Quantum Extension of Kruskal Black Holes

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Motivation

- Symmetry reduced models (FRW, Bianchi, Schwarzschild, Kerr, ...) allow us to carry out **concrete calculations** (local and global properties of the spacetime geometry, primordial power spectrum, Hawking radiation and black hole evaporation, ...).

- Within loop quantum gravity, it takes lots of effort to obtain similar reduced results ... and efforts have begun (Alesci, Cianfrani, Dapor, Liegener, Pawłowski, ...).

- The (loop) quantization of **symmetry reduced models** of GR is simpler and has **provided very useful lessons** about the physics and mathematics applicable into the full theory (singularity resolution, semiclassical sectors, potential predictions, ...).

- The most fruitful example is loop quantum cosmology. **Rigorous quantization of** various **cosmological spacetimes** based on a difference (discrete) quantum Hamiltonian operator.

Motivation

- Spherically symmetric spacetimes in vacuum provide **the simplest models for black holes**.
- They have important applications in quantum gravity. For instance:
  - **Singularity resolution** and its role in the understanding of **information loss paradox** (LQG viewpoint).
  - Other approaches argue that BH singularities may persist.
- A lot of literature in the past years focused on the application of LQC ideas (based on the analogs of μ_o and $\bar{\mu}$ schemes) to the interior of black holes. But detailed predictions have physically undesirable features.
- Other alternatives (adopting an Abelian constraint) need to be further explored (Bojowald, Brahma, Campiglia, Corichi, Gambini, O, Pullin, Saeed, ...).

The model

- Main ideas:
  - We focus on spacetimes with **spherical symmetry in vacuum** (black holes) using **homogeneous slicings** (trivial constraint algebra) and adopt **loop quantum cosmology techniques**.
  - We limit ourselves to **effective descriptions** (classical evolution equations modified to incorporate quantum geometry corrections).
- What is new:
  - Extension to the **exterior region**.
  - **Judicious choices of plaquettes** to define the curvature operator in the Hamiltonian constraint (in the spirit of *improved dynamics proposals*).
- Consequences: We provide the **quantum** extension of the **macroscopic Kruskal black holes**. The **singular regions** are **replaced by regular transition surfaces** that separate trapped and anti-trapped regions. There, **curvature reaches universal upper bounds**. **Away from the Planck regime**, the **spacetime metric** (and then the curvature) is **well approximated by the classical theory**. **ADM masses are the same.**

Kinematical setting

- Topology of homogeneous Cauchy slices is $\mathbb{R} \times \mathbb{S}^2$. The fiducial metric $\mathring{q}_{ab}$ on Σ is
$$\mathring{q}_{ab}\mathrm{d}x^a\mathrm{d}x^b = \mathrm{d}x^2 + r_o^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2), \quad x \in (-\infty, \infty), \quad r_o = \text{const.} \tag{1}$$
- Reduced connection and triad can be expressed as
$$A_a^i\,\tau_i\,\mathrm{d}x^a = \bar{c}\,\tau_3\,\mathrm{d}x + \bar{b}\,r_o\,\tau_2\mathrm{d}\theta - \bar{b}\,r_o\,\tau_1\sin\theta\,\mathrm{d}\phi + \tau_3\cos\theta\,\mathrm{d}\phi, \tag{2}$$
$$E_i^a\,\tau^i\partial_a = \bar{p}_c\,r_o^2\,\tau_3\,\sin\theta\,\partial_x + \bar{p}_b\,r_o\,\tau_2\,\sin\theta\,\partial_\theta - \bar{p}_b\,r_o\,\tau_1\,\partial_\phi. \tag{3}$$
- The symplectic structure (fiducial cell $x \in [0, L_o]$) takes the form
$$\bar{\Omega} = \frac{L_o r_o^2}{2G\gamma}\left(2\mathrm{d}\bar{b}\wedge\mathrm{d}\bar{p}_b + \mathrm{d}\bar{c}\wedge\mathrm{d}\bar{p}_c\right). \tag{4}$$
- New suitable variables $b = r_o\bar{b}$, $c = L_o\bar{c}$, $p_b = L_o r_o\bar{p}_b$ and $p_c = r_o^2\bar{p}_c$ satisfying the Poisson brackets:
$$\{c,\, p_c\} = 2G\gamma, \quad \{b,\, p_b\} = G\gamma. \tag{5}$$
These variables, under the transformation $r_o \to \beta r_o$, are invariant. Under a rescaling of fiducial length L_o the combinations c/L_o and p_b/L_o, and b, p_c are invariant.
- The spacetime metric is
$$g_{ab}\mathrm{d}x^a\mathrm{d}x^b \equiv \mathrm{d}s^2 = -N_t^2\mathrm{d}t^2 + \frac{p_b^2}{p_c L_o^2}\mathrm{d}x^2 + p_c(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2), \tag{6}$$

