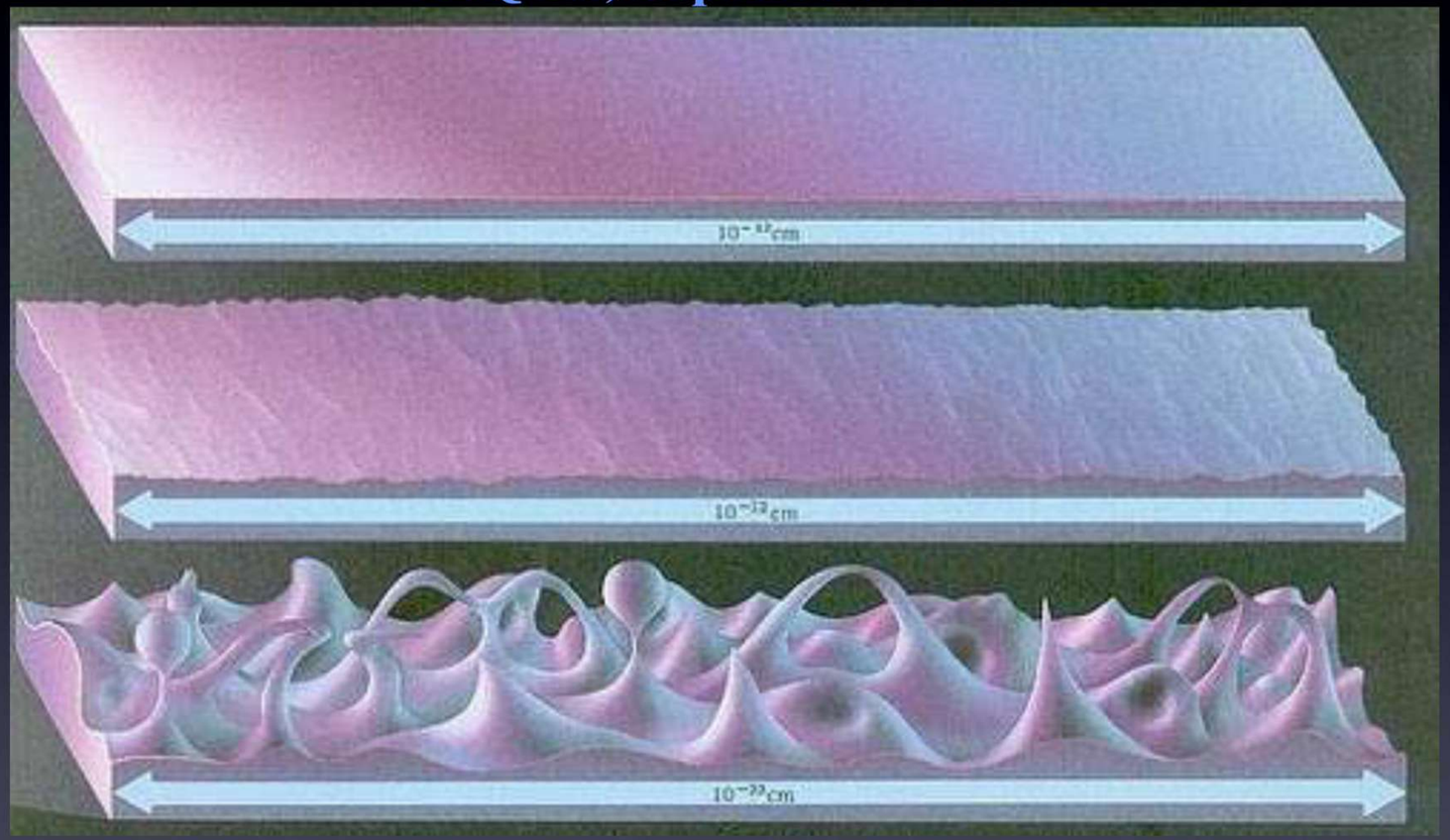
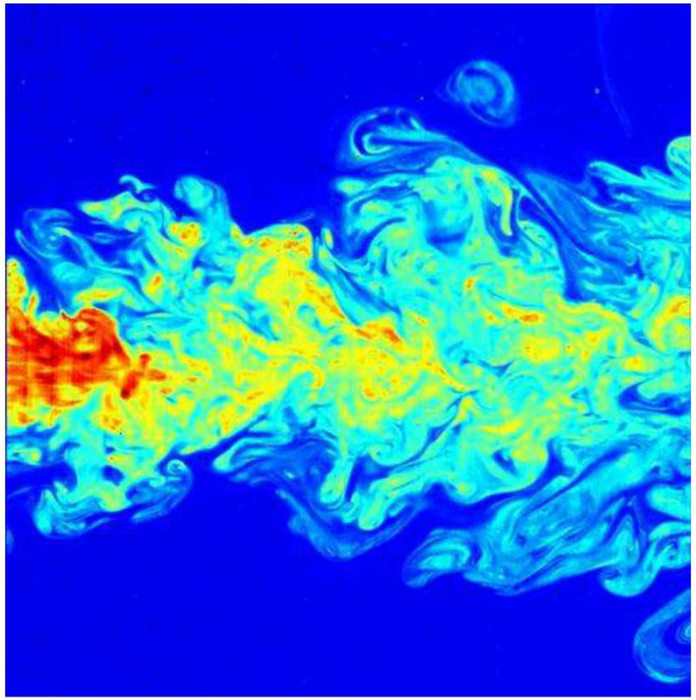
Inflationary phenomenology from Planckian discreteness

ILQGS, September 2022



Alejandro Perez
Centre de Physique Théorique,
Marseille, France.

Mathematical description of fluids
(**Navier-Stokes**)

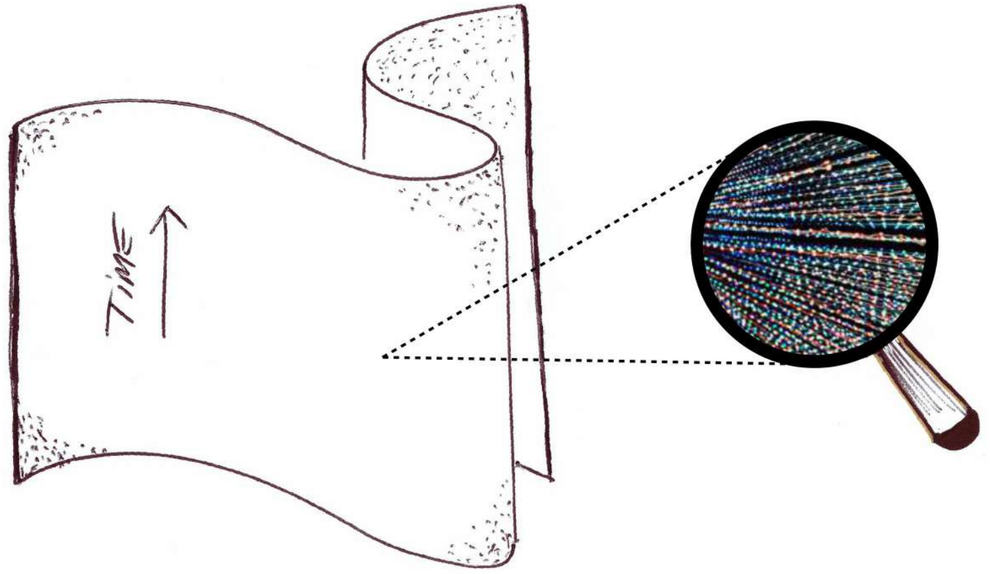


Continuous fluid description breaks down at molecular scales.

Interesting phenomenology:

- Diffusion
- Brownian motion (fluctuation/dissipation)
- ...

Mathematical description of gravity
General Relativity



The continuum spacetime description breaks down at the Planck scale.

Can we produce an interesting phenomenology?

The cosmological constant problem

$$\mathbf{R}_{ab} - \frac{1}{2} g_{ab}\mathbf{R} = 8\pi \mathbf{T}_{ab} - \Lambda g_{ab}$$

$$\Lambda_{\rm obs} \approx 1.19\ 10^{-52}\ {\rm m}^{-2}$$

How does the vacuum gravitate?

$$\langle \mathbf{T}_{ab} \rangle = \frac{\Lambda_{vac}}{8\pi G}\ g_{ab}$$

$$\rho_{vac} \equiv \frac{\Lambda_{vac}}{8\pi G} \approx m_p^4$$

$$\rho_{\Lambda_{obs}} \approx 10^{-120} m_p^4 \approx (10^{-2} eV)^4$$

Three related ideas by Einstein on gravity

1) General Relativity (1916)

$$\mathbf{R}_{ab} - \frac{1}{2} g_{ab}\mathbf{R} = 8\pi \mathbf{T}_{ab}$$

Bianchi Identities

$$\nabla^a \mathbf{T}_{ab} = 0$$

Bianchi Identities

2) The cosmological constant (1917)

$$\mathbf{R}_{ab} - \frac{1}{2} g_{ab}\mathbf{R} = 8\pi \mathbf{T}_{ab} - \Lambda g_{ab}$$

3) Unimodular Gravity (1919)

$$\mathbf{R}_{ab} - \frac{1}{4} g_{ab}\mathbf{R} = 8\pi \left(\mathbf{T}_{ab} - \frac{1}{4} g_{ab}\mathbf{T} \right) \quad \text{with} \quad \nabla^a \mathbf{T}_{ab} = 0$$

Weinberg 1987

$$\langle \mathbf{T}_{ab} \rangle = \frac{\Lambda_{vac}}{8\pi G}\, g_{ab}$$

The vacuum does not gravitate in UG

Three related ideas by Einstein on gravity

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A naturalness problem remains: if initial data determine the cosmological constant why it is not Planck scale?

Unimodular Gravity:

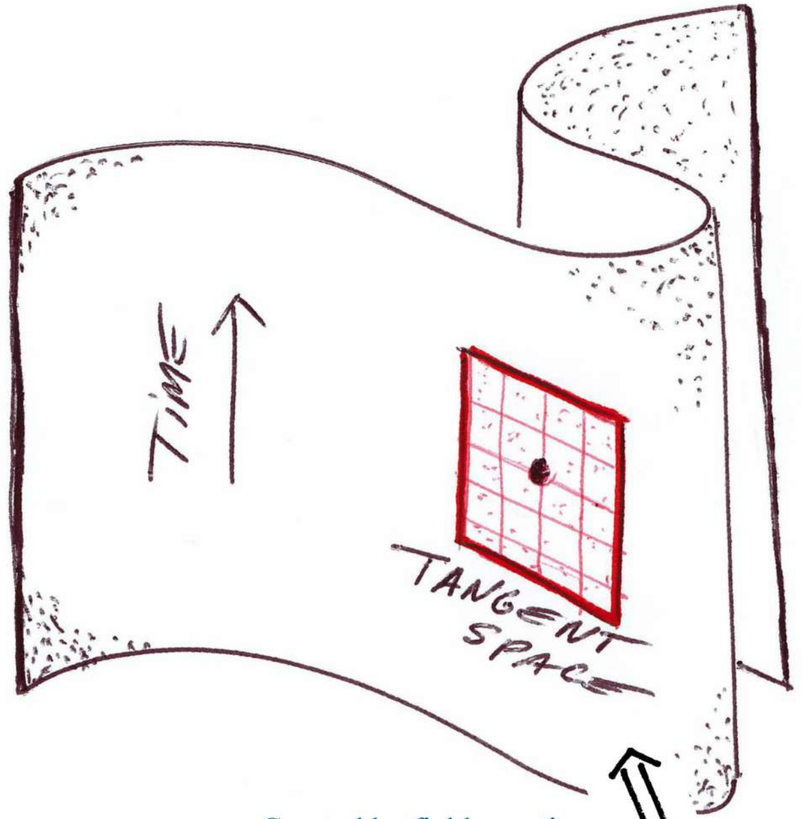
Equivalent to General Relativity:
with a cosmological constant as a constant of integration!

$$\langle \mathbf{T}_{ab} \rangle = \frac{\Lambda_{vac}}{8\pi G}\, g_{ab}$$

The vacuum does not gravitate in UG

$\Longleftarrow$

Conservation of Energy:
symmetries of tangent space



Granted by field equations due to symmetries of the tangent space.

