# Black Hole Perturbations with Backreaction

## Symposium on Non-singular Black Holes


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arXiv: 2404.18956, 2404.18958, 2404.18959, 2405.01430


ILQGS, February 4th 2025

**Big open question of black hole physics**:

## What is the fate of evaporating black holes?

Many ideas proposed in the literature:

- black hole explosions [Barrau, Hawking, Rovelli, ...]
- black hole remnants [Bianchi, Christodoulou, D'Ambrosio, Haggard, Rovelli, Vidotto, ...]
- black hole white hole transition [Bianchi, Dona, Haggard, Han, Qu, Rovelli, Speziale, Zhang ...]
- ...

$\Rightarrow$ Need to work towards development of a rigorous and first-principle quantum gravity calculation

# Motivation

In Loop Quantum Gravity (LQG) black holes are extensively studied:
[Agullo, Ashtekar, Bodendorfer, Corichi, Engle, Gambini, Giesel, Han, Haggard, Husain, Lewandowski, Liu, Mena Marugán, Olmedo, Perez, Pullin, Rovelli, Singh, Speziale, Vidotto, Weigl, Wilson-Ewing, Zhang, ...] (see also talks by Francesco and Michal)

**Idea**: Use symmetry reduction (classical restriction to spherical symmetric degrees of freedom)
**Main Focus**: Singularity avoidance using

- Tools of Loop Quantum Cosmology (LQC) (Kantowski-Sachs cosmology)
- Modified classical equations of motion and analysis of dust collapse scenarios [Fazzini, Giesel, Husain, Kelly, Liu, Santacruz, Weigl, Wilson-Ewing, ...]
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In this talk: Consistent framework for black hole perturbation theory including backreaction

- Full phase space of general relativity (+ electromagnetic matter)
- Exact treatment of the symmetric sector and perturbative analysis of non-symmetric degrees of freedom
- Backreaction effects: Interaction between non-symmetric and symmetric variables
- Reduced phase space approach to obtain well-defined quantum theory without anomalies
- Applications to investigation of Hawking evaporation process

Starting point: Canonical formulation of General Relativity [JN & TT]

1. Consider spherical symmetry (rotation group $SO(3)$):

- Angular momentum radiated away faster than mass [Page 1976] $\rightarrow$ good approximation
- Canonical variables: induced metric $m$ and conjugate momentum $W$
- Expand into scalar, vector, tensor spherical harmonics $L_{lm}$ [Thorne 1980; Freeden, Gervens, Schreiner 1993]
- $l = 0$ modes: symmetric variables ("background"), notation: $(q, p, Q, P)$
- $l > 0$ modes: non-symmetric variables ("perturbations"), notation: $(x, y, X, Y)$

# Reduced Phase Space Approach to Black Hole Perturbation Theory

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- Decompose full Hamiltonian constraint $V_v$ and diffeomorphism constraints $V_h, V_A$

$$C_{v/h} := \int_{S^2} V_{v/h} \, \mathrm{d}\Omega$$

$$Z^{lm}_{v/h} := \int_{S^2} V_{v/h} L_{lm} \, \mathrm{d}\Omega \, , \quad (l \geq 1)$$

$$Z^{lm}_{e/o} := \int_{S^2} V_A L^A_{e/o,lm} \, \mathrm{d}\Omega \, , \quad (l \geq 1)$$

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2. Split symmetric and non-symmetric variables into observable (true) and non-observable (gauge) degrees of freedom. Notation:

| | **Observable** | **Non-observable** | |
|---|---|---|---|
| **Symmetric** | $(Q, P)$ | $(q, p)$ | "background" |
| **Non-Symmetric** | $(X, Y)$ | $(x, y)$ | "perturbations" |

3. Reduced phase space formulation

- Apply Gullstrand-Painlevé gauge fixing $q = q_*$ and $x = x_*$ (valid for interior and exterior)
- Solve constraints perturbatively: $C = 0$ as $p = p_*(X, Y)$ and $Z = 0$ as $y = y_*(X, Y)$
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4. Boundary term analysis [Regge, Teitelboim 1974]

- Impose asymptotic flat boundary conditions on the fields
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5. Physical Hamiltonian $H$: For any function of the observables $F(Q, P, X, Y)$:

$$\{H, F\} = \{C(f) + Z(g) + B(f, g), F\}_{p=p_*,q=q_*,f=f_*,y=y_*,x=x_*,g=g_*}$$

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An explicit calculation gives:

$$H = M + \sum_{\text{modes } k} \int_{\mathbb{R}^+} \mathrm{d}r \sqrt{\frac{2M}{r}} Y_k \partial_r X^k + \frac{1}{2}\Big(Y_k^2 + (\partial_r X^k)^2 + V_k (X^k)^2\Big),$$

where $V_k$ Regge-Wheeler-Zerilli potentials

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**Remark**:

- Black Hole perturbation theory well established to second order both in Lagrangian [Regge, Wheeler 1957; Zerilli 1970,...] and Hamiltonian formulation [Moncrief 1974; Brizuela, Martín-García 2009,...]
- Agreement of $H$ with these works after transforming from Gullstrand-Painlevé to Schwarzschild coordinates and neglecting backreaction

**Key Advantage**:

- Formalism generalisable to many situations: cosmology, Schwarzschild and Kerr black holes
- Disentangle the definition of observables from perturbation theory
- No need to discuss gauge invariance at every order (no consensus in the literature)
- Hamiltonian $H$ computable in $X, Y$ to any order: $H = H_0 + H_1 + H_2 + \dots$
- Full reduction and derivation of a physical Hamiltonian $\rightarrow$ No constraints in quantum theory
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**Next step:** Quantisation of the observables with respect to Gullstrand Painlevé free-falling observer [work in progress]

- Quantise the perturbations using a Fock quantisation
- Mode functions: eigenvalue equation similar to Schrödinger equation for singular potential
- Possible regularisation at the singularity ($r = 0$):

$\rightarrow$ New orthonormal basis for singular Schrödinger operators [JN & TT]

$\rightarrow$ Methods of LQC type quantisation of Kantowski Sachs [Ashtekar, Bodendorfer, Gambini, Haggard, Olmedo, Pullin, Rovelli, Singh, Vidotto]

$\rightarrow$ Methods of dust collapse models [Fazzini, Giesel, Husain, Kelly, Liu, Santacruz, Weigl, Wilson-Ewing]

**Summary**:
Development of a novel, first-principle approach to black hole evaporation including:

backreaction, exterior and interior, quantised gravity, perturbation-independent definition of observables, expansion to arbitrary orders

# Summary and Outlook

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Development of a novel, first-principle approach to black hole evaporation including:

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**Future Work**:

- Generalisation to higher order perturbations: Interacting gravitational waves [work in progress]
- Study of boundary observables at infinity (BMS group) to constrain quantum theory
- Extension to Standard Model matter (electromagnetic field [JN, Thiemann], neutrinos, ...)
- Generalisation to axial symmetry (Kerr black hole)
- Study quantisation of background and perturbations
- Application of the framework to evaporating black holes

Thank You!