Classical dynamics of the interior region

- **Classical dynamics in the interior** dictated by the Hamiltonian constraint

$$H_{\rm cl}[N_{\rm cl}] = -\frac{1}{2G\gamma}\left(2c\,p_c + \left(b + \frac{\gamma^2}{b}\right)p_b\right), \quad N_{\rm cl} = \gamma\, b^{-1}\, p_c^{1/2}. \tag{7}$$

- The solutions to the dynamical equations and Hamiltonian constraint (well adapted to Schwarzschild geometry) are

$$b(T_{\rm cl}) = \gamma\,\left(e^{-T_{\rm cl}} - 1\right)^{1/2}, \quad p_b(T_{\rm cl}) = -2mL_0\, e^{T_{\rm cl}}\left(e^{-T_{\rm cl}} - 1\right)^{1/2}, \tag{8}$$

$$c(T_{\rm cl}) = \tfrac{\gamma L_o}{4m}\, e^{-2T_{\rm cl}}, \qquad p_c(T_{\rm cl}) = 4m^2\, e^{2T_{\rm cl}}, \tag{9}$$

where $-\infty < T_{\rm cl} \leq 0$ and

$$\frac{cp_c}{L_o\gamma} = m = -\frac{1}{2L_o\gamma}\left(b + \frac{\gamma^2}{b}\right)p_b. \tag{10}$$

- With $\tau = 2me^{T_{\rm cl}}$ we get

$${\rm d}s^2 = -\left(\frac{2m}{\tau} - 1\right)^{-1}{\rm d}\tau^2 + \left(\frac{2m}{\tau} - 1\right){\rm d}x^2 + \tau^2({\rm d}\theta^2 + \sin^2\theta{\rm d}\phi^2). \tag{11}$$

- **Singularity located at $\tau = 0$ (or $T_{\rm cl} = -\infty$). Horizon located at $\tau = 2m$ (or $T_{\rm cl} = 0$).**

Extension to the exterior

- We keep a homogeneous slicing, i.e. time-like hypersurfaces with $(-++)$ 3-metric.
- The connection and triad are now $SU(1,1)$ valued (instead of $SU(2)$).

$$A_a^i\,\tilde{\tau}_i\,\mathrm{d}x^a \;=\; \frac{\tilde{c}}{L_o}\,\tilde{\tau}_3\,\mathrm{d}x + \tilde{b}\,\tilde{\tau}_2\mathrm{d}\theta - \tilde{b}\,\tilde{\tau}_1\sin\theta\,\mathrm{d}\phi + \tilde{\tau}_3\cos\theta\,\mathrm{d}\phi, \tag{12}$$

$$E_i^a\,\tilde{\tau}^i\partial_a \;=\; \tilde{p}_c\,\tilde{\tau}_3\,\sin\theta\,\partial_x + \frac{\tilde{p}_b}{L_o}\,\tilde{\tau}_2\,\sin\theta\,\partial_\theta - \frac{\tilde{p}_b}{L_o}\,\tilde{\tau}_1\,\partial_\phi, \tag{13}$$

or equivalently:

$$\tilde{\tau}_1 \to i\tau_1,\; \tilde{\tau}_2 \to i\tau_2,\; \tilde{\tau}_3 \to \tau_3; \qquad b \to i\tilde{b},\; p_b \to i\tilde{p}_b; \qquad c \to \tilde{c},\; p_c \to \tilde{p}_c \tag{14}$$

- The Poisson brackets are now given by:

$$\{\tilde{c},\,\tilde{p}_c\} \;=\; 2G\gamma, \quad \{\tilde{b},\,\tilde{p}_b\} \;=\; -G\gamma. \tag{15}$$

