Classical limit of spinfoams on arbitrary triangulations

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References

- *Curvature in spinfoams*, Magliaro-CP, Mar 2011 (CQG Highlight)
- *Regge gravity from spinfoams*, Magliaro-CP, May 2011 (to app. CQG)
- *Emergence of gravity from spinfoams*, Magliaro-CP, Aug 2011 (EPL)
- *Einstein-Regge equations in spinfoams*, CP, Oct 2011 (JPCS)
- *The flipped limit of EPRL spinfoam*, CP, Feb 2012 (to app.)

Main inspiration

- *Lorentzian spin foam amplitudes: graphical calculus and asymptotics*, Barrett-Dowdall-Fairbairn-Hellmann-Pereira
- *On the semiclassical limit of 4d spin foam models*, Conrady-Freidel

Related directions

- *Asymptotics of Spinfoam Amplitude on Simplicial Manifold: Lorentzian Theory*, Han-Zhang
- *Perturbative quantum gravity with the Immirzi parameter*, Benedetti-Speziale

Plan of the talk

The theory

- LQG kinematics
- LQG dynamics in Spinfoam language
- truncated amplitudes vs. continuum limit

Low-energy behavior

- semiclassicality: the flipped limit
- stationary phase in two diff. schemes
- emergence of classical geometry
- application: structure of graviton propagator

To be done: homeworks

The theory

(tentative, of course)

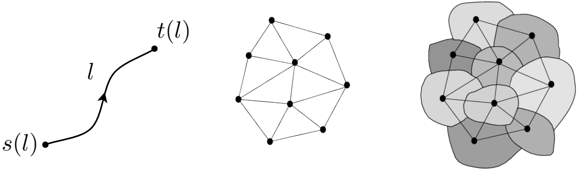
LQG in a nutshell

KINEMATICS: spin-networks

Gauge-invariant $\mathcal{H} = \oplus_\Gamma \mathcal{H}_\Gamma$ does not depend on a lattice $\rightsquigarrow$ continuum theory

$$\mathcal{H}_\Gamma = L^2(SU(2)^{\#\text{links}}/SU(2)^{\#\text{nodes}}) \quad \text{comp. to Yang-Mills} \tag{1}$$

Pictorially, $\underline{\Gamma = \text{quanta of space}}$ with adjacency relations



Use Peter-Weyl decomposition in $SU(2)$ irreps:

$$\mathcal{H}_\Gamma = \oplus_{j_l} \otimes_N \mathcal{H}_N, \quad \mathcal{H}_N = \text{Inv} \otimes_{l \supset N} \mathcal{H}^{j_l} \tag{2}$$

Choosing Bloch $SU(2)$ coh. states for $\mathcal{H}^{j_l}$ have (overcomplete) basis $|\Gamma, j_l, \vec{n}_{Nl}\rangle$ of spin-nets

KINEMATICS: geometry operators and semiclassical interpretation

Based on flux-holonomy algebra: $\boxed{L_l = \int_{l^*} E \in su(2), \quad g_l = \mathcal{P}e^{\int_l A} \in SU(2)}$

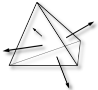
$$\{\vec{L}_l, g_{l'}\} = \gamma\hbar\, \vec{\tau} g_{l'} \delta_{ll'} \qquad \vec{\tau}\ su(2) \text{ gen.} \qquad \gamma \text{ Barbero-Immirzi parameter} \tag{3}$$

Quantization:

- $\mathcal{H}_\Gamma$
- flux $\longrightarrow \vec{L}_l f(g_l) = \gamma\hbar \left.\frac{d}{d\vec{\alpha}}\right|_{\alpha=0} f(e^{\vec{\alpha}\cdot\vec{\tau}} g_l)$
- holonomy $\longrightarrow g_l f(g_l) = g_l f(g_l)$

The gauge-invariant Barbieri-Penrose operator acts on each $\mathcal{H}_N$ $(N = s(l) = s(l'))$

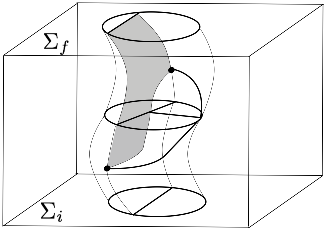
$$G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$$



$l = l'$ is area operator: $A_l = \sqrt{\vec{L}_l \cdot \vec{L}_l} \quad \rightsquigarrow \quad A_l\, |\Gamma, j, \vec{n}\rangle = \gamma\hbar\sqrt{j_l(j_l+1)}\, |\Gamma, j, \vec{n}\rangle$

$l \neq l'$ is rel. to angle operator. But non commuting, thus tetrahedron only in s.c. sense!

