h^{(0)}_{ab} = O(1) \tag{16}$$

$$\mathcal{E}_{ab} = O(e^{-3Hu}). \tag{17}$$

However, we don't need such strong boundary conditions! Indeed, we have:

- $\mathcal{E}_{ab} = O(1)$ to ensure finite Q_T
- $\mathcal{E}_{ab}$ integrable at $u = \infty$ to ensure finite Q_S
- $e^{Hu}\mathcal{E}_{ab}$ integrable at $u = \infty$ to ensure finite $Q_{T_{(i)}}$.

On the side note: Schwarzschild–de Sitter has $\mathcal{E}_{uu} = \mathrm{const.} \neq 0$.

Gauge dependence

It seemed to us that the best way to fix boundary terms in the definition of ΔE was imposing gauge invariance. Gauge transformations on the initial data act as follows[1]:

$$\delta h_{ab} = \mathring{\nabla}_{(a}\xi_{b)} \tag{18}$$

$$\delta \mathcal{E}_{ab} = 0, \tag{19}$$

where ξ is a vector field on $\mathcal{I}^+$. Now we can calculate $\delta(E(u_1) - E(u_2))$

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$$\delta(E(u_1) - E(u_2)) = \frac{1}{16\pi H^3} \int_{\mathbb{S}^2} dx^A \sqrt{\mathring{\gamma}} \mathcal{E}_{ad} \mathcal{L}_T \xi^d (\partial_u)^a |_{u_1}^{u_2} \tag{20}$$

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There are no boundary terms which would kill it entirely. However, δQ_T vanishes even when ξ is not of compact support.
One should establish residual gauge transformations and check $\delta \Delta E$ then.

Residual gauge transformations

We look for such X that $h + \mathcal{L}_X g$ is still written in the Bondi gauge. A short calculations gives the following form of the residual gauge transformations generators:

$$X = \left(\mathring{X}^u + \frac{1}{2}\int du \mathring{D}_A \mathring{X}^A\right)\partial_u + \left(-\frac{1}{2} r \mathring{D}_A \mathring{X}^A + \frac{1}{2}\Delta_{\mathring{\gamma}} \mathring{X}^u + \frac{1}{2}\int du \Delta_{\mathring{\gamma}} \mathring{D}_A \mathring{X}^A\right) \\ + \left(\mathring{X}^B - \frac{1}{r}\mathring{D}^B \mathring{X}^u - \frac{1}{r}\int du \mathring{D}^B \mathring{D}_A \mathring{X}^A\right)\partial_B, \tag{22}$$

where $\mathring{X}^u = \mathring{X}^u(x^A)$ and $\mathring{X}^A$ is u–dependent conformal Killing vector of $\mathring{\gamma}$.

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where $\mathring{X}^u = \mathring{X}^u(x^A)$ and $\mathring{X}^A$ is u–dependent conformal Killing vector of $\mathring{\gamma}$. Too many of them!

Rigid transport

We can impose one additional gauge condition – namely that our cones do not rotate with respect to each other. It is a little bit technical but the result is veery simple – residual gauge transformations are:

$$X = f\partial_u + \frac{1}{2}\Delta_{\mathring{\gamma}} f\partial_r - \frac{1}{r}\mathring{D}^B f\partial_B + K^a\partial_a, \tag{23}$$

where K^a is a Killing vector of g and $f = f(x^A)$. Our energy-loss formula IS invariant with respect to those transformations!
Remark: this additional condition in the $\Lambda = 0$ case reduces to $h_A^{(0)} = 0$ so it is implicit in every calculation.

Superpseudotranslations

It is tempting to call

$$X = f\partial_u + \frac{1}{2}\Delta_{\mathring{\gamma}} f\partial_r - \frac{1}{r}\mathring{D}^B f\partial_B \tag{24}$$

supertranslation.

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supertranslation. However, it would be a misnomer since (spatial) translations are not of this form! Thus, we are going to call them superpseudotranslations.

Note that they commute with each other but not with Killing vectors of g.

Mass-loss formula

We thus propose the following mass-loss formula:

$$E(u_1) - E(u_2) = \frac{1}{32\pi} \int_{u_1}^{u_2} du \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi$$
$$\mathring{\gamma}^{BE} \left(\mathring{\gamma}^{FC} h^{(-1)}_{BC,u} h^{(-1)}_{EF,u} - 6 h^{(-3)}_{B} h^{(0)}_{E,u} \right) \quad (25)$$

as the only expression which has correct $\Lambda \to 0$ limit and is invariant with respect to all superpseudotranslations.
Remark: this was also obtained by the Noether theorem [Chruściel, Hoque, Smołka 2020] but with the help of a priori arbitrary regularization – its uniqueness is fixed by gauge invariance.