- The spacetime metric is

$$\tilde{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b \equiv \mathrm{d}\tilde{s}^2 = -\frac{\tilde{p}_b^2}{\tilde{p}_c L_o^2}\mathrm{d}x^2 + \tilde{N}_\tau^2\mathrm{d}\tau^2 + \tilde{p}_c(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{16}$$

Classical dynamics of the exterior region

- **Classical dynamics in the exterior** dictated by the Hamiltonian constraint

$$\tilde{H}_{\rm cl}[\tilde{N}_{\rm cl}] = -\frac{1}{2G\gamma}\left(2\tilde{c}\,\tilde{p}_c + \left(-\tilde{b} + \frac{\gamma^2}{\tilde{b}}\right)\tilde{p}_b\right), \quad \tilde{N} = \gamma\tilde{b}^{-1}\tilde{p}_c^{1/2}\,. \tag{17}$$

- Solutions to the dynamical equations and Hamiltonian constraint are given by

$$\tilde{b}(T_{\rm cl}) = \pm\gamma\,\left(1 - e^{-T_{\rm cl}}\right)^{1/2}, \quad \tilde{p}_b(T_{\rm cl}) = -2mL_0\,e^{T_{\rm cl}}\left(1 - e^{-T_{\rm cl}}\right)^{1/2}, \tag{18}$$

$$\tilde{c}(T_{\rm cl}) = \tfrac{\gamma L_o}{4m}\,e^{-2T_{\rm cl}}, \qquad \tilde{p}_c(T_{\rm cl}) = 4m^2\,e^{2T_{\rm cl}}, \tag{19}$$

where $0 \leq T_{\rm cl} < \infty$ and

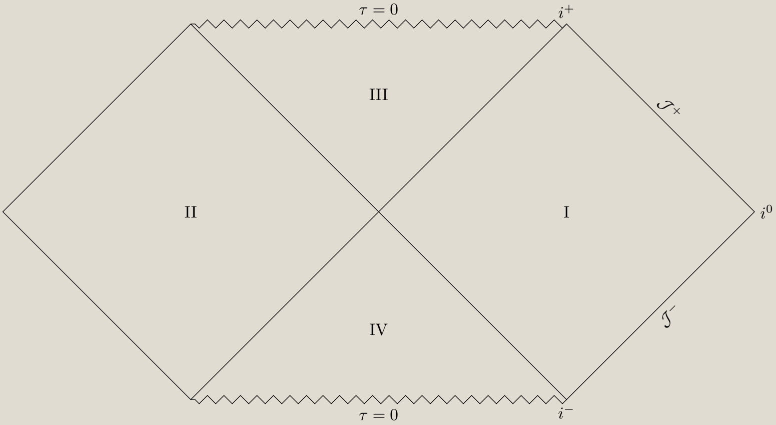
$$\frac{\tilde{c}\tilde{p}_c}{L_o\gamma} = m = -\frac{1}{2L_o\gamma}\left(-\tilde{b} + \frac{\gamma^2}{\tilde{b}}\right)\tilde{p}_b. \tag{20}$$

- The exterior and interior regions match smoothly across the horizon ($T_{\rm cl} = 0$). Spatial infinity is located at $T_{\rm cl} = \infty$.

- With $\tau = 2me^{T_{\rm cl}}$ we get

$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{\tau}\right)\mathrm{d}x^2 + \left(1 - \frac{2m}{\tau}\right)^{-1}\mathrm{d}\tau^2 + \tau^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{21}$$

Classical Penrose diagram of the Kruskal spacetime



Quantum corrected dynamics for the interior

- **Quantum corrections** to the classical theory.
  - i) $H[N]$ contains curvature F^i_{ab} of the gravitational connection A^i_a.
  - ii) F^i_{ab} can be obtained from $(h_\Box/\mathcal{A}_\Box)$, where $h_\Box$ is a loop $\Box$ of holonomies of A^i_a and $\mathcal{A}_\Box$ the area enclosed by $\Box$ (we choose it to be equal to the LQG area gap $\Delta = 4\sqrt{3}\pi\gamma\ell^2_{\rm Pl}$).
  - iii) curvature operators $\hat{F}_{\theta,\phi}$, $\hat{F}_{x,\theta}$, $\hat{F}_{x,\phi}$ are determined by the *fractional lengths* of the links in these plaquettes: δ_c for the x-directional link and δ_b for links in the 2-spheres.
- **LQC effective dynamics** dictated by

$$H_{\rm eff}[N] = -\frac{1}{2G\gamma}\left[2\frac{\sin(\delta_c c)}{\delta_c}\, p_c + \left(\frac{\sin(\delta_b b)}{\delta_b} + \frac{\gamma^2\delta_b}{\sin(\delta_b b)}\right) p_b\right], \quad N = \frac{\gamma\ p_c^{1/2}\ \delta_b}{\sin(\delta_b\, b)}. \tag{22}$$