DYNAMICS: spin-foams (2-complex, t-evolution of spin-network with branching points)



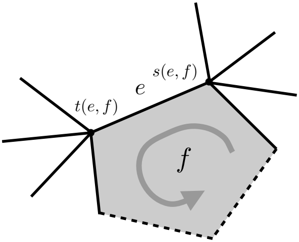
Can be thought as dual to spacetime cellular decomposition: v, e, f dual of v^*, e^*, f^*

Transition amplitude for the boundary spin-network:

$$W(j_{\text{ext}}, \vec{n}_{\text{ext}}) = \sum_{j_{\text{int}}, \vec{n}_{\text{int}}} \prod_f F(j) \prod_e E(j, \vec{n}) \prod_v V(j, \vec{n}) \tag{4}$$

Can be interpreted both as a set of Feynman rules for the vertices and propagators of the 2-complex, or as a Misner-Hawking path integral over virtual geometries

Decoration of oriented 2-complex



$$
\begin{array}{ccl}
f & \rightsquigarrow & j_f \\
(ev) & \rightsquigarrow & g_{ev} = g_{ve}^{-1} \in SL(2,\mathbb{C}) \\
(ef) & \rightsquigarrow & \vec{n}_{ef}
\end{array}
$$

Intuitively:

- j_f area of f^*
- $\vec{n}_{ef}$ unit normal to f^* in the frame of e^*
- g_{ev} parallel transport from v to e, $g_{v'v} = g_{v'e} g_{ev}$ for the full edge

The simple form of the dynamics (EPRL/FK)

$$W(j_{\text{ext}}, \vec{n}_{\text{ext}}) = \sum_{j_{\text{int}}} \int dg \int d\vec{n}_{\text{int}} \prod_f (2j_f + 1)\, P_f \tag{5}$$

Face amplitude:

$$P_f = \text{tr} \overrightarrow{\prod}_{e \subset f} P_{ef} \tag{6}$$

$$P_{ef} = g_{t(e,f),e} Y |j_f, \vec{n}_{ef}\rangle\langle j_f, \vec{n}_{ef}| Y^\dagger g^\dagger_{s(e,f),e} \quad \text{(half for ext. faces)} \tag{7}$$

Y is the embedding of $SU(2)$ irreps j into lowest weight $k = j$ of $SL(2,\mathbb{C})$ irreps $(j, \gamma j)$

$$\mathcal{H}^{(j,\gamma j)} = \bigoplus_{k \geq j} \mathcal{H}_k^{(j,\gamma j)} \tag{8}$$

Path integral-like representation (Freidel-Conrady, Barrett ..., CP-Magliaro)

$$W(j_{\text{ext}}, \vec{n}_{\text{ext}}) = \sum_{j_{\text{int}}} \int dg \int d\vec{n}_{\text{int}} \exp(S(j, g, \vec{n})) \tag{9}$$

the action S is sum of local terms (omitting extra variables $z_{vf} \in \mathbb{CP}^1$)

Truncated amplitudes vs continuum limit (taken from Rovelli's paper)

$$
\begin{tikzcd}
\text{amplitude } W(\sigma_\infty) \arrow[r, "\hbar \to 0"] & e^{iS_\infty} \\
\text{truncated amplitude } W(\sigma_N) \arrow[r, "\hbar \to 0"] \arrow[u, "N \to \infty"] & e^{iS_N} \arrow[u, "N \to \infty"]
\end{tikzcd}
$$

Low-energy behavior

Concepts of semiclassicality: the flipped limit

Area spectrum:

$$a = \gamma\hbar\sqrt{j(j+1)} \sim \gamma\hbar\, j \tag{10}$$

1. large quantum numbers $j \to \infty$, large spacetime atoms
2. alternatively, keep the size fixed but disregard quantum geometry

$$\boxed{\text{flipped limit} \quad j \to \infty,\ \gamma \to 0,\ \gamma j = \text{fix}}$$

Dynamics breaks equivalence. Restoring the Planck constant,

$$W = \int e^{\frac{1}{\hbar} S^0 + \frac{1}{\gamma\hbar} S'} \tag{11}$$

1. is the usual classical limit $\hbar \to 0$ (oscillations same magnitude)
2. the second (Holst) term oscillates more rapidly

can also combine 1. and 2., similar to earlier Bojowald's proposal in LQC.