$$\nabla^a \mathbf{T}_{ab} = 0$$

$\Uparrow$

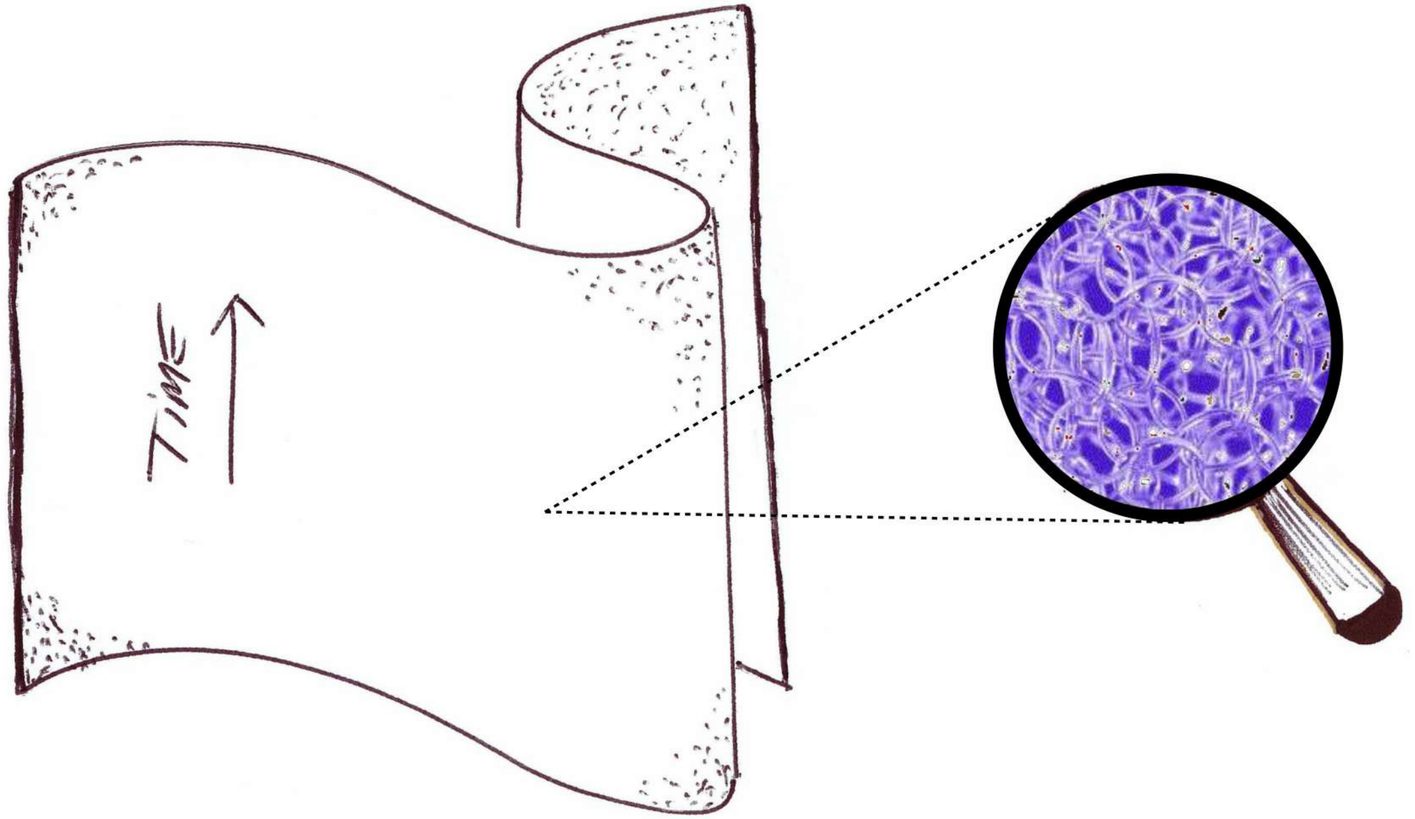
Traceless Einstein's Equations:

$$\mathbf{R}_{ab} - \frac{1}{4} g_{ab}\mathbf{R} = 8\pi \left(\mathbf{T}_{ab} - \frac{1}{4} g_{ab}\mathbf{T} \right)$$

Conservation of Energy: fails if spacetime is not smooth at the Planck scale

Unimodular Gravity

UG is a natural generalization of GR as an open system



Ingredient 1

$$\mathbf{R}_{ab} - \frac{1}{4} g_{ab} \mathbf{R} = 8\pi \left(\mathbf{T}_{ab} - \frac{1}{4} g_{ab} \mathbf{T} \right)$$

Ingredient 2

~~$\nabla^a \mathbf{T}_{ab} = 0$~~

Unimodular Gravity: symmetries

Action principle

$$S = \int \left(\sqrt{g}\,\mathbf{R} + \lambda\left[\sqrt{g} - 1\right]\right) dx^4 + S_m$$

$$\nabla^a \xi_a = 0 \quad \Longleftrightarrow \quad \xi^a = \epsilon^{abdc}\nabla_b \omega_{cd}$$

$$\mathbf{J}_b \equiv (8\pi G)\nabla^a \mathbf{T}_{ab}$$

$$\begin{aligned} 0 &= \delta S_m = \int_M \sqrt{-g}\mathbf{T}_{ab}\nabla^a \xi^b dx^4 \\ &= -\int_M \sqrt{-g}\nabla^a \mathbf{T}_{ab}\xi^b dx^4 = \int_M \sqrt{-g}\nabla_c(\nabla^a \mathbf{T}_{ab}\epsilon^{bcde})\omega_{de} dx^4 \end{aligned}$$

$$d\mathbf{J} = 0 \quad \Longleftrightarrow \quad \mathbf{J}_a = \nabla_a Q$$

Anderson-Finkelstein American Journal of Physics (1971)

Unimodular gravity should emerge as effective description as Planckian granularity breaks SCALE INVARIANCE

$$\delta g_{ab} = 2\nabla_{(a}\xi_{b)} = \frac{\theta}{4} g_{ab} + \sigma_{ab}$$

On shell, the broken symmetries are conformal transf.

$$\delta g_{ab} = 2\nabla_{(a}\xi_{b)}$$

$$g_{ab} \to (1 + \frac{\theta}{4}) g_{ab}$$

Unimodular gravity is a natural low energy gravitational theory

Unimodular gravity as a gravitational open system

Unimodular gravity emerges as the low energy limit in the thermodynamical arguments of Jacobson (Jacobson 1995 and 2015) as well as in formulations where 4d discrete building blocks of spacetime are posited (Sorkin's causal sets approach). Anderson-Finkelstein American Journal of Physics (1971)

In **Unimodular gravity** diffeomorphisms are broken down to volume preserving diffeomorphisms.

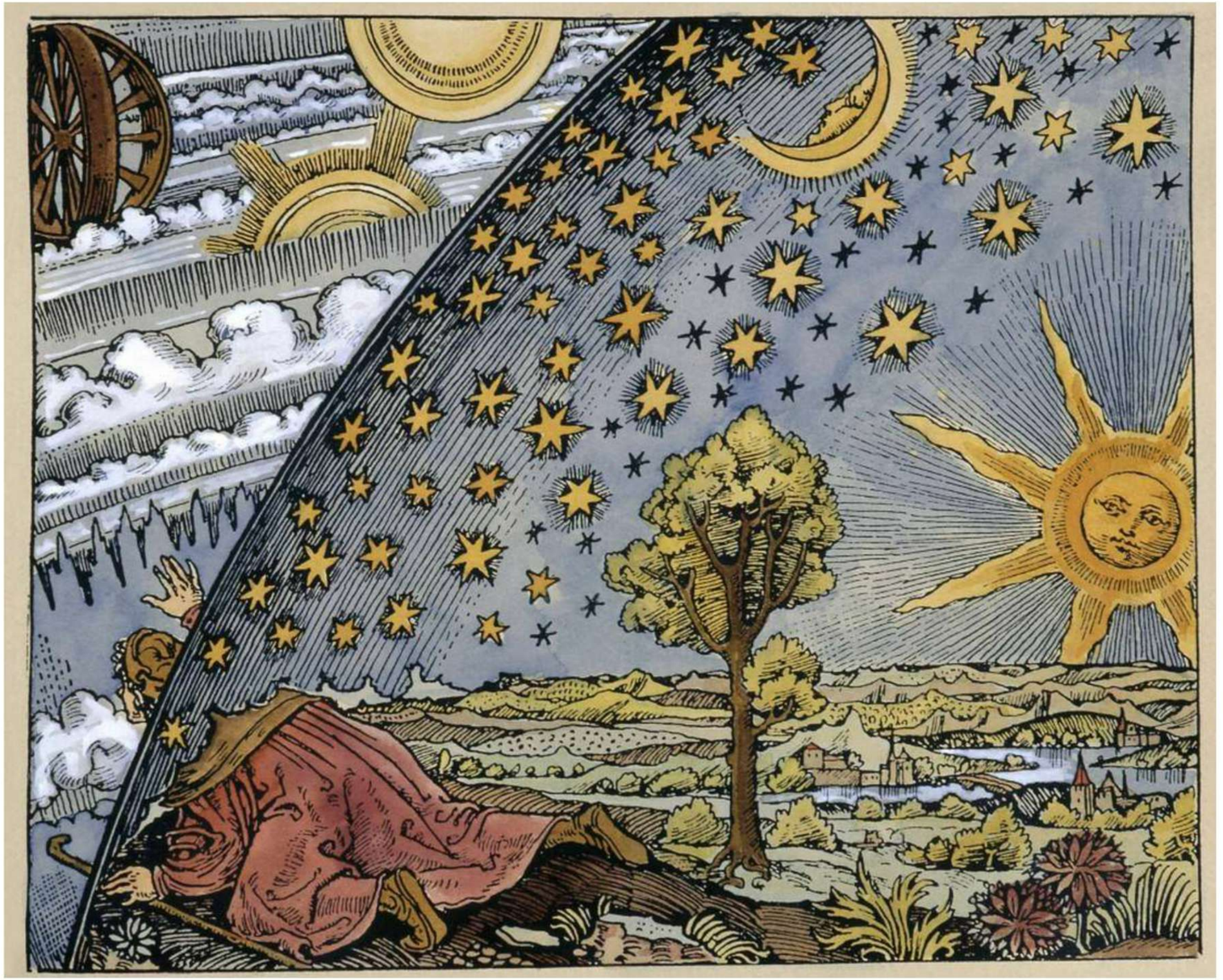
Unimodular gravity is the natural semiclassical theory when renormalizing the energy momentum tensor on curved spacetimes. There is no trace-anomaly but diffeo-anomaly due to quantum fluctuations. AP. arXiv:2104.08881

In **Unimodular gravity** vacuum fluctuations do not gravitate. There is no vacuum energy contributions to the cosmological constant. S. Weinberg, *Rev.Mod.Phys.* 61 (1989) 1-23

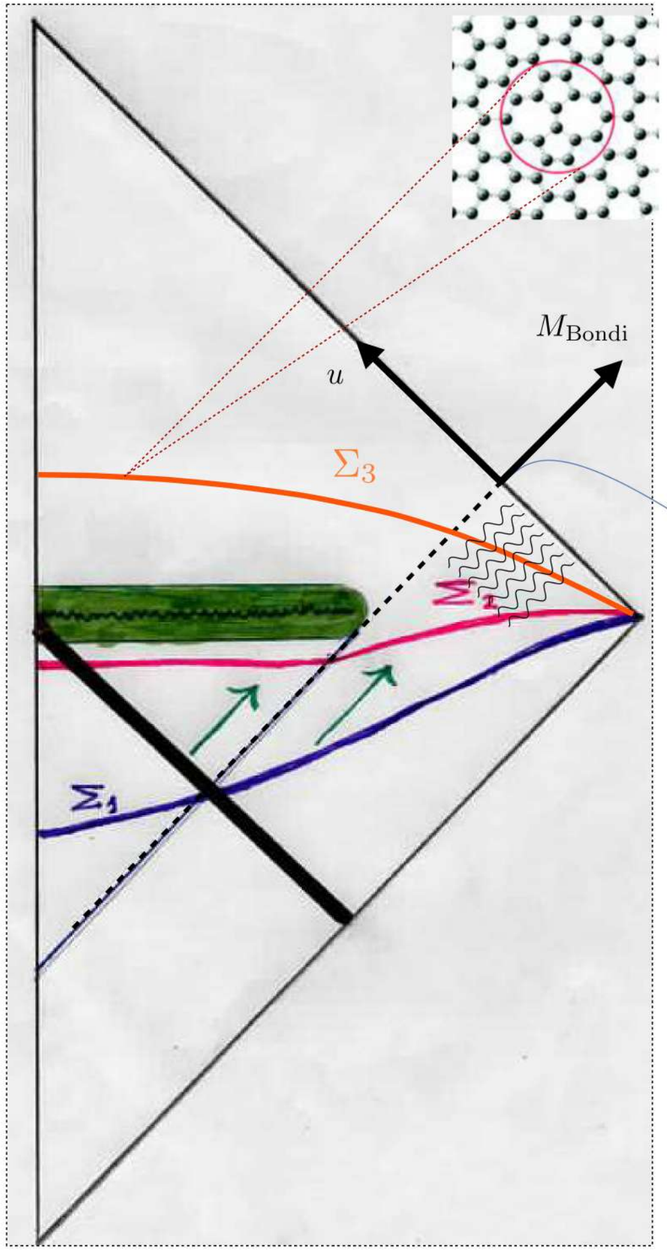
Apart from differences in the dark energy sector **Unimodular gravity** is equivalent to **general relativity**. It passes all the tests of GR: 2 local degrees of freedom, gravitational waves, etc are just the same as in GR.

