Quantum gravity, probabilities and general boundaries

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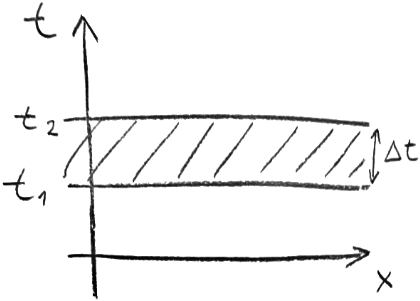
Outline

Interpretational problems in quantum gravity

- The most popular approaches to quantum gravity assume that somehow standard quantum theory holds.
- The "output" of these approaches is usually either:
  - an S-matrix (e.g. string theory)
  - or transition amplitudes (e.g. LQG)
- In QFT such quantities can be directly converted to observable probabilities.
- In QG several problems appear when trying to relate such quantities to observable probabilities. I will focus on transition amplitudes here and consider:
  - the problem of time
  - the quantum cosmology problem

The problem of time (I)

Consider transition amplitudes in a background spacetime.



We prepare a state ψ at t_1, wait for a time Δt, then measure if we obtain the state η at t_2. The probability for this depends on Δt:

$$P = |\langle\eta|U(\Delta t)|\psi\rangle|^2$$

Recall properties of U:

- Composition: $U(\Delta t)U(\Delta t') = U(\Delta t + \Delta t')$
- Unitarity: $U^\dagger = U^{-1}$

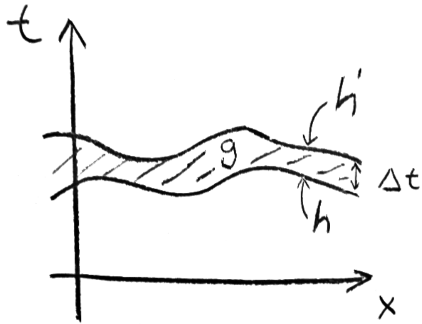
The problem of time (II)

In quantum gravity no background time is available on which the "evolution operator" U can depend. By composition, we expect $U^2 = U$. Then unitarity yields $U = 1$. The transition probability from ψ to η should hence be merely the inner product:

$$P = |\langle \eta | \psi \rangle|^2$$

But, it is then unclear how the operational information of the time difference Δt can be encoded into the expression for the transition probability.

The problem of time (III)



(Semi)classical regime:

Thick sandwich conjecture

Given an initial 3-metric h and a (similar) final 3-metric h' there is generically one interpolating 4-metric g (up to equivalence).

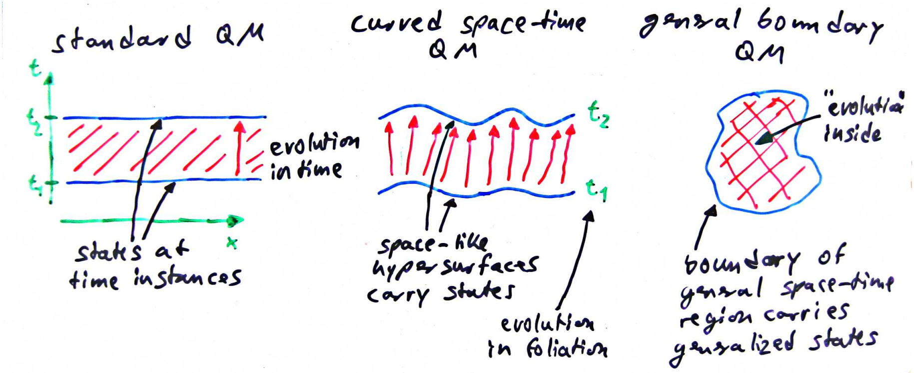
Let ψ_h and $\psi_{h'}$ be semiclassical states associated to h and h'. The tentative transition probability $P = |\langle\psi_{h'}|\psi_h\rangle|^2$ contains the information about Δt in the states.

But, we should have $|\langle\psi_{h'}|\psi_h\rangle|^2 = 1$ and also $|\langle\psi_h|\psi_h\rangle|^2 = 1$. Hence $\psi_h = \psi_{h'}$ up to a phase. However, h and h' are generally physically different states (not related by a 3-diffeomorphism).

