What can we learn from shape dynamics?

International Loop Quantum Gravity Seminar

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Outline

1. Motivation from 2+1 diemsnional qunatum gravity to consider conformal evolution as fundamental
2. Conformal evolution is different from spacetime (i.e. abandon spacetime)
3. Generic dynamical emergence of spacetime in the presence of matter (i.e. regain spacetime)

Introduction and Motivation

Motivation

Canonical metric path integral in 2+1 (only known metric path integral)

necessary: 2+1 split and CMC gauge condition $g_{ab}\pi^{ab} - t\sqrt{g} = 0$

$$Z = \int [dg_{ab}][d\pi^{ab}][dN][d\xi^a]\delta[\tfrac{\pi}{\sqrt{g}} - t]\delta[F^c]\,\det[FP]e^{i\int dt d^2x(\dot{g}_{ab}\pi^{ab} - S(N) - H(\xi))}$$
$$= \int [d\tau_A][dp^A]e^{i\int dt d^2x(\dot{\tau}_A p^A - V_o(\tau,p;t))}$$ [Carlip: CQG 12 (1995) 2201, Seriu PRD 55 (1997) 781]

where: τ_A are Teichmüller parameters, $V_o(\tau, p; t)$ denotes on-shell volume, which depends explicitly on time t

$\Rightarrow$ QM on Teichmüller space, phys. Hamilt. $V_o(\tau, p; t)$ [Moncrief: JMP 30 (1989) 2907]

Fradkin-Vilkovisky theorem: [cf. Henneaux/Teitelboim: "Quantization of Gauge Systems"]

"Partition function depends on gauge fixing cond. **only** through the gauge equivalence class of gauge fixing conditions."

for a discussion and some examples of non-equivalence [see Govaerts, Scholtz: J.Phys. A 37 (2004) 7359]

Why does CMC equivalence class appear special?

particularly, since CMC condition is unnatural from spacetime perspective

$\Rightarrow$ Take 2+1 split and CMC-cond. seriously for quantization

CMC class is equivalent to Shape Dynamics description

Shape Dynamics description

"gravity can be described as evolution of spatial conformal geometry"

$\Rightarrow$ Shape Dynamics path integral (coincides with ADM in special conformal gauge):

$$Z_{SD} = \int [dg_{ab}][d\pi^{ab}][d\rho][d\xi^a]\delta[gf.]\det[FP]e^{i\int dtd^2x(\dot{g}_{ab}\pi^{ab}-\rho(\pi-t\sqrt{g})-H(\xi))}$$
$$= \int [d\tau_A][dp^A]e^{i\int dtd^2x(\dot{\tau}_A p^A - V_{g.f.}(\tau,p,t))}$$

reduction to QM system works for **all** conformal gauge fixings

where $V_{g.f.}(\tau,p;t)$ is volume that solves gauge fixing cond., for spirit see [T.K. IJMP A 28(2013) 1330017]

Gravity as generic effective field theory:

Renormalization group:

requires *theory space* (metric field cont. + diffeomorph.+conf. gauge inv.)

preserves even number of space derivatives and time reflection

dimensionally irrelevant terms disappear at low energy

$\Rightarrow$ generic low energy limit of even gauge fixing

$$\chi = a\left(\frac{\pi_T^{ab}\pi_{ab}^T}{\sqrt{g}} + bR\sqrt{g} + c(t^2+d)\sqrt{g}\right) \approx 0$$

Generic emergence of GR (opposed to perturbative non-renorm.)

$\chi = a\,(\pi^{ab}_T \pi^T_{ab}/\sqrt{g} + b\,R\sqrt{g} + c(t^2 + d)\sqrt{g})$

b defines (speed of light)^2 $\Rightarrow$ not observable
c can be removed by dynamical similarity, i.e. rescalings of c map solution curves of conf. geometries into solution curves $\Rightarrow$ not observable
$\Rightarrow$ two GR couplings G_N, Λ emerge in IR
$\Rightarrow$ IR $V_{g.f.}(\tau, p; t)$ is **in**distinguishable from GR on conf. supersp. [T.K. to appear I]

(current calculations with H. Gomes and F. Mercati for the 3+1 path integral for shape dynamics description)

Provocative conclusions:

1. Quantization of gravity suggests: fundamental description of gravity = evolution of conformal geometry
2. spacetime is an emergent IR concept

$\Rightarrow$ spacetime has to be abstracted from "how matter falls"

for reconstruction of spacetime from test matter evolution see [Gomes, T.K.: GRG 44 (2012) 1539]

Problem:

If evolution of spatial conformal geometry is the fundamental description of gravity, how does spacetime geometry emerge?