Ellis-Van Elst-Murugan-Uzan, *Class.Quant.Grav.* 28 (2011) 225007

Can we produce a phenomenology of Planckian discreteness?



A possible channel for the resolution of the BH evaporation information puzzle



No firewalls in quantum gravity: the role of discreteness of quantum geometry in resolving the information loss paradox , AP; Class.Quant.Grav. 32 (2015) 8, 084001

Unitarity and information in quantum gravity: a simple example L. Amadei, Hongguang Liu and AP; Front.Astron.Space Sci. 8 (2021), 46

A dialog on the fate of information in black hole evaporation AP, and D. Sudarsky e-Print: 2205.08469 [gr-qc]

Diffusion from Planckian Granularity and Dark Energy

Unimodular gravity as a gravitational open system

Trace free Einstein's equations $\mathbf{R}_{ab} - \frac{1}{4}\mathbf{R}g_{ab} = 8\pi\left(\mathbf{T}_{ab} - \frac{1}{4}\mathbf{T}g_{ab}\right)$

$$\underbrace{\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab}}_{\mathbf{G}_{ab}} + \frac{1}{4}(\mathbf{R} + \mathbf{T})g_{ab} = 8\pi\mathbf{T}_{ab}$$

$$\frac{1}{4}\nabla_b\left(\mathbf{R} + 8\pi\mathbf{T}\right) = 8\pi\nabla^a\mathbf{T}_{ab}$$

$$\mathbf{J}_b \equiv 8\pi\nabla^a\mathbf{T}_{ab}$$

Need to satisfy the integrability condition

$$d\mathbf{J} = 0$$

$$\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab} = 8\pi\mathbf{T}_{ab} - \underbrace{\left[\Lambda_0 + \int_\ell \mathbf{J}\right]}_{\text{Dark Energy } \Lambda} g_{ab}$$

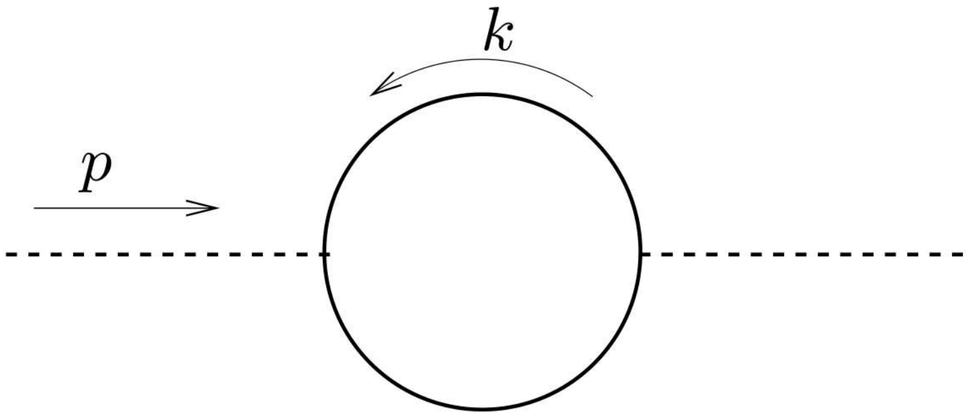
Radiative corrections make Lorentz violation percolate to low energies

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m_0^2}{2}\phi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - M_0)\psi + g_0\phi\bar{\psi}\psi.$$

$$\frac{i}{\gamma^\mu p_\mu - m_0 + i\epsilon} \to \frac{if(|\mathbf{p}|/\Lambda)}{\gamma^\mu p_\mu - m_0 + \Delta(|\mathbf{p}|/\lambda) + i\epsilon},$$

$$\frac{i}{p^2 - M_0^2 + i\epsilon} \to \frac{i\tilde{f}(|\mathbf{p}|/\Lambda)}{p^2 - M_0^2 + \tilde{\Delta}(|\mathbf{p}|/\lambda) + i\epsilon}.$$

Collins, AP, Sudarsky, Urrutia, Vusetich;
Phys. Rev. Letters. 93 (2004).

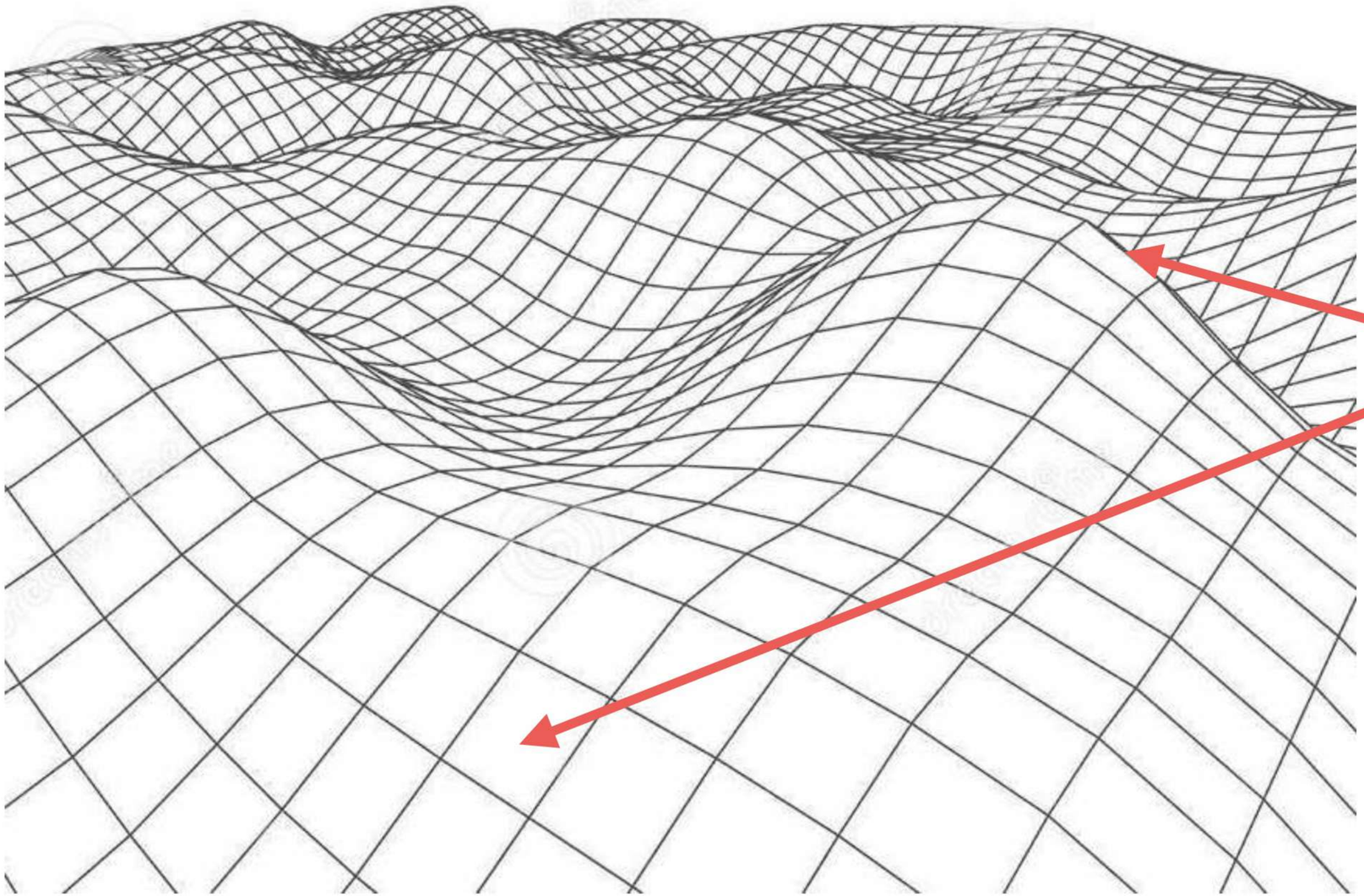


$$\Pi(p) = A + p^2 B + p^\mu p^\nu W_\mu W_\nu \tilde{\xi} + \Pi^{(\mathrm{LI})}(p^2) + \mathcal{O}(p^4/\Lambda^2)$$

$$\tilde{\xi} = \frac{g^2}{6\pi^2}\left[1 + 2\int_0^\infty dx x f'(x)^2\right]$$

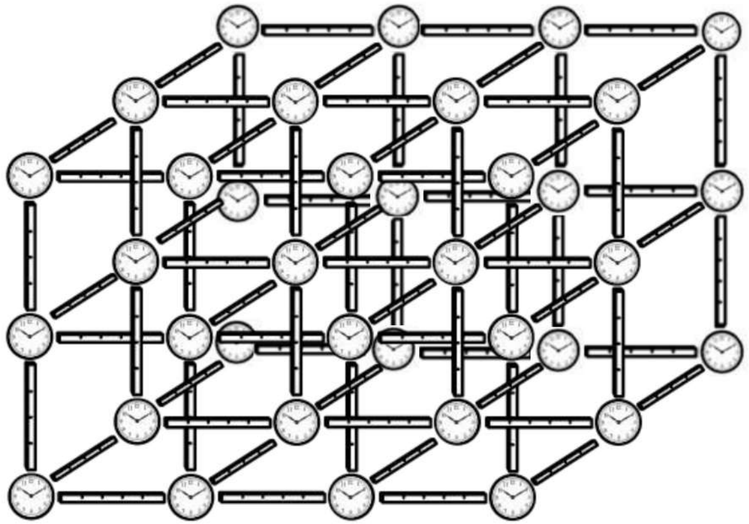
WAY OUT: Observables in QG are relational,
discreteness must be relational

Discreteness and Lorentz invariance



Discreteness **could** manifest itself in regions of non trivial curvature

To probe **Planck scale**
we need a **breaking of**
scale invariance
(need a ruler!)



Scale-invariance-breaking degrees of freedom are those where the phenomenology of granularity should more easily manifest.