The quantum cosmology problem

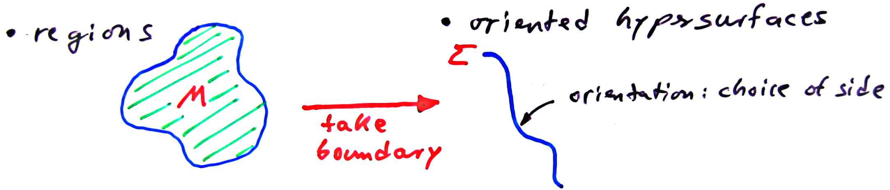
- States are associated with spacelike hypersurface. These extend over the whole universe. Hence a state is a priori a state of the universe.
- In quantum field theory we can avoid talking about the hole universe by noticing that distant systems (with respect to the background metric) are independent (causality, cluster decomposition etc.). We can thus successfully describe a local system as if it was alone in a Minkowski universe.
- In quantum gravity there is no background to separate systems. Worse, diffeomorphism symmetry makes any kind of localization difficult. Hence, we need to worry about the global structure of space(time) and it seems hard to avoid having to do quantum cosmology.

General boundary formulation: Basic idea



Basic structures

Basic spacetime structures:



Basic algebraic structures:

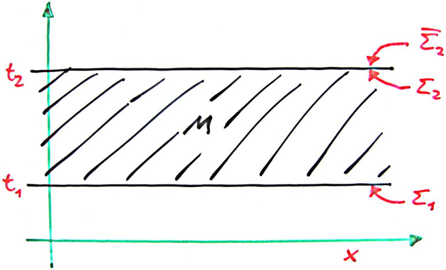
- To each hypersurface Σ associate a Hilbert space $\mathcal{H}_\Sigma$ of states.
- To each region M with boundary Σ associate a linear amplitude map $\rho_M : \mathcal{H}_\Sigma \to \mathbb{C}$.

The structures are subject to a number of rules. For example:

- $\overline{\Sigma}$ is Σ with opposite orientation. Then $\mathcal{H}_{\overline{\Sigma}} = \mathcal{H}_\Sigma^*$.
- $\Sigma = \Sigma_1 \cup \Sigma_2$ is a disjoint union of hypersurfaces. Then $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\Sigma_2}$.

Recovering standard quantum mechanics

Consider the geometry of a standard transition amplitude.



- region: $M = [t_1, t_2] \times \mathbb{R}^3$
- boundary: $\partial M = \Sigma_1 \cup \overline{\Sigma}_2$
- state space: $\mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\overline{\Sigma}_2} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}^*_{\Sigma_2}$

- Via time-translation symmetry identify $\mathcal{H}_{\Sigma_1} \cong \mathcal{H}_{\Sigma_2} \cong \mathcal{H}$, where $\mathcal{H}$ is the state space of standard quantum mechanics.
- Write the amplitude map as $\rho_M : \mathcal{H} \otimes \mathcal{H}^* \to \mathbb{C}$.
- The relation to the standard amplitude is:

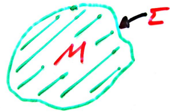
$$\rho_M(\psi \otimes \eta) = \langle \eta | U(t_2 - t_1) | \psi \rangle$$

The need for a generalized interpretation

- So far we have only extended the technical part of quantum theory. The physically interpretable quantities are the same as before. This alone is not sufficient to address the conceptual problems.
- We want to use local regions of spacetime to address the quantum cosmology problem. Hence, we need a physical interpretation of amplitudes for local regions.
- In general, the boundary state space does not split into a tensor product of initial and final state spaces. Hence, we cannot apply the standard probability formula, but need something new.
- This will also allow us to avoid the problem of time.

Generalized probability interpretation

Consider the context of a general spacetime region M with boundary Σ.



Probabilities in quantum theory are generally conditional probabilities. They depend on two pieces of information. Here these are:

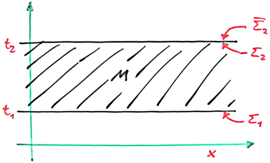
- $\mathcal{S} \subset \mathcal{H}_\Sigma$ representing preparation or knowledge
- $\mathcal{A} \subset \mathcal{H}_\Sigma$ representing observation or the question

The probability that the system is described by $\mathcal{A}$ given that it is described by $\mathcal{S}$ is:

$$P(\mathcal{A}|\mathcal{S}) = \frac{|\rho_M \circ P_\mathcal{S} \circ P_\mathcal{A}|^2}{|\rho_M \circ P_\mathcal{S}|^2}$$

- $P_\mathcal{S}$ and $P_\mathcal{A}$ are the orthogonal projectors onto the subspaces.
- Let $\alpha : \mathcal{H} \to \mathbb{C}$ be a bounded linear map. Then there exists $\xi \in \mathcal{H}$ such that $\alpha(\psi) = \langle \xi, \psi \rangle \quad \forall \psi \in \mathcal{H}$. Define $|\alpha| := |\xi|$.

Recovering standard probabilities



Recall the geometry for standard transition amplitudes with $\mathcal{H}_{\partial M} = \mathcal{H} \otimes \mathcal{H}^*$ and $\rho_M(\psi \otimes \eta) = \langle \eta | U(t_2 - t_1) | \psi \rangle$.