- The solutions to the dynamical equations, provided that δ_b and δ_c are chosen to be appropriate constants of motion, take the form

$$\cos\left(\delta_b\, b(T)\right) = b_o \tanh\left(\frac{1}{2}\left(b_o T + 2\tanh^{-1}\left(\frac{1}{b_o}\right)\right)\right),\ \ p_b(T) = -2\frac{\sin(\delta_c\, c(T))}{\delta_c}\,\frac{\sin(\delta_b\, b(T))}{\delta_b}\,\frac{p_c(T)}{\frac{\sin^2(\delta_b\, b(T))}{\delta_b^2}+\gamma^2}, \tag{23}$$

$$\tan\left(\frac{\delta_c\, c(T)}{2}\right) = \mp\frac{\gamma L_o\delta_c}{8m}e^{-2T}, \quad p_c(T) = 4m^2\left(e^{2T} + \frac{\gamma^2 L_o^2\delta_c^2}{64m^2}e^{-2T}\right), \quad b_o = (1+\gamma^2\delta_b^2)^{1/2}, \tag{24}$$

where

$$\left[\frac{\sin(\delta_c\, c)}{\gamma L_o\delta_c}\right] p_c = m = -\frac{1}{2}\left[\frac{\sin(\delta_b\, b)}{\delta_b} + \frac{\gamma^2\delta_b}{\sin(\delta_b\, b)}\right]\frac{p_b}{\gamma L_o} \tag{25}$$

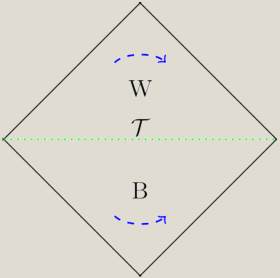
Causal structure

- The spacetime has the Killing vector field $X^a\partial_a = \partial/\partial_x$ that becomes null when $p_b = 0$ (horizons at $T = 0$ and $T = -4/b_o \tanh^{-1}(1/b_o)$).

- The expansions $\theta_\pm$ of the null normal vectors $\ell_a^\pm$ to the 2-spheres vanish at $\dot{p}_c = 0$, i.e. at $T_{\mathcal{T}} = \frac{1}{2}\ln\left(\frac{\gamma L_o \delta_c}{8m}\right)$. Each solution has one and only one transition surface $\mathcal{T}$ that separates a trapped region in the past (BH region) and an anti-trapped region to the future (WH region).

- The entire geometry is smooth since the transition surface $\mathcal{T}$ replaces the classical singularity by a regular high-curvature region. Here $p_c(T_{\mathcal{T}}) = \frac{1}{2}\gamma\,(L_o\delta_c)$.

- In summary, the geometry consists of a boundary $\mathcal{T}$ between a trapped region (BH type) in the past and an anti-trapped region (WH type) to the future.



Determination of δ_b and δ_c

- The quantum parameter δ_b has the interpretation of the fractional length of each link constituting the plaquette within the θ-ϕ 2-spheres, and δ_c, of the fractional length of the links in the x-direction within the plaquettes in the θ-x and ϕ-x planes in a fiducial cell.