The endothermic case

$$\mathbf{J}_b \equiv 8\pi \nabla^a \mathbf{T}_{ab}$$

Matter energy is lost

$$\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab} = 8\pi \mathbf{T}_{ab} - \underbrace{\left[\Lambda_0 + \int_\ell \mathbf{J}\right]}_{\text{Dark Energy } \Lambda} g_{ab}$$

Grows

A model predicting the observed cosmological constant

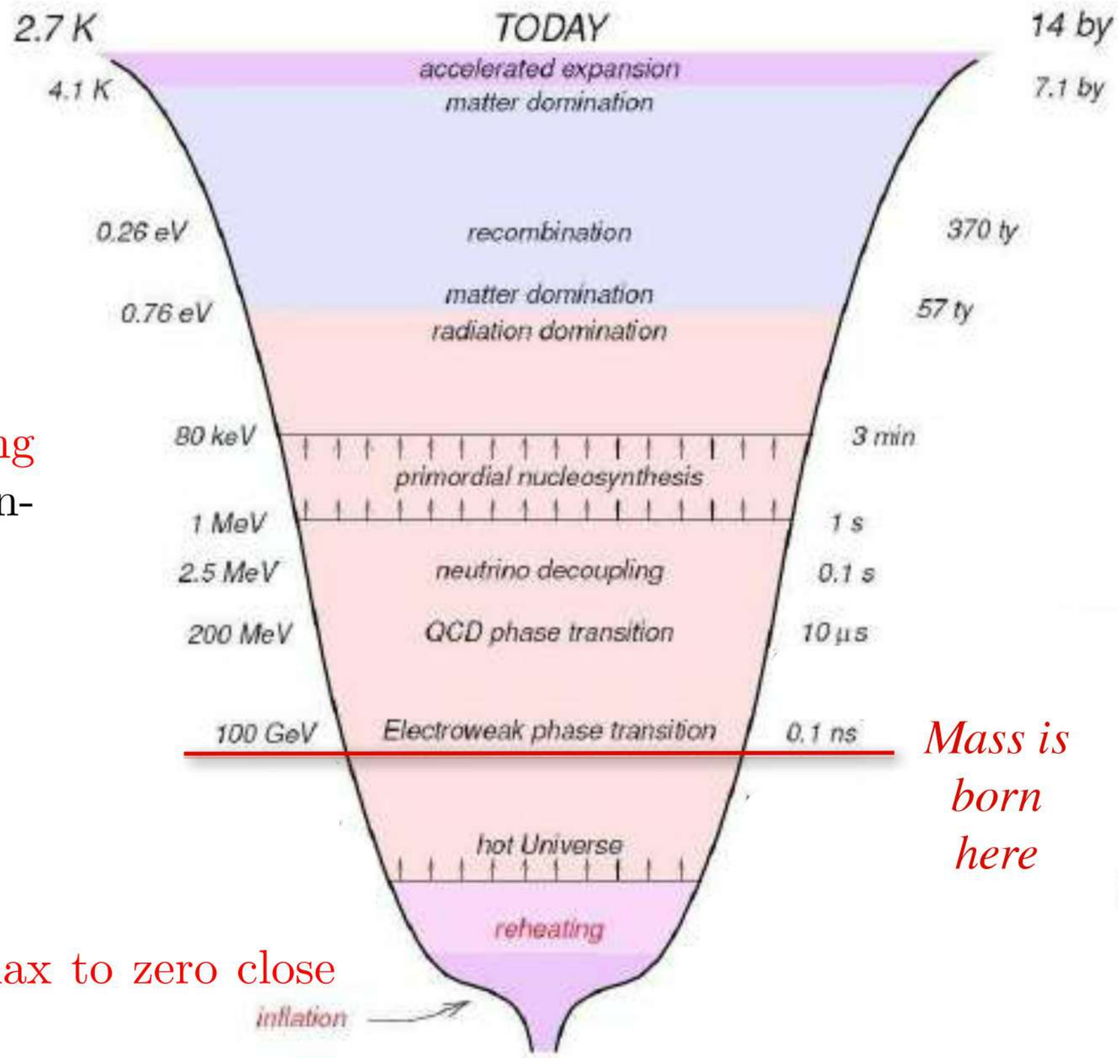
T. Josset, AP and D. Sudarsky, *Phys.Rev.Lett.* 118 (2017) 2, 021102.
AP. D. Sudarsky and J.D. Bjorken *Int.J.Mod.Phys.D* 27 (2018) 14, 1846002
AP and D. Sudarsky, *Phys.Rev.Lett.* 122 (2019) 22, 221302

$$\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab} = 8\pi\mathbf{T}_{ab} - \underbrace{\left[\Lambda_0 + \int_\ell \mathbf{J}\right]}_{\text{Dark Energy } \Lambda} g_{ab}$$

Assuming that the cosmological constant $\Lambda = 0$ at the big-bang then the diffusion effect generates it during the electro-weak transition when massive-spinning particles first appear.

$$\Lambda \approx \frac{\overline{m}_t^4 T_{ew}^3}{m_p^7} m_p^2 \approx \underbrace{\left(\frac{T_{ew}}{m_p}\right)^7}_{10^{-120}} m_p^2$$

Diffusion can make an initially large cosmological constant relax to zero close to the big bang. L. Amadei, AP (see rest of the talk)

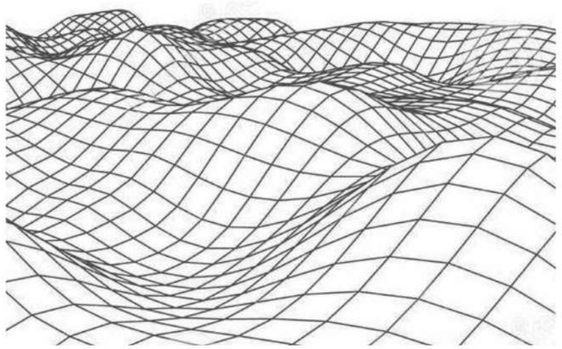


Mass is born here

What effects could be observable after the EW era?

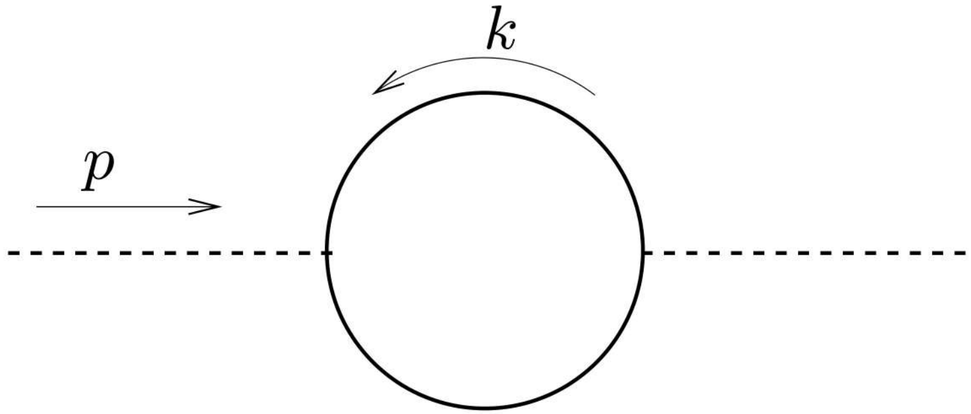
Collins, AP, Sudarsky, Urrutia, Vusetich;
Phys. Rev. Letters. 93 (2004).

Lorentz violating operators in EQFT must be suppressed by the scalar curvature. The leading operators dimensionally allowed are:



$$O_1 = \lambda_1 \, \xi^\mu \nabla_\mu \phi \mathbf{R} = \lambda_1 \dot{\phi} \mathbf{R}$$

$$O_2 = \lambda_2 \xi^\mu \bar{\psi} \gamma_\mu \psi \frac{\mathbf{R}}{m_p}$$



Constraints from present experiments and observations

$$T > 10^{-8} T_p = 10^{11} \text{GeV}$$

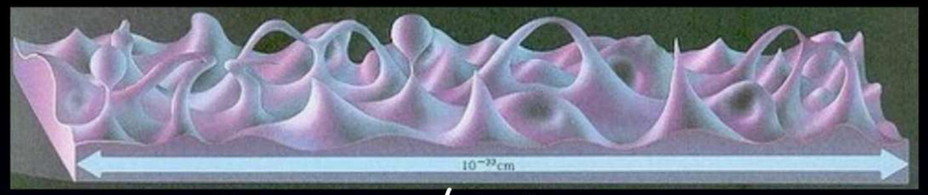
V. Alan Kostelecky and Neil Russell. Data Tables for Lorentz and CPT Violation. Rev. Mod. Phys., 2011.

The exothermic case

$$\mathbf{J}_b \equiv 8\pi \nabla^a \mathbf{T}_{ab}$$ **Matter energy is produced**

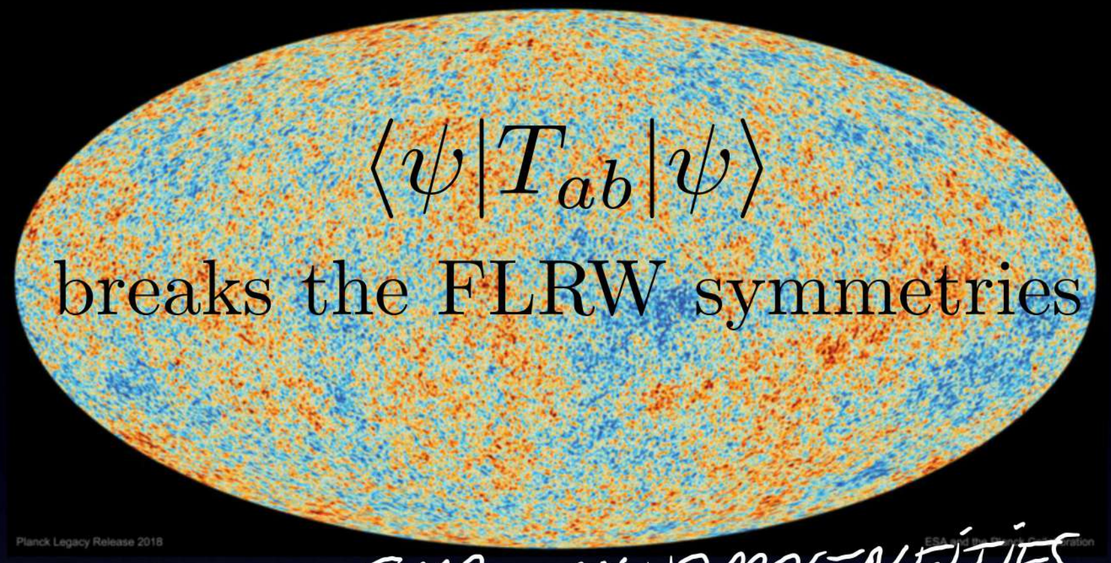
$$\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab} = 8\pi \mathbf{T}_{ab} - \underbrace{\left[\Lambda_0 + \int_\ell \mathbf{J}\right]}_{\text{Dark Energy } \Lambda} g_{ab}$$

Decays



PLANCKIAN GRANULARITY

INFLATION



CMB INHOMOGENEITIES

A COSMOLOGICAL MODEL WHERE:

1) THE COSMOLOGICAL CONSTANT DECAYS FROM ITS INITIAL PLANCK SCALE NATURAL VALUE

2) INHOMOGENEITIES IN THE CMB ARISE FROM PLANCKIAN DISCRETENESS

The exothermic relaxation: background dynamics

$$\Lambda(t) = \Lambda_0 \exp(-\beta m_p t)$$

$$ds^2 = -a^{-6}dt^2 + a^2 d\vec{x}^2$$

$$dt = a^3 d\tau$$

$$\Lambda(a) \approx \Lambda_0 \exp(-\frac{m_p}{3H_0}\beta(a^3 - 1))$$

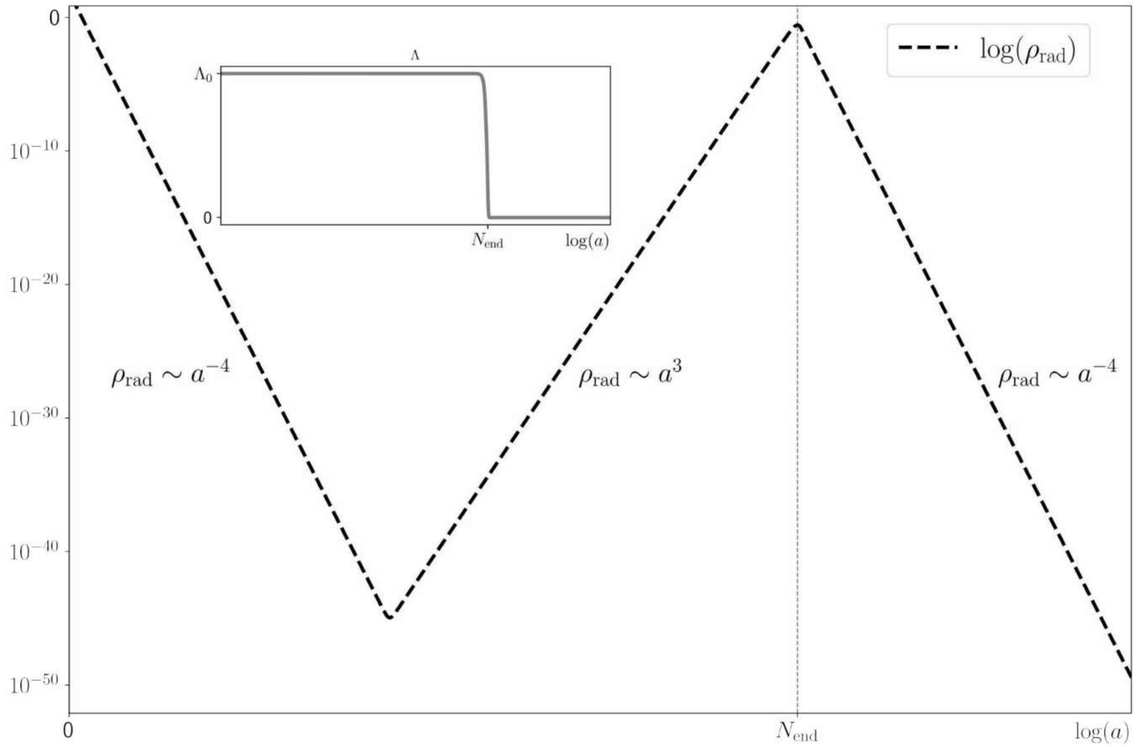


Figure 1: Numerical solution of (24) with $\beta = 10^{-80}$, we plot the cosmological constant Λ (inserted panel in linear scale), and the radiation energy density ρ_{rad} (in log scale) in terms of the number of e-folds $\log(a)$. Λ behaves effectively as a constant until about when condition (27) is satisfied and abruptly decays to zero thereafter. The radiation density decays exponentially from its initial Planckian value until the energy injection from the relaxation of Λ starts winning over the expansion. By the end of inflation radiation density grows back (*reheating*) to about Planckian density again (the reheating temperature is estimated in equation (39)).