We want to compute the probability of measuring η at t_2 given that we prepared ψ at t_1. This is encoded via

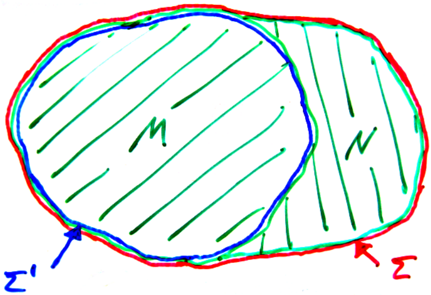
$$\mathcal{S} = \psi \otimes \mathcal{H}^*, \quad \mathcal{A} = \mathcal{H} \otimes \eta.$$

The resulting expression yields correctly

$$P(\mathcal{A}|\mathcal{S}) = \frac{|\rho_M \circ P_{\mathcal{S}} \circ P_{\mathcal{A}}|^2}{|\rho_M \circ P_{\mathcal{S}}|^2} = \frac{|\rho_M(\psi \otimes \eta)|^2}{1} = |\langle \eta | U(t_2 - t_1) | \psi \rangle|^2.$$

Probability conservation

Probability conservation in time is generalized to probability conservation in spacetime.

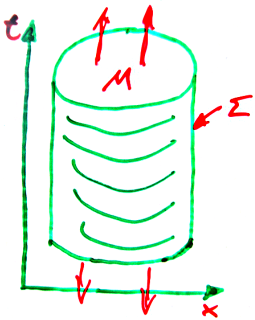


Consider a region M and a region N “deforming” it. Call Σ the boundary of $M \cup N$ and Σ' the boundary of M.

- The amplitude map for N induces a unitary map $\tilde{\rho} : \mathcal{H}_\Sigma \to \mathcal{H}_{\Sigma'}$.
- Let $\mathcal{S} \subset \mathcal{H}_\Sigma$ and $\mathcal{A} \subset \mathcal{H}_\Sigma$. Define $\mathcal{S}' := \tilde{\rho}(\mathcal{S})$ and $\mathcal{A}' := \tilde{\rho}(\mathcal{A})$.
- Then, probability is conserved, $P(\mathcal{A}|\mathcal{S}) = P(\mathcal{A}'|\mathcal{S}')$.

Scattering on a hypercylinder in QFT (I)

To go beyond the realm of standard quantum theory, we consider an example with a connected boundary.



Consider a region with the shape of a solid hypercylinder in Minkowski space, $M = \mathbb{R} \times B^3$. Its boundary is $\Sigma = \partial M = \mathbb{R} \times S^2$. In Klein-Gordon theory the state space $\mathcal{H}_\Sigma$ turns out to be a Fock space. Particles can be characterized by energy-momentum quantum numbers. Unusually, these also determine if a particle is in- or out-going.

We want to describe a 1-1 scattering process. Say a particle with quantum numbers p_{in} goes in and we want to test if a particle with quantum numbers q_{out} comes out.

Scattering on a hypercylinder in QFT (II)

To obtain a probability, we need to specify:

- The knowledge about the experiment is that exactly one particle p_{in} goes in, but we don't know what comes out. $\mathcal{S}$ is the subspace of $\mathcal{H}_\Sigma$ of states with this property.
- The question about the experiment is if one particle q_{out} comes out, while we do not care about what goes in. $\mathcal{A}$ is the subspace of $\mathcal{H}_\Sigma$ of states with this property.

We get

$$P(\mathcal{A}|\mathcal{S}) = |\rho_M(|p_{\text{in}}, q_{\text{out}}\rangle)|^2,$$

where $|p_{\text{in}}, q_{\text{out}}\rangle$ is the state with one in-particle p_{in} and one out-particle q_{out}. As we should expect in a free theory this probability is a kind of delta function $\delta(p - q)$.

On graviton scattering in quantum gravity

To see how the problem of time is avoided we consider a graviton scattering problem.

Consider a ball shaped region in spacetime, $M = B^4$, with boundary $\Sigma = \partial M = S^3$. Suppose the state space $\mathcal{H}_\Sigma$ has a sector $\mathcal{H}_{\text{lin}}$ that (approximately) describes gravitons on Minkowski spacetime, i.e., $\mathcal{H}_\Sigma = \mathcal{H}_{\text{lin}} \oplus \mathcal{H}_{\text{rest}}$.