- Our proposal: Choose the plaquettes to lie on the transition surface **where the space-time has largest curvature**. One then has:

$$2\pi\, \delta_c \delta_b\, |p_b|\big|_{\mathcal{T}} = \Delta, \qquad (26)$$

(Physical (fractional) area of the annulus around equators)

$$4\pi\, \delta_b^2\, p_c\big|_{\mathcal{T}} = \Delta. \qquad (27)$$

(Physical (fractional) area of the sphere)

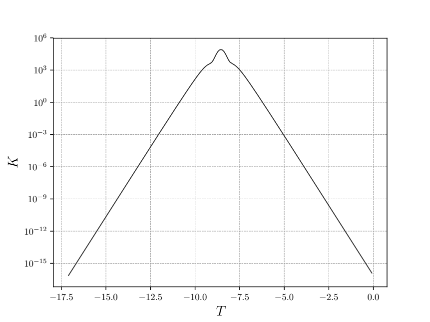
- The solutions can be written in a closed form for macroscopic BHs ($m \gg \ell_{\rm Pl}$):

$$\delta_b = \Big(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m}\Big)^{1/3}, \qquad L_o\delta_c = \frac{1}{2}\Big(\frac{\gamma\Delta^2}{4\pi^2 m}\Big)^{1/3}. \qquad (28)$$

Physical consequences

- Several curvature invariants are bounded above, reaching a maximum value at the transition surface that is mass independent, namely

$$C_{abcd}C^{abcd}\,|_{\mathcal{T}} = \frac{1024\pi^2}{3\gamma^4\Delta^2} + \mathcal{O}\left(\left(\frac{\Delta}{m^2}\right)^{\frac{1}{3}}\,\ln\frac{m^2}{\Delta}\right), \quad K\,|_{\mathcal{T}} = \frac{768\pi^2}{\gamma^4\Delta^2} + \mathcal{O}\left(\left(\frac{\Delta}{m^2}\right)^{\frac{1}{3}}\,\ln\frac{m^2}{\Delta}\right). \quad (29)$$

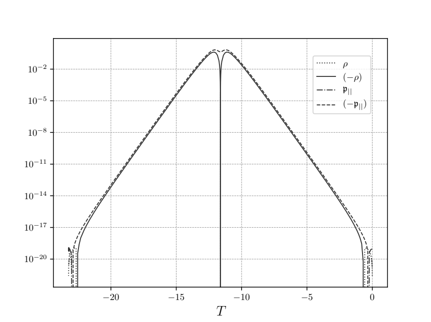


Curvature invariant for $m = 10^6$.

Physical consequences

- Quantum correction can be seen as an effective fluid with a stress-energy tensor $\mathfrak{T}_{ab}$ characterized by the energy density ρ and the radial and tangential pressures $\mathfrak{p}_x$ and $\mathfrak{p}_{||}$, respectively. One can easily see that, at the transition surface, the strong energy condition is violated

$$\left(\mathfrak{T}_{ab} - \frac{1}{2} g_{ab} \mathfrak{T}\right) T^a T^b \Big|_{\mathcal{T}} < 0. \tag{30}$$



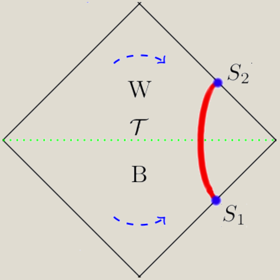
Curvature invariant for $m = 10^6$.

Physical consequences

- A puzzling aspect arises from considerations of the Komar mass at the horizons ($T = 0$ and $T = -4/b_o \tanh^{-1}(1/b_o)$) with respect to the Killing vector field $X = X^a\partial_a = \partial_x$:

$$K[S] := -\frac{1}{8\pi G} \oint_S \epsilon_{ab}{}^{cd} \nabla_c X_d \, \mathrm{d}S^{ab}, \tag{31}$$

$$K[S_2] - K[S_1] = 2 \int_{\bar{\Sigma}} \big(\mathfrak{T}_{ab} - (\mathfrak{T}/2) g_{ab}\big) X^a \mathrm{d}V^b. \tag{32}$$



- The 3-surface integral is large and negative because of properties of $\mathfrak{T}_{ab}$ in the interior. But it is also delicately balanced to make the volume integral precisely equal to $-2K[S_1]$ (in the large m limit). Therefore,

$$K[S_2] = -K[S_1]. \tag{33}$$

- Since ADM mass refers to the unit future pointing asymptotic time-like Killing vector field, this is the sufficient condition for $r_{\mathrm{B}} = r_{\mathrm{W}}$ when $m^2 \gg \Delta$.

- This shows that there are highly non-trivial constraints that our effective geometry satisfies: singularity resolution and (nearly) unit amplification factor for the mass in the transition from the trapped to the anti-trapped region.