The exothermic relaxation: simplified solvable model

$$\nabla_b \Lambda = -(8\pi G)\nabla^a \mathbf{T}_{ab} \qquad \frac{d\rho}{da} + 4\frac{\dot{a}}{a}\rho = -\frac{\dot{\Lambda}}{8\pi G}$$

The continuity equation (3) can be written using the number of e-folds $N \equiv \log(a)$ as a time variable as

$$\frac{d\rho}{dN} + 4\rho = -\frac{1}{8\pi G}\frac{d\Lambda}{dN},$$

The result of the paper follow from the simplistic model where

$$\Lambda(N) = \Lambda_0(1 - \Theta(N - N_0)),$$

where $\Theta(N - N_0)$ is the Heaviside step function. This is an extremely simplified model that will have the same predictions of the one in our paper. Here the cosmological constant is Λ_0 for $N < N_{\rm end}$ and it vanishes afterwards. One can solve the continuity equation for this 'sudden' relaxation of dark energy and one finds that

$$\Delta\rho|_{N=N_0} = \frac{\Lambda_0}{32\pi G},$$

writing the result in terms of $\rho(a)$ one has

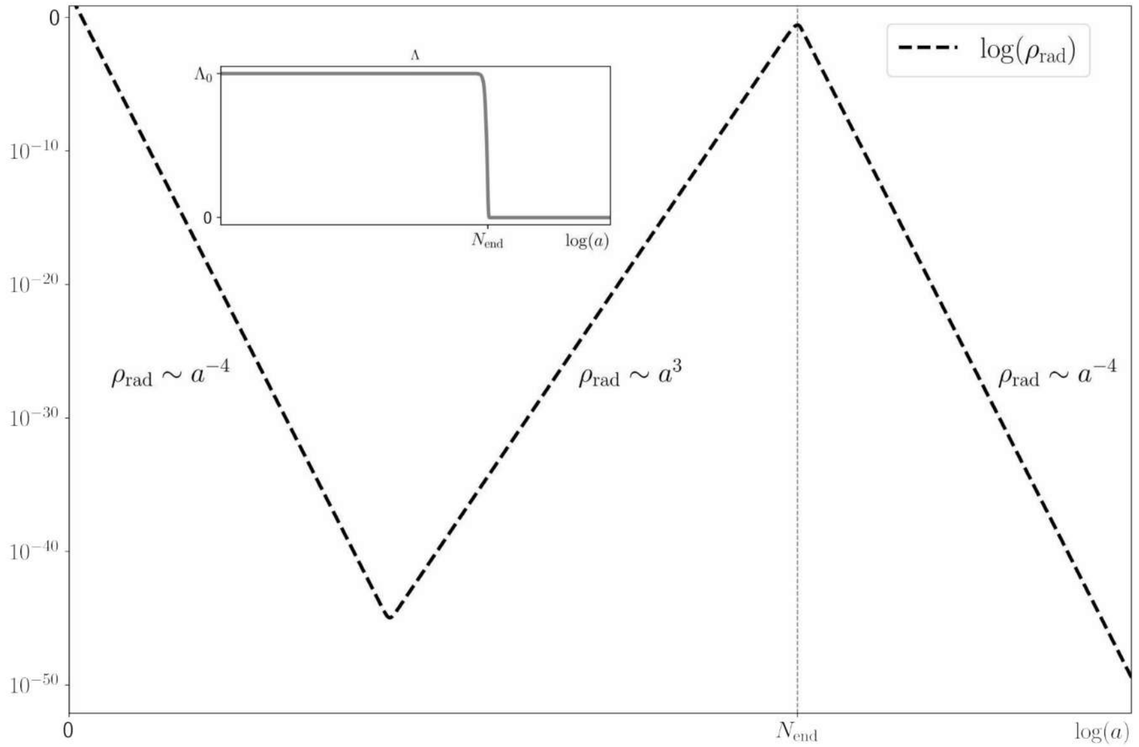
$$\rho = \frac{\rho_0 + a_{\rm end}^4 \frac{\Lambda_0 m_p^2}{32\pi}\Theta(a - a_{\rm end})}{a^4},$$

$$\Lambda(t) = \Lambda_0 \exp(-\beta m_p t)$$

$$ds^2 = -a^{-6}dt^2 + a^2 d\vec{x}^2$$

$$dt = a^3 d\tau$$

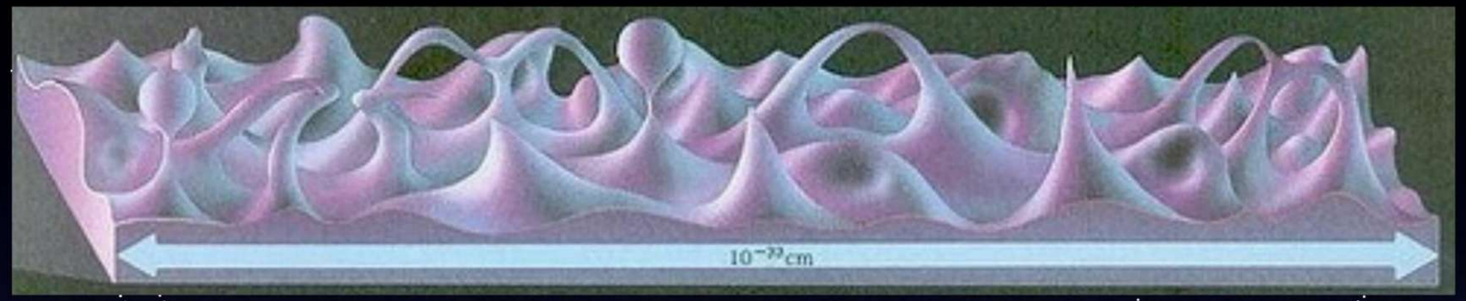
$$\Lambda(a) \approx \Lambda_0 \exp(-\frac{m_p}{3H_0}\beta(a^3 - 1))$$



J. Valiviita, E. Majerotto, and R. Maartens, "Instability in interacting dark energy and dark matter fluids," JCAP **07** (2008) 020, arXiv:0804.0232.

M. de Cesare and E. Wilson-Ewing, "Interacting dark sector from the trace-free Einstein equations: Cosmological perturbations with no instability," Phys. Rev. D **106** (2022), no. 2, 023527, arXiv:2112.12701.

PLANCKIAN DISCRETENESS INDUCING INHOMOGENEITIES

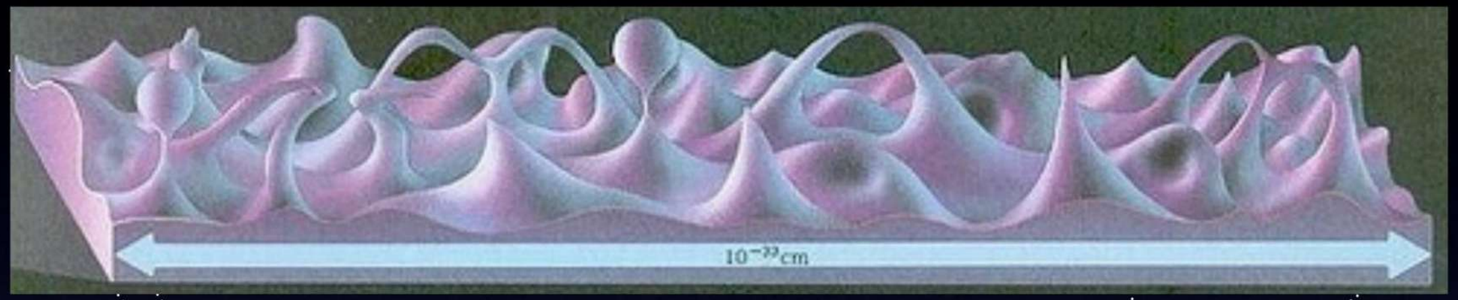


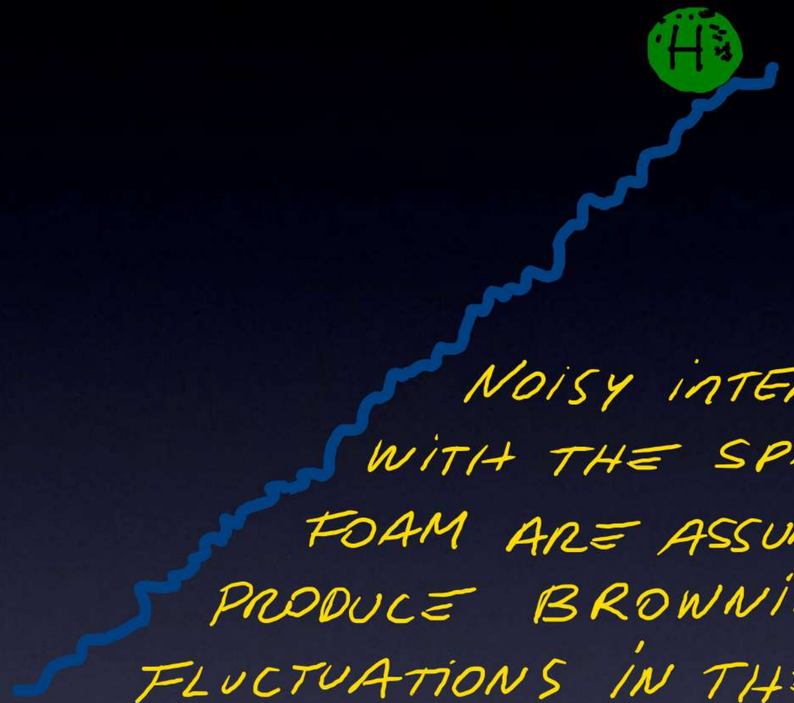
QUANTUM GRAVITY DISCRETENESS BREAKS COSMOLOGICAL SYMMETRIES AT THE PLANCK SCALE

DESITTER EXPANSION

AT LARGE SCALES THE SPACETIME IS HOMOGENEOUS AND ISOTROPIC

THE HIGGS SCALAR EVOLVES IN SUCH SPACETIME AND INHERITS PERTURBATIONS FROM PLANCKIAN INHOMOGENEITIES.