Again, we want to describe a 1-1 scattering process, with p_{in} the in-particle (prepared) and q_{out} the out-particle (to be measured). The subspaces $\mathcal{S}$ and $\mathcal{A}$ are set up in analogy to the previous example. However, they are now subspaces not only of $\mathcal{H}_\Sigma$ but of $\mathcal{H}_{\text{lin}}$. Supposing that the dynamics in $\mathcal{H}_{\text{lin}}$ is near to that of a free field theory we would get similarly as before:

$$P(\mathcal{A}|\mathcal{S}) \sim |\rho_M(|p_{\text{in}}, q_{\text{out}}\rangle)|^2.$$

Graviton scattering and the problem of time

- This result cannot be obtained as a standard transition probability.
- The key to avoiding the problem of time is that the subspace $\mathcal{S}$ fixes the time "Δt".
- We could apply the same reasoning to a region with spacelike boundaries. Then, crucially, $\mathcal{S}$ and $\mathcal{A}$ would not factorize as subspaces of $\mathcal{H}_\Sigma = \mathcal{H}_{\text{initial}} \otimes \mathcal{H}_{\text{final}}$. Neither would the decomposition $\mathcal{H}_\Sigma = \mathcal{H}_{\text{lin}} \oplus \mathcal{H}_{\text{rest}}$.
- The fact that we consider a region of spacetime with connected boundary is useful (to avoid using the Thick Sandwich conjecture) but not essential to avoiding the problem of time.

Relation to recent LQG/SF calculations

- Using creation and annihilation operators in $\mathcal{H}_{\text{lin}}$ on a "Minkowski vacuum state" ψ_0 we get

  $$P(\mathcal{A}|\mathcal{S}) \sim |\rho_M(a^\dagger(p_{\text{in}})a^\dagger(q_{\text{out}})\psi_0)|^2 = |\rho_M(\phi(p_{\text{in}})\phi(q_{\text{out}})\psi_0)|^2$$

  In this sense the recent computations of a graviton propagator in LQG/SF could be interpreted as yielding 1-1 graviton scattering probabilities.
- It is important to remember that $|\rho_M(\psi)|^2$ for some ψ does not in general have the interpretation of a probability. This is true above only due to special circumstances and only approximately.
- The details will depend on how exactly we choose $\mathcal{H}_{\text{lin}}$ in $\mathcal{H}_\Sigma$, in which way it approximates a Fock space, up to which energies, etc. These ambiguities might be related to the renormalization ambiguities of perturbative quantum gravity.

Conclusions

- The probability formula enables us to encode the information about "Δt", thus avoiding the problem of time.
- State spaces, amplitudes and probabilities for local regions in spacetime allow us to describe their physics independent of the physics outside, thus avoiding the quantum cosmology problem. We do not need to invoke such principles as causality, cluster decomposition etc.
- The generalized probability formula may be used to interpret recent computations of a graviton propagator from LQG/SF in terms of scattering probabilities.

Outlook

- The general boundary formulation is still work in progress, develop by application to known physics (QFT):
  - more general QFTs and more general geometries
  - derive the S-matrix of QFT
  - develop suitable quantization prescriptions
- Application to LQG/SF:
  - cast spin foam models in general boundary form
  - LQG is based on spacelike hypersurfaces, generalize this
- Apply to other quantum gravity approaches (e.g. string theory).
- Develop understanding of non-standard probabilities, especially in non-perturbative contexts.
- Understand implications for interpretational issues, e.g. “collapse of the wavefunction”.

References on the general boundary formulation I

- Initial ideas, partial formalism:
  - R. O., *Schrödinger's cat and the clock: Lessons for quantum gravity*, CQG **20** (2003) 5371-5380, gr-qc/0306007.
  - R. O., *A "general boundary" formulation for quantum mechanics and quantum gravity*, PLB **575** (2003) 318-324, hep-th/0306025.
  - F. Conrady, L. Doplicher, R. O., C. Rovelli, M. Testa, *Minkowski vacuum in background independent quantum gravity*, PRD **69** (2004) 064019, gr-qc/0307118.
- Probability interpretation, formalism:
  - R. O., *General boundary quantum field theory: Foundations and probability interpretation*, hep-th/0509122.
- Application to QFT:
  - R. O., *States on timelike hypersurfaces in quantum field theory*, PLB **622** (2005) 172-177, hep-th/0505267.
  - R. O., *General boundary quantum field theory: Timelike hypersurfaces in Klein-Gordon theory*, PRD **73** (2006) 065017, hep-th/0509123.

References on the general boundary formulation II

- R. O., *Two-dimensional quantum Yang-Mills theory with corners*, hep-th/0608218.

- Generalized Tomonaga-Schwinger quantization:
  - F. Conrady, C. Rovelli, *Generalized Schroedinger equation in Euclidean field theory*, IJMPA **19** (2004) 4037-4068, hep-th/0310246.
  - L. Doplicher, *Generalized Tomonaga-Schwinger equation from the Hadamard formula*, PRD **70** (2004) 064037, gr-qc/0405006.
- LQG/SF graviton propagator:
  - $\longrightarrow$ see recent talks by C. Rovelli and S. Speziale and references therein.