In the semiclassical region the exponent is rapidly oscillating $\rightsquigarrow$ suppressed unless stationary phase: classical paths are generated from constructive interference

Setup of the stationary phase

Def. the partial amplitude $W(j_{\text{ext}}, \vec{n}_{\text{ext}}; j_{\text{int}})$, still no sum over internal spins

$$W(j_{\text{ext}}, \vec{n}_{\text{ext}}) = \sum_{j_{\text{int}}} W(j_{\text{ext}}, \vec{n}_{\text{ext}}; j_{\text{int}}) \tag{12}$$

PRO well-defined problem, rigorous results,

CONS partial information on equations of motion

Intuition: eom of partial amplitude analogous to variation of the connection in first order gravity

The classical limit of the partial amplitude can be studied in the asymptotic expansion

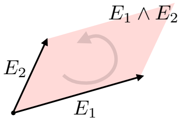
1. $W_\gamma(\alpha j_{\text{int}}, \alpha j_{\text{ext}})$, $\quad \alpha \to \infty$ (Conrady-Freidel)
2. $W_{\gamma/\alpha}(\alpha j_{\text{int}}, \alpha j_{\text{ext}})$, $\quad \alpha \to \infty$ (CP-Magliaro)

Equations of motion

1. $\text{Re}\, S = \delta_g S = \delta_{\vec{n}} S = 0$
2. $\text{Re}\, S' = \delta_g S' = \delta_{\vec{n}} S' = 0$

Result: the eom are the same 1.=2. ! for most of following consider the two cases at once

Eom in terms of 4D geometry variables



From $(j_f, g_{ev}, \vec{n}_{ef})$ define the space-like bivectors in $\mathbb{R}^{1,3}$

$$A_{ef}(v) := a_f g_{ve} * (N_e \wedge \tilde{N}_e)$$

have used a null decomposition in terms of

$$N = (1, \vec{n}), \quad \tilde{N} = (1, -\vec{n}) \tag{13}$$

Eom

- Transport $\quad A_f(v) := A_{ef}(v) = A_{e'f}(v) \qquad v \subset e, e'$
- Closure $\quad \sum_{f \supset e} \mathrm{sgn}(v, e, f) A_f(v) = 0$

plus, by construction

- Simplicity $\quad A_f(v) \wedge A_f(v) = 0$
- Cross-simplicity $\quad A_f(v) \wedge A_{f'}(v) = 0 \qquad f, f' \supset e$

These four constraints define a bivector geometry.

Result: bivector geometries are the stationary phase configurations of path integral

Emergence of classical geometry: the 4-simplex (Barrett et al.)

Given oriented Lorentzian 4-simplex (σ_v, μ_v), its area bivectors

$$A_f(\sigma_v, \mu_v) := \mu_v \, a_f * \frac{N_{s(f,v)} \wedge N_{t(f,v)}}{|N_{s(f,v)} \wedge N_{t(f,v)}|} \tag{14}$$

are nondegenerate (n.d.) and satisfy bivector geometry constraints.

Theorem (Barrett and Crane). Given a n.d. bivector geometry $A_f(v)$ there exists a unique oriented 4-simplex (σ_v, μ_v) in $\mathbb{R}^{1,3}$, up to inversion $x^\mu \to -x^\mu$ and translation, such that

$$A_f(v) = A_f(\sigma_v, \mu_v) \tag{15}$$

and μ_v is independent from ambiguity in σ_v.

Useful property: parity $P : \vec{x} \to -\vec{x}$ flips μ_v. Indeed

$$P A_f(v) = P A_f(\sigma_v, \mu_v) = A_f(P\sigma_v, -\mu_v) \tag{16}$$

Consequences: given a arbitrary n.d. stationary configuration $(j_f, g_{ev}, \vec{n}_{ef})$, there is always a P-related with μ_v constant on the 2-complex.