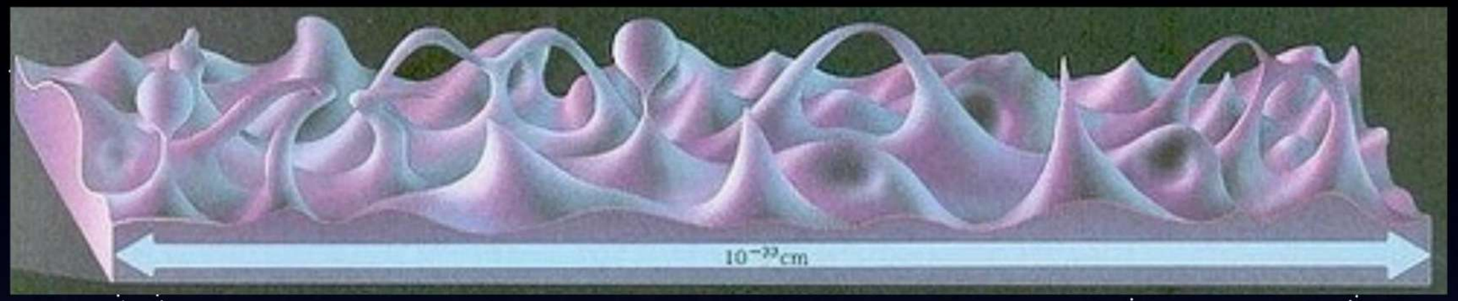


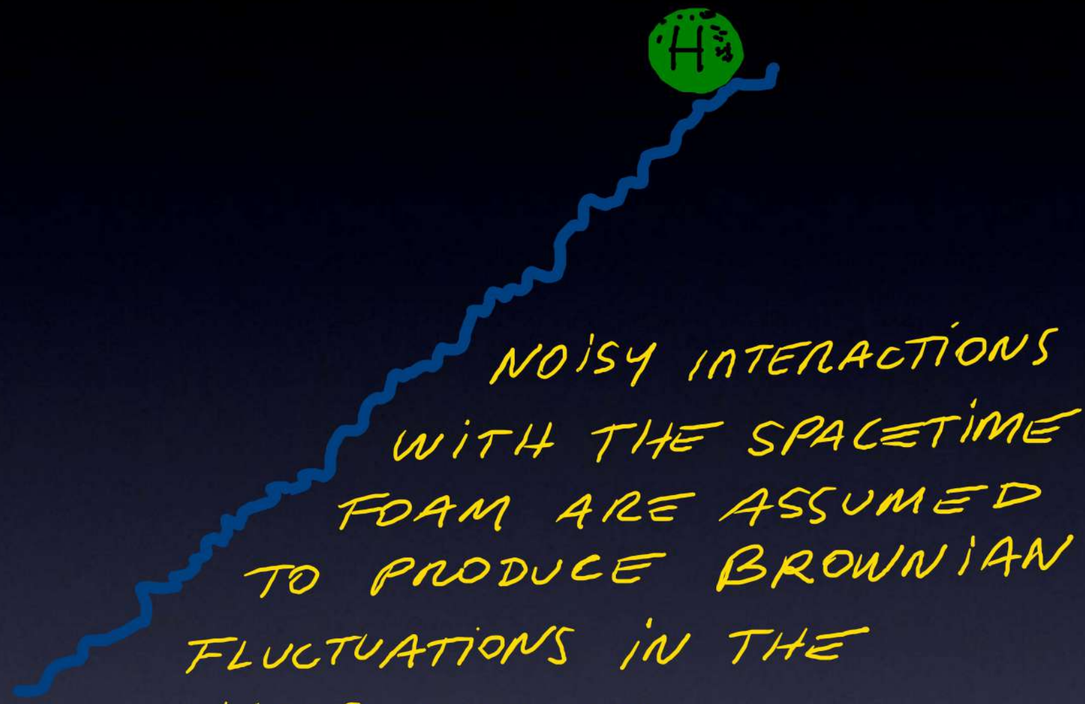
NOISY INTERACTIONS WITH THE SPACETIME FOAM ARE ASSUMED TO PRODUCE BROWNIAN FLUCTUATIONS IN THE HIGGS SCALAR.

BUT NOISE IS AT THE PLANCK SCALE

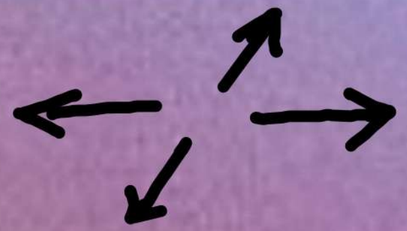
... WHICH BREAKS SCALE INVARIANCE

THE HIGGS SCALAR EVOLVES IN SUCH SPACETIME AND INHERITS PERTURBATIONS FROM PLANCKIAN INHOMOGENEITIES.





De Sitter expansion



RESTORES SCALE INVARIANCE TO PERTURBATIONS!!

BUT NOISE IS AT THE PLANCK SCALE

ADDITIONAL ASSUMPTION: THE STANDARD MODEL OF PARTICLE PHYSICS IS VALID UP TO CLOSE TO THE PLANCK SCALE. THE HIGGS SCALAR STARTS WITH LARGE INITIAL VALUE!

$$\phi = \phi_0 + \delta\phi(x^\mu)$$

$\phi_0 \approx m_p$

$$\langle \delta\phi(x^\mu) \rangle = 0$$

$\langle\ \rangle$ denote ensemble average as perturbations will be generated by a stochastic process

$$\delta\phi(t,\vec{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int dk^3 \delta\phi_{\vec{k}}(t) \exp(i\vec{k}\cdot\vec{x}) \qquad \overline{\delta\phi_{\vec{k}}} = \delta\phi_{-\vec{k}}$$

$$\begin{aligned} \xi_\phi(\vec{r}) &\equiv \langle \delta\phi(\vec{x})\delta\phi(\vec{x}+\vec{r}) \rangle \\ &= \frac{1}{(2\pi)^3} \int dk^3 dq^3 \, \langle \delta\phi_{\vec{k}} \delta\phi_{\vec{q}} \rangle \exp(i(\vec{k}+\vec{q})\cdot\vec{x}) \end{aligned}$$

SOME STANDARD DEFINITIONS

$$\langle \delta\phi_{\vec{k}} \delta\phi_{\vec{q}} \rangle = P_{\delta\phi}(k) \delta^{(3)}(\vec{k}+\vec{q})$$

$$\langle \delta\phi(\vec{x})\delta\phi(\vec{x}) \rangle = \frac{1}{(2\pi)^3} \int dk^3 P_{\delta\phi}(k)$$

THE POWER SPECTRUM (RELATED TO OBSERVABLE QUANTITY IN THE CMB)

Energy cost of inhomogeneities

$$\begin{aligned}\langle \delta T_{ab} \rangle &= \langle \nabla_a \delta\phi \nabla_b \delta\phi \rangle - \frac{1}{2} g_{ab} \left(\langle \nabla_\alpha \delta\phi \nabla^\alpha \delta\phi \rangle + \frac{d^2 V(\phi_0)}{d\phi^2} \langle \delta\phi^2 \rangle \right) \\ &= \frac{\langle \delta\dot{\phi}^2 \rangle + \frac{1}{a^2} \langle (\vec{\nabla}\delta\phi)^2 \rangle + \frac{d^2 V(\phi_0)}{d\phi^2} \langle \delta\phi^2 \rangle}{2} u_a u_b + \frac{\langle \delta\dot{\phi}^2 \rangle - \frac{1}{3a^2} \langle (\vec{\nabla}\delta\phi)^2 \rangle - \frac{d^2 V(\phi_0)}{d\phi^2} \langle \delta\phi^2 \rangle}{2} h_{ab},\end{aligned}$$

$$\begin{aligned}\langle \delta\rho^{(2)} \rangle &= \frac{\langle \delta\dot{\phi}^2 \rangle + \frac{1}{a^2} \langle (\vec{\nabla}\delta\phi)^2 \rangle + \frac{d^2 V(\phi_0)}{d\phi^2} \langle \delta\phi^2 \rangle}{2} \approx \frac{1}{2a^2} \langle (\vec{\nabla}\delta\phi)^2 \rangle + \frac{1}{2} \frac{d^2 V(\phi_0)}{d\phi^2} \langle \delta\phi^2 \rangle \\ \langle \delta P^{(2)} \rangle &= \frac{\langle \delta\dot{\phi}^2 \rangle - \frac{1}{3a^2} \langle (\vec{\nabla}\delta\phi)^2 \rangle - \frac{d^2 V(\phi_0)}{d\phi^2} \langle \delta\phi^2 \rangle}{2} \approx -\frac{1}{6a^2} \langle (\vec{\nabla}\delta\phi)^2 \rangle - \frac{1}{2} \frac{d^2 V(\phi_0)}{d\phi^2} \langle \delta\phi^2 \rangle ,\end{aligned}$$

$$\begin{aligned}\frac{dW^{\text{pert.}}(P_{\delta\phi})}{da} &\equiv \frac{1}{\dot{a}} \left(\langle \delta\dot{\rho}^{(2)} \rangle + 3H_0 \left(\langle \delta\rho^{(2)} \rangle + \langle \delta P^{(2)} \rangle \right) \right) \\ &= \frac{d \langle \delta\rho^{(2)} \rangle}{da} + \frac{3}{a} \left(\langle \delta\rho^{(2)} \rangle + \langle \delta P^{(2)} \rangle \right)\end{aligned}$$

$$\frac{dW^{\text{pert.}}(P_{\delta\phi})}{da} \equiv \frac{d \langle \delta\rho^{(2)} \rangle}{da} + \frac{2}{a} \langle \delta\rho^{(2)} \rangle - \frac{1}{a} \frac{d^2 V(\phi_0)}{d\phi^2} \langle \delta\phi^2 \rangle$$

Energy cost of inhomogeneities: Einstein detail balance

$$\frac{dW^{\rm pert.}(P_{\delta\phi})}{da} = \text{stochastic source}$$

$$\frac{dW^{\rm pert.}(P_{\delta\phi})}{da} \equiv \frac{d\left\langle\delta\rho^{(2)}\right\rangle}{da} + \frac{2}{a}\left\langle\delta\rho^{(2)}\right\rangle - \frac{1}{a}\frac{d^2V(\phi_0)}{d\phi^2}\left\langle\delta\phi^2\right\rangle \quad = \quad \gamma\frac{H^4}{a}$$

$$\left\langle\delta\rho^{(2)}\right\rangle \equiv \left\langle\delta T_{00}\right\rangle \quad \approx \quad \frac{1}{2\pi^2}\int_{\mu}^{am_p} dk k^2 \left(\frac{k^2}{2a^2} + \frac{1}{2}\frac{d^2V(\phi_0)}{d\phi^2}\right) P_{\delta\phi}(k)$$

Einstein detail balance (fluctuation/dissipation theorem)

Consider a free particle moving in a viscous medium. The equation of motion is given by the Langevin equation

$$\ddot{x} + \gamma \dot{x} = \delta x(t),$$

where $\delta x(t)$ is a stochastic noise source representing the fluctuations of x due to the action of the microscopic elements in the environment. This stochastic variable (which is the analog of the inhomogeities $\delta\phi$ in our cosmological context) has a noise dynamics dictated by a probability distribution such that

$$\langle \delta x(t) \rangle = 0, \quad \langle \delta x(t) \delta x(t') \rangle = P_{\delta x} \delta(t, t').$$

The Langevin equation has a simple solution in the large time asymptotics such that

$$\dot{x}(t) = e^{-\gamma t} \int_0^t e^{\gamma t'} \delta x(t') dt'$$

Consider now the kinetic energy of the particle $E = m\dot{x}^2/2$, in the long time asymptotic regime conservation of energy requires $\langle \dot{E} \rangle = 0$. This condition can be written using the Langevin equation as

$$\langle \dot{E}(t) \rangle \quad = \quad m \langle \dot{x}(t) \ddot{x}(t) \rangle = m \langle \dot{x}(t) (\delta x(t) - \gamma \dot{x}(t)) \rangle = m \langle \dot{x}(t) \delta x(t) \rangle - 2\gamma \langle E(t) \rangle \quad \Longrightarrow \quad m P_{\delta x} = 2\gamma \langle E(t) \rangle .$$

The result is Einstein's detail balance equation relating the expectation value of the energy of the particle (which does not vanish due to the action of the Brownian collisions of the microscopic constituents of the environment), the friction coefficient γ (encoding dissipation), and the power 'spectrum' $P_{\delta x}$ of the fluctuations $\delta x(t)$.