Comparison with other proposals

i) Constant μ_0 schemes (physical results depend on fiducial cell)

$$\delta_b \propto \frac{\sqrt{\Delta}}{\ell_{\rm Pl}}, \quad L_o\delta_c \propto \sqrt{\Delta}. \tag{34}$$

ii) Improved dynamics $\bar{\mu}$ schemes (BV scheme)

$$\delta_b \propto \sqrt{\frac{\Delta}{p_c}}, \quad L_o\delta_c \propto \frac{\sqrt{\Delta p_c}}{(p_b/L_o)}. \tag{35}$$

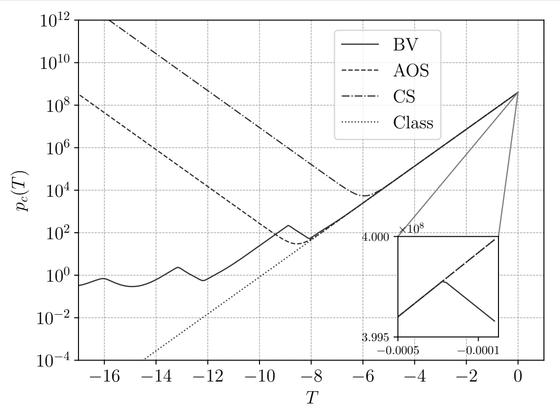
iii) Schemes based on dimensional arguments (CS scheme)

$$\delta_b \propto \frac{\sqrt{\Delta}}{2m}, \quad L_o\delta_c \propto \sqrt{\Delta}. \tag{36}$$

iv) Our proposal (AOS scheme)

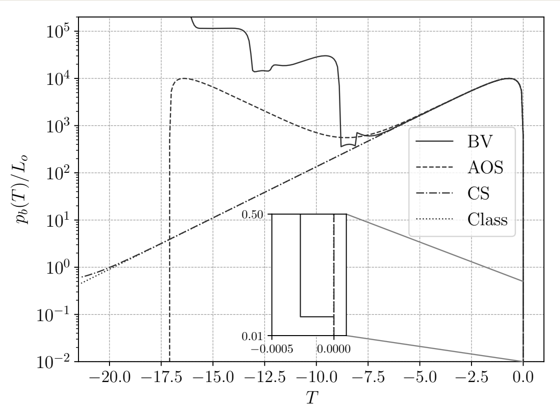
$$\delta_b = \Big(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m}\Big)^{1/3}, \qquad L_o\delta_c = \frac{1}{2}\Big(\frac{\gamma\Delta^2}{4\pi^2 m}\Big)^{1/3}. \tag{37}$$

Comparison with other proposals



Comparison for $m = 10^4$. In the classical theory, p_c decreases steadily corresponding to the monotonic decrease in the radius of the round 2-spheres. In 'CS' and 'AOS', it undergoes precisely one bounce, with a trapped region to the past of the bounce and anti-trapped to the future. In BV, it undergoes several bounces. The anti-trapped region after the first bounce is very short lived. After the second bounce, this $\bar{\mu}$ scheme cannot be trusted because its underlying assumptions are violated.

Comparison with other proposals



Comparison for $m = 10^4$. This triad component signals the emergence of a black or white hole type horizon when it vanishes. The white hole type horizon emerges much much later in the CS approach (large mass amplification in the CS approach) than in AOS one (no amplification in the large m limit). The BV approach becomes unreliable after $T \sim -12$. Besides, very near the black hole type horizon, the BV dynamics deviates from the classical theory. The AOS dynamics is indistinguishable from classical dynamics near this horizon.

Extension to the exterior

- We follow the same ideas discussed in the classical theory (homogeneous slicing and $SU(1,1)$ valued triad and connection)
- **The dynamics in the exterior** is dictated by

$$\tilde{H}_{\rm eff}[\tilde{N}] = -\frac{1}{2G\gamma}\left[2\frac{\sin(\delta_{\tilde{c}}\,\tilde{c})}{\delta_{\tilde{c}}}\,|\tilde{p}_c| + \left(-\frac{\sinh(\delta_{\tilde{b}}\,\tilde{b})}{\delta_{\tilde{b}}} + \frac{\gamma^2\delta_{\tilde{b}}}{\sinh(\delta_{\tilde{b}}\,\tilde{b})}\right)\tilde{p}_b\right], \quad \tilde{N} = \frac{\gamma\tilde{p}_c^{1/2}\,\delta_{\tilde{b}}}{\sinh(\delta_{\tilde{b}}\,\tilde{b})}. \tag{38}$$