The rest of this talk:

1. Conformal evolution differs in (subtly) form spacetime that solves Einstein's equations (two explicit examples)
2. Spacetime emerges spontaneously and generically from some dynamical laws (in Newtonian limit)

Conformal evolution differs from spacetime

Conformal evolution differs form spacetime

Formal reconstruction of spacetime

Formal problem: Given $\hat{g}_{ab}(t), \xi^a(t)$ find scale $\Omega(t)$ and lapse $N(t)$
s.t. spacetime metric $g_{\mu\nu}[N, \xi, \Omega^4 \hat{g}_{ab}]$ solves Einsteins equations
This is a **non**dynamical elliptic problem:

$\Delta N = \left(\frac{\tau^2}{3} + R + matt.\right) N$ CMC-lapse equation + normalization $\langle \dot{\pi} \rangle = 3/2$

$8\Delta\Omega = R\,\Omega - \frac{\pi_T^{ab}\pi^T_{ab}}{|g|}\Omega^{-7} + (\frac{3}{8}\tau^2 - 2\Lambda)\Omega^5 + matt.$ Lichnerowicz-York equation

these equations have solutions for physical metrics (3 positive eigenvalues), but *may* require Ω, N to vanish or diverge somewhere
$\Rightarrow$ there are conformal evolutions without corresponding GR-spacetime

Two tractable examples: [T.K.: to appear II]

1. Bianchi I on torus (pre- big bang and post-freeze out)
2. irrotational thin shell collapse (after the end of proper time)

Bianchi I (torus topology)

homogeneous metric $g_{ab}(V, \tau_A)$ (volume V and Teichmüller parameters τ_A)

- symmetry reduced CMC system $H = \sqrt{\pi^{ab}\pi_{ab}}$, conformal constraint $\pi \approx 0$
- dyn. constraint $H_o - \sqrt{\pi^{ab}\pi_{ab}} \approx 0$ reparametrizes Teichmüller geodesics

  (i.e. no reference to any time parametrization)

Teichmüller geodesics in spacetime

- Einstein's equations require lapse $N = (\frac{3}{8}\tau^2 - 2\Lambda)^{-\frac{1}{2}}$

  and conformal factor $\Omega = \left(\frac{\pi^{ab}\pi_{ab}}{\frac{3}{8}\tau^2 - 2\Lambda}\right)^{\frac{1}{12}}$

$\Rightarrow$ generic trajectory has:

(1) big bang $\Omega = 0$ and

(2) asymptotic freeze out $N \to \infty$

- big bang and freeze out can occur at **generic** points in Teichmüller space

$\Rightarrow$ pure shape dynamics evolves through big bang and past the end of time

(3 dim analogue of Penrose's cyclic Weyl cosmology)

Thin irrotational shell in AF spacetime

shperically symmetric SD is nondynamical

spherical symmetry + asymptot. flat = spatial geometry is conformally flat
$\Rightarrow$ no gravitational evolution (in SD description)

physical evolution of thin irrotational shell

- all d.o.f. are shell d.o.f., e.g. R, p_R
- Hamiltonian $H = N_{CMC} H_{matt}(\Omega_{CMC}) \Rightarrow$ evolution sticks at $N_{CMC} = 0$
- irrotational setup $\Rightarrow$ ADM-mass conserved

$\Rightarrow$ reparametrization generator $M_o - M_{ADM}(p_R, R) \approx 0$
this parametrization constraint generates thin-shell collapse

Explicit parametrization cond. $ln(R/r_o) - t \approx 0$

parametrization cond. evolves constraint through $N = 0$ region
Einstein's equations require:

conf. fact. $\Omega = (1 + \frac{2M_o}{r})\theta(r - R) + (1 + \frac{2M_o}{R})\theta(R - r)$

and lapse $N = \frac{1-\frac{2M_o}{r}}{1+\frac{2M_o}{r}}\theta(r - R) + \frac{1-\frac{2M_o}{R}}{1+\frac{2M_o}{R}}\theta(R - r)$

$\Rightarrow$ dynamical generation of isotropic line element through matter collapse

Summary (equivalence vs. difference)

Equivalence between spacetime and conformal evolution needs:

1. spacetime → conformal: (1) global hyperbolicity and
   (2) CMC foliability
2. conformal → spacetime: (1) lapse is finite and positive
   (2) conformal factor is finite and positive

⇒ despite local indistinguishability there are global differences

Conformal evolution avoids focusing (it is pure gauge)

⇒ focusing arguments do not predict breakdown of conformal evolution
⇒ avoids many singularity theorems
N.B.: conformal evolution still reaches singularities:
e.g. Bianchi I: shape singularity (boundary of Teichmüller space) may be reached by conformal evolution!