Solving detail balance

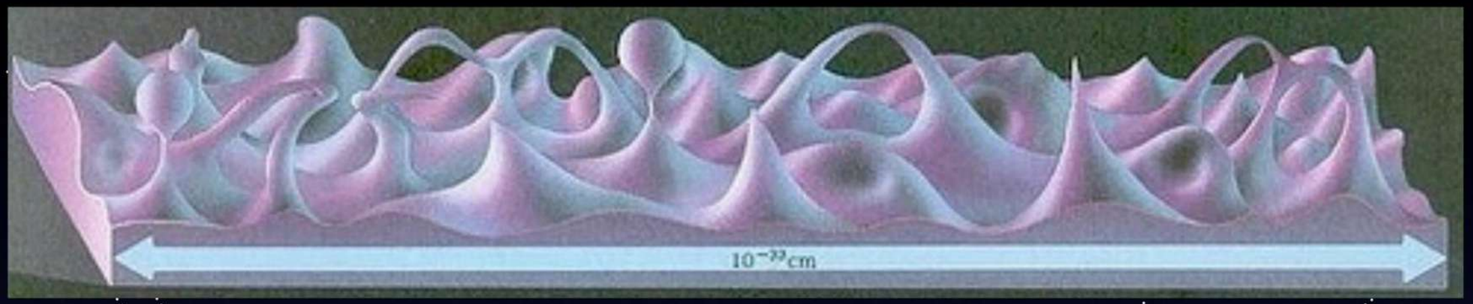
$$\frac{dW^{\text{pert.}}(P_{\delta\phi})}{da} = \text{stochastic source}$$

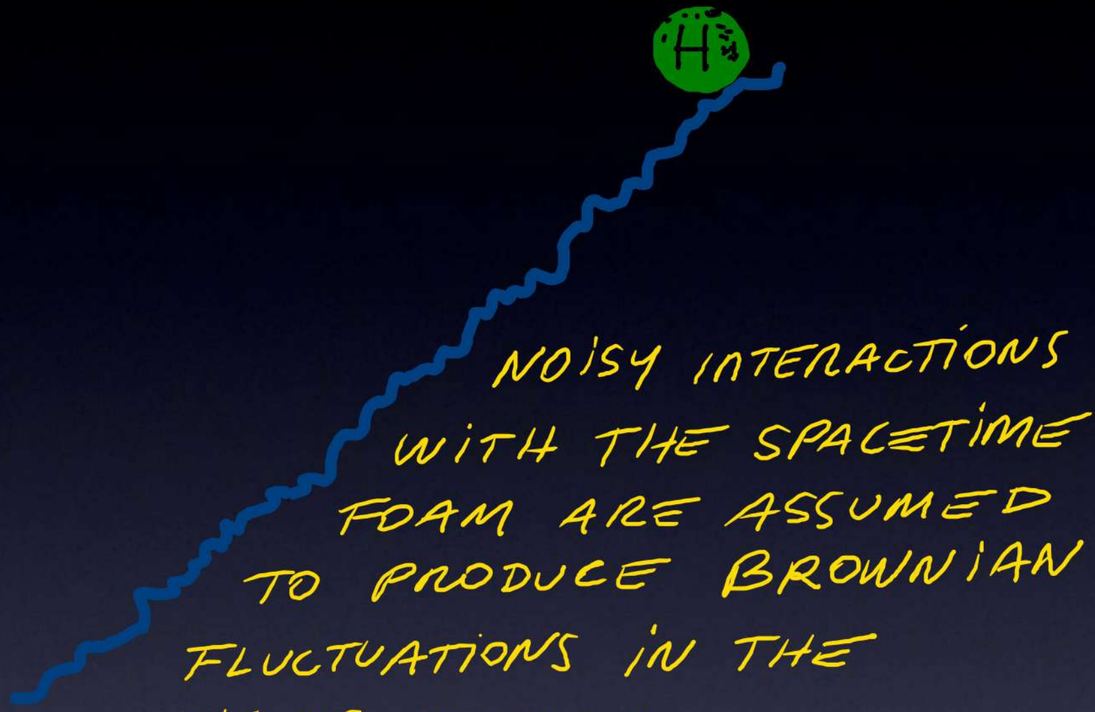
$$\frac{dW^{\text{pert.}}(P_{\delta\phi})}{da} \equiv \frac{d\left\langle\delta\rho^{(2)}\right\rangle}{da} + \frac{2}{a}\left\langle\delta\rho^{(2)}\right\rangle - \frac{1}{a}\frac{d^2V(\phi_0)}{d\phi^2}\left\langle\delta\phi^2\right\rangle \quad = \quad \gamma\frac{H^4}{a}$$

$$\left\langle\delta\rho^{(2)}\right\rangle \equiv \left\langle\delta T_{00}\right\rangle \quad \approx \quad \frac{1}{2\pi^2}\int_{\mu}^{am_p} dk k^2 \left(\frac{k^2}{2a^2} + \frac{1}{2}\frac{d^2V(\phi_0)}{d\phi^2}\right) P_{\delta\phi}(k)$$

$$P_{\delta\phi}(k) = \frac{P_0}{k^3} \qquad\qquad P_0 = 4\pi^2\gamma\frac{H_0^3}{m_p}$$

THE HIGGS SCALAR EVOLVES IN SUCH SPACETIME AND INHERITS PERTURBATIONS FROM PLANCKIAN INHOMOGENEITIES.

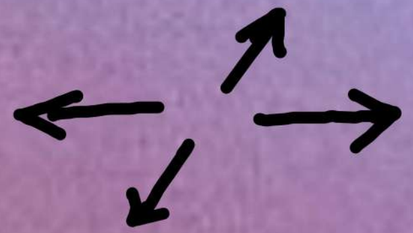




NOISY INTERACTIONS WITH THE SPACETIME FOAM ARE ASSUMED TO PRODUCE BROWNIAN FLUCTUATIONS IN THE HIGGS SCALAR.

De Sitter expansion RESTORES SCALE INVARIANCE TO PERTURBATIONS!!

BUT NOISE IS AT THE PLANCK SCALE



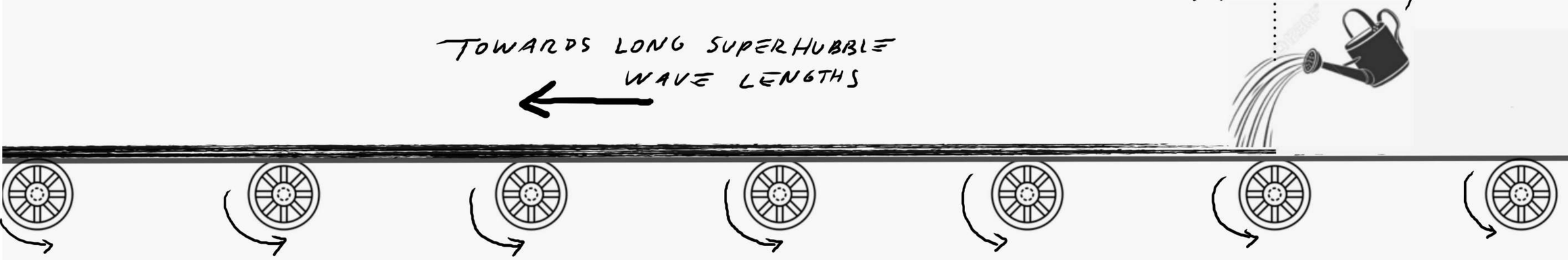
INFLATIONARY "CONVEYOR BELT"

SUB PLANCKIAN SCALES

TRANS PLANCKIAN SCALES

$\lambda \sim H_0^{-1} \sim l_p$

TOWARDS LONG SUPERHUBBLE WAVE LENGTHS



$$\frac{dW^{\text{pert.}}}{d\tau} \equiv \frac{d\left\langle \delta\rho^{(2)} \right\rangle}{d\tau} + 2\frac{\dot{a}}{a}\left\langle \delta\rho^{(2)} \right\rangle - \frac{\dot{a}}{a}\frac{d^2V(\phi_0)}{d\phi^2}\left\langle \delta\phi^2 \right\rangle = \gamma H^5$$

APROX. CONSTANT (GEOMETRIC) ENERGY INPUT into SCALAR field perturbations

$$\left\langle \delta\rho^{(2)} \right\rangle \equiv \left\langle \delta T_{00} \right\rangle = \frac{1}{2\pi^2}\int_{\mu}^{aH_0} dk k^2 \left(\frac{k^2}{2a^2} + \frac{1}{2}\frac{d^2V(\phi_0)}{d\phi^2} \right) P_{\delta\phi}(k)$$

$$P_{\delta\phi}(k) = \frac{4\pi^2\gamma H^2}{k^3}$$

The Weinberg Theorem: connecting fluctuations of the Higgs during inflation to inhomogeneities seen at the CMB

$$\frac{\delta\rho_k^{(1)}}{\dot{\rho}_0} = -\frac{\mathcal{R}_k}{a(\tau)}\int_{\mathcal{T}}^{\tau} a\left(\tau'\right) d\tau' \approx -\frac{\mathcal{R}_k}{H_0} \qquad \delta\mathbf{R}^{(3)} = -4\vec{\nabla}^2\mathcal{R}$$

$$\Phi_k = \Psi_k = \mathcal{R}_k\left(-1 + \frac{H(\tau)}{a(\tau)}\int_{\mathcal{T}}^{\tau} a\left(\tau'\right) d\tau'\right) \approx 0$$

$$\mathcal{R}_k = -H_0\frac{\delta\rho_k^{(1)}}{\dot{\rho}_0}$$

$$\boxed{P_{\mathcal{R}} \approx \frac{9\pi^2\gamma}{k^3\lambda^2}\left(1 + 4\lambda\log\left(\frac{k}{k_0}\right)\right).}$$

Relating predictions to observations

$$N^2 \approx \frac{9\pi^2\gamma}{\lambda^2} \approx 1.9\times 10^{-10}$$

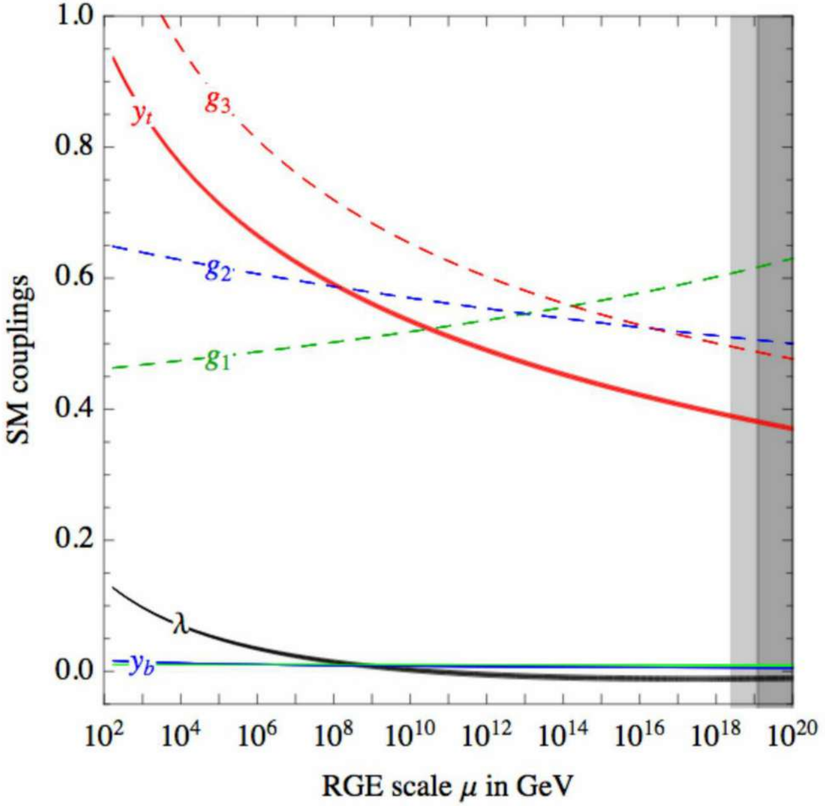
$$n_{\mathrm{s}} - 1 \equiv \frac{d\log(k^3 P_{\mathcal{R}})}{d\log k} \approx 4\lambda + \mathscr{O}\left[\lambda^2\log\left(\frac{k_{\mathrm{max}}}{k_0}\right)\right]$$

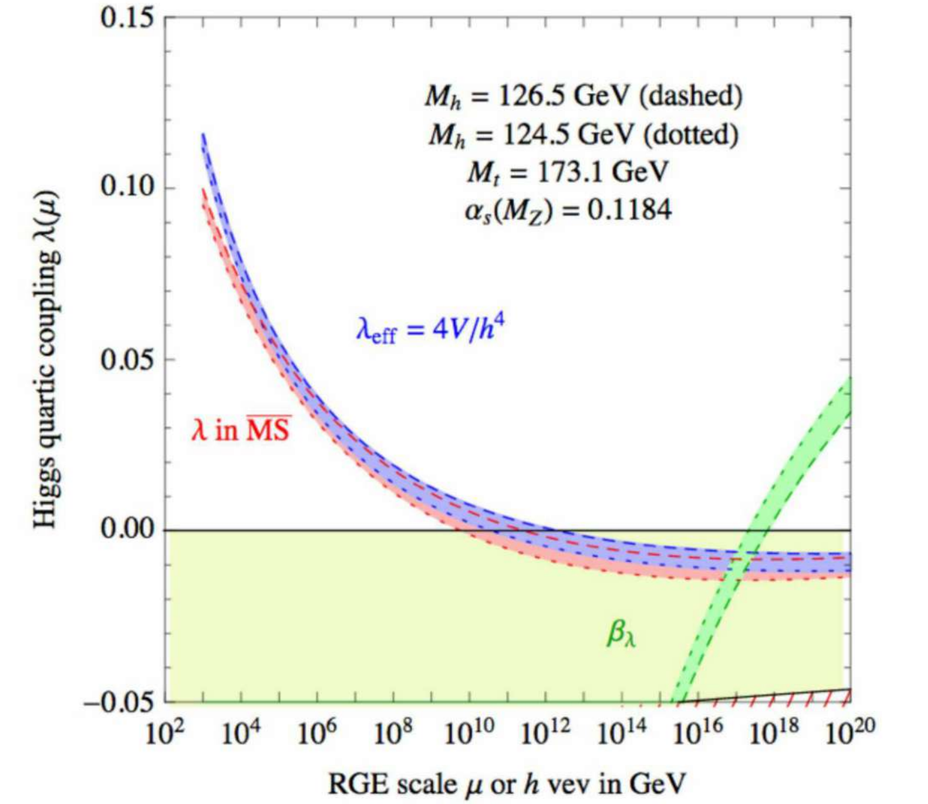
Planck, Y. Akrami et al., "Planck 2018 results. X. Constraints on inflation," Astron. Astrophys. 641 (2020) A10, arXiv: 1807.06211.

$$\gamma \approx 10^{-16} \approx \frac{m_{\mathrm{H}}}{m_p}$$

$$\lambda = -(1.3 \pm 0.7)\times 10^{-2}$$

Renormalization Group Flow of the Couplings of the Standard Model





From G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, and A. Strumia, "Higgs mass and vacuum stability in the Standard Model at NNLO," JHEP 08 (2012) 098, arXiv:1205.6497.