- The solutions to the dynamical equations take the form

$$\cosh\left(\delta_{\tilde{b}}\,\tilde{b}(T)\right) = \tilde{b}_o \tanh\left(\frac{1}{2}\left(\tilde{b}_o T + 2\tanh^{-1}\left(\frac{1}{\tilde{b}_o}\right)\right)\right), \quad \tilde{p}_b(T) = -2\,\frac{\sin(\delta_{\tilde{c}}\,\tilde{c}(T))}{\delta_{\tilde{c}}}\,\frac{\sinh(\delta_{\tilde{b}}\,\tilde{b}(T))}{\delta_{\tilde{b}}}\,\frac{\tilde{p}_c(T)}{\gamma^2 - \frac{\sinh^2(\delta_{\tilde{b}}\,\tilde{b}(T))}{\delta_{\tilde{b}}^2}}, \tag{39}$$

$$\tan\left(\frac{\delta_{\tilde{c}}\,\tilde{c}(T)}{2}\right) = \mp\frac{\gamma L_o\delta_{\tilde{c}}}{8m}e^{-2T}, \quad \tilde{p}_c(T) = 4m^2\left(e^{2T} + \frac{\gamma^2 L_o^2\delta_{\tilde{c}}^2}{64m^2}e^{-2T}\right), \quad \tilde{b}_o = (1+\gamma^2\delta_{\tilde{b}}^2)^{1/2}. \tag{40}$$

where

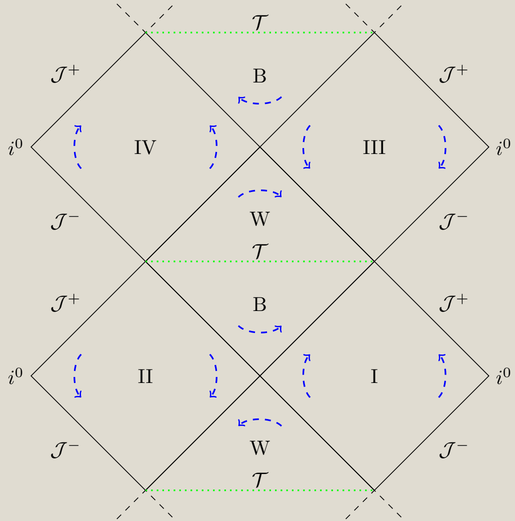
$$\frac{\sin(\delta_{\tilde{c}}\,\tilde{c})}{\gamma L_o\delta_{\tilde{c}}}\tilde{p}_c = m = -\frac{1}{2}\left[-\frac{\sinh(\delta_{\tilde{b}}\,b)}{\delta_{\tilde{b}}} + \frac{\gamma^2\delta_{\tilde{b}}}{\sinh(\delta_{\tilde{b}}\,\tilde{b})}\right]\frac{\tilde{p}_b}{\gamma L_o} \tag{41}$$

- The spacetime metric is

$$\tilde{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b \equiv \mathrm{d}\tilde{s}^2 = -\frac{\tilde{p}_b^2}{\tilde{p}_c L_o^2}\mathrm{d}x^2 + \tilde{N}_\tau^2\mathrm{d}\tau^2 + \tilde{p}_c(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \tag{42}$$

- **The horizon is located at $T = 0$** and the **spatial infinity at $T = +\infty$**. The **exterior and interior** regions match **smoothly across the horizon**.

Quantum extension of the Kruskal diagram



Summary

- We provide a quantum extension (effective description a la LQC) of an eternal black hole with the following properties.
  i) Physical quantities are insensitive to the fiducial structures necessary to construct the classical phase space.
  ii) It admits an infinite number of trapped, anti-trapped and asymptotic regions.
  iii) Consecutive asymptotic regions of macroscopic black holes have same ADM mass (macroscopic BHs).
  iv) High-curvature regions are regular (singularity resolution).
  v) The simplest curvature scalars have mass-independent upper bounds (macroscopic BHs).
  vi) At low curvatures, quantum effects are small.
- None of the previously proposed models so far meet all these properties simultaneously.