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Flux through the horizon

We can repeat the whole calculation at $r = \sqrt{\frac{3}{\Lambda}}$ (it means, at the horizon) to obtain flux. Symplectic current yields:

$$
\begin{aligned}
&-16\pi\omega^r(h_1, h_2) = && (26)\\
&h_{2BC}\nabla_D h_{1uA} g^{CD} g^{AB} - h_{2Bu}\nabla_r h_{1uA} g^{AB} - \frac{1}{2} h_{2BD}\nabla_u h_{1AC} g^{AB} g^{CD} - 1 \longleftrightarrow 2
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and as before $h_2 = \mathcal{L}_T h$.

It it not very illuminating form...

$\Lambda \to 0$ limit

We have the expansion:

$$\begin{aligned} \frac{h_{AB}(r,u,x)}{r^2} &= \frac{h^{(-1)}_{AB}(u,x)}{r} + \frac{h^{(-3)}_{AB}(u,x)}{r^3} + \dots \\ \frac{h_{Au}(r,u,x)}{r^2} &= h^{(0)}_{Au}(u,x) + \frac{\mathring{D}^B h^{(-1)}_{AB}(u,x)}{2r^2} + \frac{h^{(-3)}_{Au}(u,x)}{r^3} + \dots \end{aligned} \tag{27}$$

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and $r = \sqrt{\frac{3}{\Lambda}}$ so it is like we are going with $r \to \infty$...
The key question is how all those h_A and h_{AB} depends upon Λ. We can make it:

$$\begin{aligned}h^{(0)}_{A} &= O(\Lambda) \\ h^{(-1)}_{AB} &= O(1)\end{aligned} \tag{28}$$

and the rest is at least $O(1)$. This implies that only terms containing $h^{(-1)}_{AB,u}$ contribute, reproducing (up to the boundary terms) Trautman–Bondi formula.

$\Lambda \to 0$ limit cnd

Let us notice that this is not a simple large r expansion because scaling of coefficients with Λ is non-trivial. In particular, we have:

$$
\begin{aligned}
h_{AB} &= h_{AB}^{(-1)}\sqrt{\frac{3}{\Lambda}} + O(\Lambda^{\frac{1}{2}}) \\
h_B &= -P^{AC}_{\ \ B} h_{AC}^{(0)} + \frac{1}{2}\mathring{D}^A h_{AB}^{(-1)} + O\left(\Lambda^{\frac{1}{2}}\right),
\end{aligned}
\tag{29}
$$

where $P^{AC}_{\ \ B}$ is (non-unique) inverse to the constraints.

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Commutators

Let us start with a trivial observation:

$$[T, T_{(i)}] = HT_{(i)} \tag{30}$$

It follows that energy is not invariant if we translate our solution:

$$\delta_{T_{(i)}} Q_T = HQ_{T_{(i)}}. \tag{31}$$

Direct calculations shows that is indeed the case. Is that a problem?

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$$\delta_{T_{(i)}} Q_T = HQ_{T_{(i)}}. \tag{31}$$

Direct calculations shows that is indeed the case. Is that a problem? The geometrical meaning is quite clear: T is fixed by i^+ and and translations moves it. But then, can we use Q_T as a measure of PHYSICAL energy which could be used e.g. to warm up a cup of tea?

Closed universe

Trivial situation: the whole de Sitter (topologically $\mathbb{R} \times \mathbb{S}^3$) and a perturbation on it, globally defined.

Closed universe

Trivial situation: the whole de Sitter (topologically $\mathbb{R} \times \mathbb{S}^3$) and a perturbation on it, globally defined. All such perturbations must have vanishing all de Sitter charges so $Q_T = Q_{T_{(i)}} = \delta_{T_{(i)}} = 0$.

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We did not learn anything from this example. From now on, we will restrict ourselves to only one Hubble patch and we do not care about universe outside it.

Primordial GWs

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Astrophysical GWs

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Finally, let us consider GWs emitted by an astrophysical source (say merger of black holes binary). Then, clearly our previous arguments don't work. However, let us notice that the whole system BBH+GW has vanishing momenta and so their energy is invariant. It is connected with the old origin problem [Penrose '65] – we mix Coulombic and radiative modes and also their energies.

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- We calculated the flux of energy through the cosmological horizon obtaining correct $\Lambda \to 0$ limit
- We analyzed consequences of the transformation law for the canonical energy under translations

THANK YOU
FOR YOUR ATTENTION!