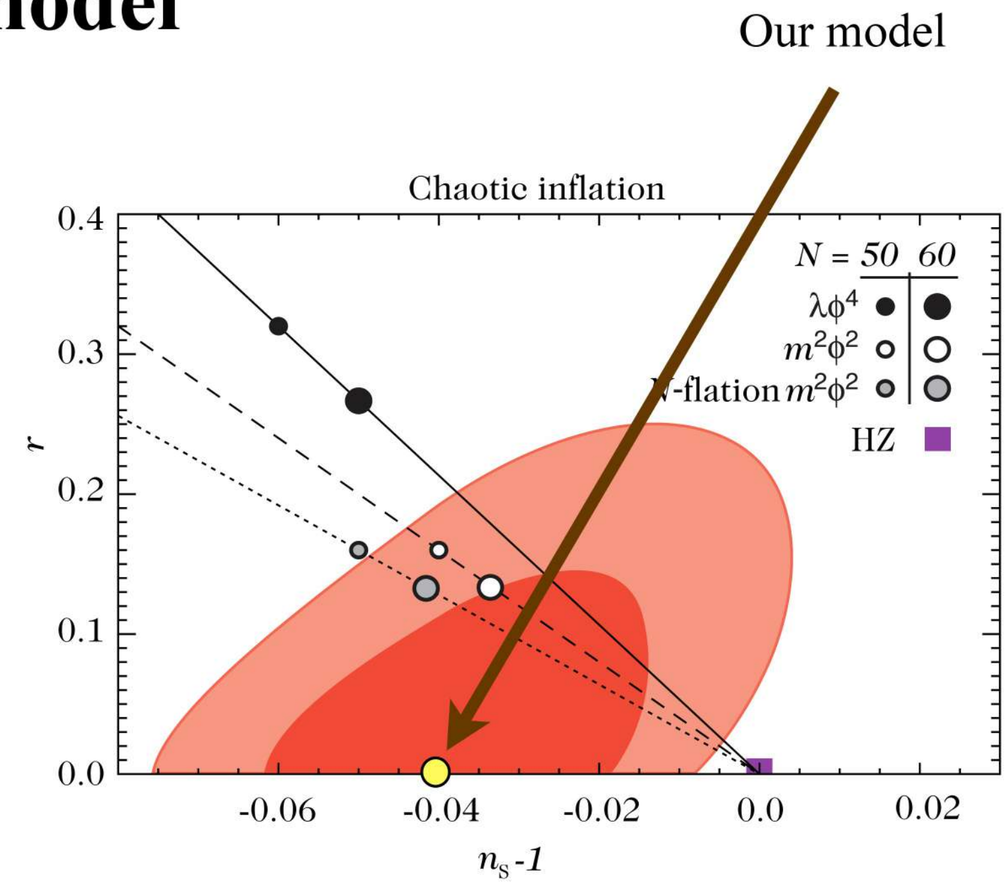
Predictions of the model

$$n_{\rm s} - 1 \equiv \frac{d\log(k^3 P_{\mathcal{R}})}{d\log k} \approx 4\lambda + \mathscr{O}\left[\lambda^2 \log\left(\frac{k_{\rm max}}{k_0}\right)\right]$$

Planck, Y. Akrami et al., "Planck 2018 results. X. Constraints on inflation," Astron. Astrophys. 641 (2020) A10, arXiv: 1807.06211.

$$\left.\begin{aligned} dn_{\rm s}/d\ln k &= -0.0045 \pm 0.0067, \\ n_{\rm s} &= 0.9641 \pm 0.0044, \\ n_{\rm s,0.002} &= 0.979 \pm 0.021, \end{aligned}\right\} \text{ 68 \%, TT,TE,EE +lowE+lensing,}$$

$$\left.\begin{aligned} dn_{\rm s}/d\ln k &= -0.0041 \pm 0.0067, \\ n_{\rm s} &= 0.9659 \pm 0.0040, \\ n_{\rm s,0.002} &= 0.979 \pm 0.021, \end{aligned}\right\} \text{ 68 \%, TT,TE,EE +lowE+lensing +BAO,}$$



Running of the spectral index

$$\frac{dn_{\rm s}}{d\log k} = -0.0005 + \mathscr{O}(\lambda^3).$$

Nearly vanishing tensor to scalar ratio

$$r \approx 0$$

Dark matter as primordial Planckian BHs

At the end of the inflationary era reheating raises the temperature to close to the Planck temperature and Planck mass remnants could be created via thermal fluctuations if thermal equilibrium density is achieved. In order for this to happen one needs the remnant interaction rate $\Gamma_{\rm pbh} > H$, where the interaction rate is given by $\Gamma = n\sigma v$ with n the number density, σ the interaction cross section, and v the velocity. For remnants of mass $m_{\rm pbh}$ the interaction cross section is $\sigma_{\rm pbh} \approx m_{\rm pbh}^2/m_p^4$ while their number density n while in thermal equilibrium goes like $n \approx T^3$. Using that in the radiation dominated era $H \approx (T/m_p)T$, we conclude that remnants decouple from thermal equilibrium when

$$T \lesssim \frac{m_p^2}{m_{\rm pbh}^2}\, m_p \equiv T_{\rm D}.$$

If thermal equilibrium can hold up to $T_{\rm D} \lesssim T_{\rm end}$ then the thermal remnant abundance of dark matter today can be estimated to be about (see equation 4.38 in Peter-Uzan)

$$\frac{\rho_{\rm pbh}^{\rm thermal}(T_{\rm D})}{m_p^4} \approx \left(\frac{m_{\rm pbh}}{m_p}\right)^4 \left(\frac{T_{\rm today}}{T_{\rm D}}\right)^3 \left(\frac{T_{\rm D}}{m_{\rm pbh}}\right)^{\frac{3}{2}} e^{-\frac{m_{\rm pbh}}{T_{\rm D}}}.$$

One can easily check that it is possible to obtain a remnant density compatible with dark energy density today—which would correspond to evaluating the previous line to about 10^{-120}—with a $m_{\rm pbh}$ slightly larger than but of the order of m_p. This shows that the framework provided by our model could also fit dark energy genesis from the production of stable PBHs via thermal fluctuations at the end of the De Sitter phase without extreme fine tuning where the necessary suppression is brought by the standard Gibbs factor. After completion of this work we discovered that very similar arguments are put forward in A. Barrau, K. Martineau, F. Moulin, and J.-F. Ngono, "Dark matter as Planck relics without too exotic hypotheses," Phys. Rev. D **100** (2019), no. 12, 123505, arXiv:1906.09930.

GENERAL DISCUSSION: Discreteness at the Planck scale might play an important role in phenomenology

* It can provide a natural channel for purification of Hawking radiation. Possible nontrivial aspect of the resolution of the INFORMATION PUZZLE.

* Diffusion into microscopic granularity offers a possible perspective on the origin of a small cosmological constant.

* It opens the door for the relaxation of a primordial Λ of Planckian value driving a new kind of inflation.

* For natural initial conditions $\Lambda_0 \simeq m_p^2$ $\phi_0 \simeq m_p$ it offers a simple mechanism for the understanding of the observed inhomogeneities at the CMB (without quantum to classical symmetry breaking)

CONCLUSIONS:

- DEVIATION FROM SCALE INVARIANCE AT THE CMB is compatible with SM physics

$$n_{\rm s} - 1 \equiv \frac{d\log(k^3 P_{\mathcal{R}})}{d\log k} = 4\lambda\epsilon_\phi^2 + \mathcal{O}\left[\lambda^2\epsilon_\phi^4 \log\left(\frac{k_{\rm max}}{k_0}\right)\right] = -0.04 \pm 0.004$$

Planck, Y. Akrami et al., "Planck 2018 results. X. Constraints on inflation," Astron. Astrophys. 641 (2020) A10, arXiv: 1807.06211.

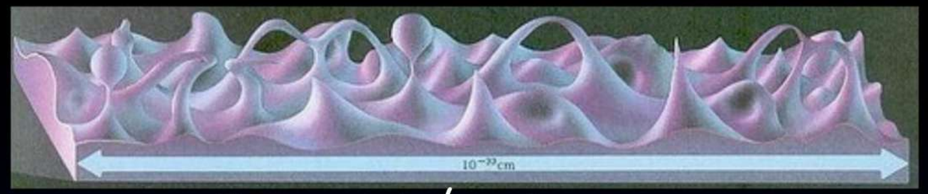
- SCALE DEPENDENCE of the scalar spectral index is fixed by SM physics and compatible with present constraints.

$$\frac{dn_s}{d\ln k} = -0.000533$$

$$\left.\begin{aligned} dn_{\rm s}/d\ln k &= -0.0045 \pm 0.0067, \\ n_{\rm s} &= 0.9641 \pm 0.0044, \\ n_{\rm s,0.002} &= 0.979 \pm 0.021, \end{aligned}\right\} \text{68 \%, TT,TE,EE +lowE+lensing,}$$

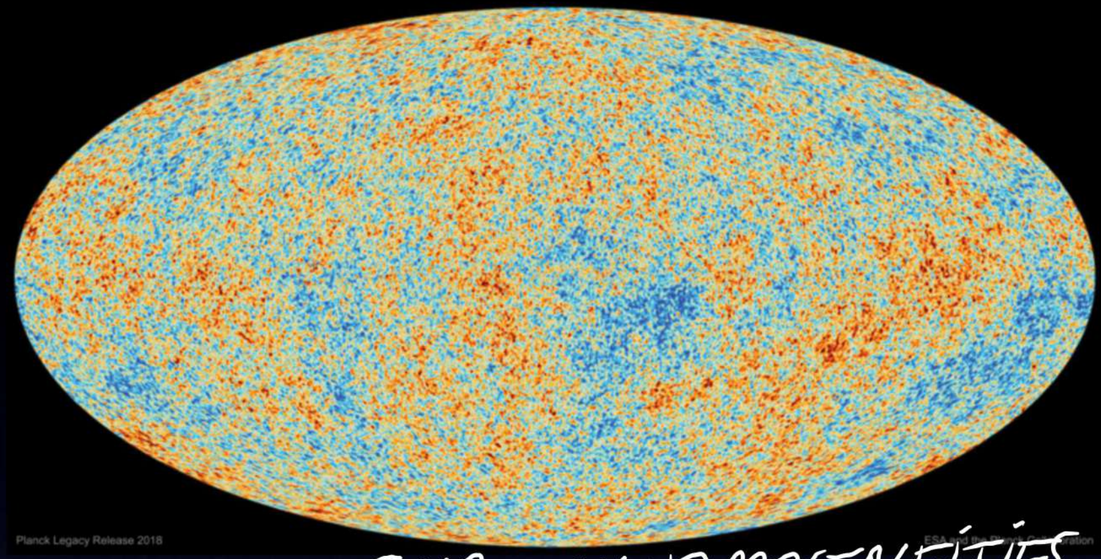
- The model predicts a very small TENSOR TO SCALAR RATIO

$$\left.\begin{aligned} dn_{\rm s}/d\ln k &= -0.0041 \pm 0.0067, \\ n_{\rm s} &= 0.9659 \pm 0.0040, \\ n_{\rm s,0.002} &= 0.979 \pm 0.021, \end{aligned}\right\} \text{68 \%, TT,TE,EE +lowE+lensing +BAO,}$$



PLANCKIAN GRANULARITY

INFLATION



CMB INHOMOGENEITIES

Thank you very much!