Limitations and future directions

- Full quantum dynamics need to be understood:
  i) The quantum Hamiltonian constraint has a complicated action. Fortunately, we have a proposal worth to be explored.
  ii) Classical and quantum Hamiltonian framework have been recently discussed in the literature for $SU(1,1)$ connections in 3+1 gravity. Interesting possibility: Application to our model.
  iii) Then, one could "derive" effective equations using the LQC strategy.
- This is not a collapsing black space-time. But many relevant calculations already use the Kruskal space-time (vacuum states of linear field theories, renormalized stress energy tensor, etc.).
- Extension to more general spacetimes.

Implementation as functions of Dirac observables

- The quantum parameters δ_b and δ_c have been judiciously chosen as Dirac observables. Subtlety: Since H depends on δ_b, δ_c and they have to be so chosen that along dynamical trajectories determined by H, they are constants of motion.
- We have found a way to meet this subtle self-consistency requirement:
  - First extend the phase space by adding δ_b, δ_c as new phase space variables, together with their conjugate momenta.
  - Then restrict δ_b, δ_c to be desired functions of the original phase space variables via new first class constraints.
  - Finally, identify suitable gauge fixing conditions that eliminate those new constraints (via gauge fixing) so that the reduced phase space is naturally isomomorphic to the original one.
- To implement this strategy, it is simplest to replace (b, p_b), (c, p_c) with other canonical coordinates (to get the solutions in closed form).

Implementation as functions of Dirac observables

- Let us consider new coordinates in the phase space Γ_o

$$O_1 := -\frac{1}{2\gamma}\left[\frac{\sin\delta_b b}{\delta_b} + \frac{\gamma^2\delta_b}{\sin\delta_b b}\right]\frac{p_b}{L_o}, \quad O_2 := \left[\frac{\sin\delta_c c}{\gamma L_o \delta_c}\right]p_c. \tag{43}$$

  and suitable conjugated momenta P_i such that $\{O_i, P_j\} = \delta_{ij}$.
- Let us extend the phase space (Γ_o, Ω_o) to (Γ, Ω) where δ_b and δ_c are now phase space variables together with their momenta $\{\delta_b, P_{\delta_b}\} = 1$ and $\{\delta_c, P_{\delta_c}\} = 1$.
- Let us now introduce the desired conditions on δ_b and δ_c as constraints (first class with the Hamiltonian).

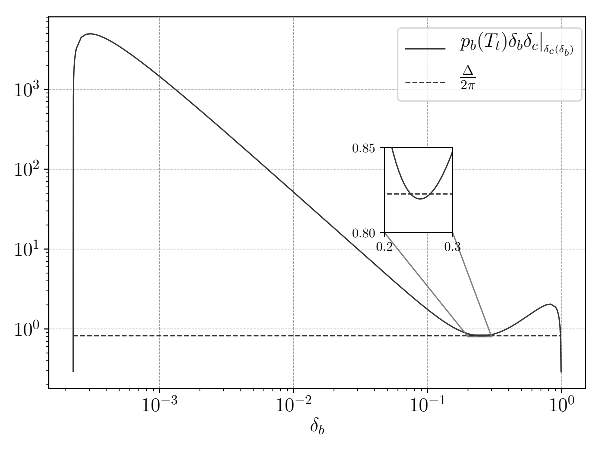
$$\Phi_1 = O_1 - F_b(\delta_b), \quad \Phi_2 = O_2 - F_c(\delta_c). \tag{44}$$

- The total Hamiltonian reads

$$H_T = -\frac{L_o\tilde{N}}{G}(O_2 - O_1) + \lambda_1\Phi_1 + \lambda_2\Phi_2, \tag{45}$$

- We now require gauge fixing conditions such that i) the Hamiltonian vector field is tangential to the constraint - gauge surfaces and ii) the pullback of Ω on those surfaces is symplectomorphic to Ω_o.
- One can see that the gauge fixing conditions that satisfy these requirements are $\phi_1 = P_{\delta_b} = 0$ and $\phi_2 = P_{\delta_c} = 0$.
- The resulting EOMs are equivalent to the ones we use in our approach.

Determination of δ_b and δ_c



The roots of δ_b obtained from solving the area conditions for $m = 10^4$. In the large m limit, the central roots are extremely well approximated by previous equations. The leftmost and rightmost roots turn out to have unphysical properties.