Generic spontaneous emergence of spacetime

work in collaboration with J. Barbour and F. Mercati: [arXiv:1310.5167] (preliminary ideas in [arXiv:1302.6264])

Newtonian Universe (N-body, $E = 0$, $\vec{J} = 0$, $\vec{P} = 0$)

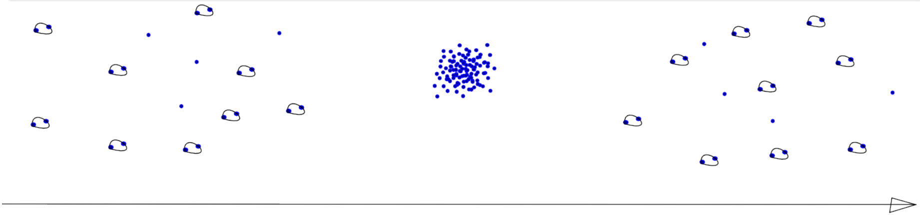
Ingredients:

(1) description of whole universe, not a subsystem

$\Rightarrow$ no extraneous frame and/or scale ($E = 0$, $\vec{J} = 0$, $\vec{P} = 0$)

(2) evolution of gravitating matter

(3) generic consequence of dynamical law, not initial cond. (no ensemble)



direction of Newtonian time

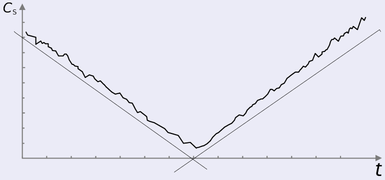
Observation:

time-reversal symmetry is broken to qualitative inversion around minimal expansion

$\Rightarrow$ "one past - two futures" scenario emergent

Arrow of time from growth of complexity

measure of complexity of configuration: $C_s = l_{rms}/l_{hm}$



- $l_{rms} = \frac{1}{m_{tot}}\sqrt{\sum_{a<b} m_a m_b r_{ab}^2}$ (dominated by average distance)
- $l_{hm} = \left(\frac{1}{m_{tot}^2}\sum_{a<b}\frac{m_a m_b}{r_{ab}}\right)^{-1}$ (dominated by short distances)

$\Rightarrow$ C_s measures how many local structures have formed and how pronounced they are

Properties of C_s

1. secular growth (linearly growing lower bound)
2. is scale invariant (as appropriate for whole universe)

$\Rightarrow$ identify arrow of time as direction of secular complexity growth
implements idea: time is deduced from local structure

Growth of dynamically generated local information

Generic late time evolution

- N-body fragments into subsystems (separt. $O(t^{2/3})$) and clusters (separat. $O(t^0)$)
- subsyst. develop energy E_i, ang. moment. $\vec{J}_i$, lin. motion $\vec{C}_i$ (wrt. cent. of mass)
- asymptotic behavior (for $t \to \infty$):

$$E_i(t) = E_i^\infty + O(t^{-5/3}), \qquad \vec{J}_i(t) = \vec{J}_i^\infty + O(t^{-2/3}), \qquad \vec{X}_i(t)/t = \vec{C}_i^\infty + O(t^{-1/3}).$$

Dynamically generated and preserved information

$I(t)$ = number of bits in $(E_1(t), \vec{J}_i(t), \vec{C}_i(t), ..., E_M(t), \vec{J}_M(t), \vec{C}_M(t))$
that will remain unchanged for all $T > t$.

Properties of $I(t)$

1. grows generically
2. measures locally accessible dynamically stored information
3. implements idea: *"Passage of time = increase of amount of records"*

Spontaneous Geometrogenesis (no frame or time parametrization in model)

Abundance of Kepler pairs forms in late evolution

1. Kepler's third law becomes increasingly accurate
   $\Rightarrow$ local clocks emerge spontaneously
2. aphelion (or perihelion) distance become more and more stable
   $\Rightarrow$ local rods emerge spontaneously

- clocks evolve more and more isochronous
- rods preserve their relative length with increasing accuracy

Newtonian spacetime emerges spontaneously from shape evolution

dynamical law (not initial condition) $\Rightarrow$ every generic solution spontaneously generates global Newtonian spacetime
(as deduced from locally available clocks and rods)

additionally:

Emergent scale is felt as "friction" by evolving shape degrees of freedom

Summary

1. 2+1 path integral can motivate conformal evolution as fundamental description of gravity (= "Shape Dynamics")
2. Conformal evolution is locally (i.e. in a neighborhood) indistinguishable from general relativity and spacetime
3. Global differences between conformal evolution and spacetime
   $\Rightarrow$ abolish spacetime if conformal evolution turns out fundamental
4. Spacetime emerges spontaneously in Newtonian limit:
   1. "one past - two futures" around minimal expansion
   2. arrow of time emerges as growths of complexity and information
   3. local clocks and rods emerge and "pretend" spacetime

   generic consequence of the dynamical law, **not** the initial condition

$\Rightarrow$ Which laws lead to dynamical emergence of spacetime?

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