Call such configuration *global orientation consistent*

Symmetries of the action and of the solution space

- Local Lorentz invariance at vertices:

$$g_{ve} \to h_v g_{ve} \tag{17}$$

- Spin lift $SO^+(1,3) \to SL(2,\mathbb{C})$:

$$g_{ve} \to -g_{ve} \tag{18}$$

- Unit vectors reparametrization at edges:

$$\vec{n}_{ef} \to u_e \vec{n}_{ef} \tag{19}$$

$$g_{ve'} \to u_e g_{ve'} \tag{20}$$

where $u_e \in SU(2)$ commutes with $g_{s(e),e}$ and $g_{t(e),e}$ and $e' \neq e$.

Symmetries of the solution space but not of the action

- Local parity (see previous slide) at vertices:

$$g_{ve} \to (g_{ve}^{\dagger})^{-1} \tag{21}$$

- Global parity of the unit vectors:

$$\vec{n}_{ef} \to -\vec{n}_{ef} \tag{22}$$

Emergence of classical geometry: glueing of 4-simplices

Apply Barrett-Crane theorem to two neighboring spinfoam vertices v, v'. Reconstruct

$$(\sigma_v, \mu_v), \ \ (\sigma_{v',\mu'})$$

Theorem.

- there is a Poincaré transformation s.t. $\tau_e \to \tau'_e$, $N \to -N'$
- its rotation part i.e. the $O(1,3)$ Levi-Civita holonomy is computed as (g in adjoint)

$$U_{v'v} := \begin{cases} \mu_e \, g_{v'e} g_{ev}, & \mu_v = \mu_{v'} \\ \mu_e \, g_{v'e} P g_{ev}, & \mu_v \neq \mu_{v'} \end{cases} \tag{23}$$

μ_v =reconstructed orientations, $\mu_e = p_v p_{v'}$ =inversion ambiguity in the reconstruction

this theorem $\rightsquigarrow$ a n.d. solution of eom determine a classical Regge geometry

this theorem+symmetries $\rightsquigarrow$ solutions in same symmetry class determine the same geometry

Observe: the spinfoam on-shell $g_{v'v}$ is *not* always the Levi-Civita connection. There are in general P, T insertions at the edge. Can use P to turn any solution into a global orientation consistent, so always in first case of (23).

Curvature

The concept of curvature in simplicial spacetime is the one of Regge calculus: curvature lives on hinges. 4-simplices are individually flat but can be glued to have deficit angle.

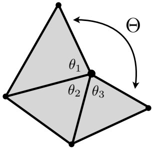


Figure: 2D analogy
Hinge=point (2D), line segment (3D), triangle (4D)

Nontrivial parallel transport around a loop, boundary of face f

$$U_f(v) := U_{vv_n} \dots U_{v'v} \tag{24}$$

For $\mu_v = const. = 1$

$$U_f(v) = (\Pi_{e \subset f} \mu_e)\, g_f(v) = e^{* A_f(v) \Theta_f} \qquad \text{(mod flip of } f^* \text{ and inversion)} \tag{25}$$

relates simply to deficit angle Θ of Regge calculus.

On-shell nondegenerate action

- For $\mu_v = 1$ configuration the stationary action gives, in the two schemes
  1. $S(a, g, \vec{n})|_{\text{on-shell}} = S^0(a, g, \vec{n})|_{\text{on-shell}} = i \sum_f a_f \Theta_f$ (Regge action)
  2. $S'(a, g, \vec{n})|_{\text{on-shell}} = 0$

  In second case, the total action is $S = S^0 + \frac{1}{\gamma} S'$, and thus get the same result as 1.
- For $\mu_v = -1$ we get the c.c., thus minus the Regge action
- For μ_v not constant (related by parity) we get only a generalized Regge action

For j_f Regge-like, in large spin and flipped limit have the asymptotic expansion

$$\boxed{W \sim A\, e^{\frac{i}{\hbar} S_{\text{Regge}}} + \text{p.r.} + \text{deg.} + \text{q-corr.}}$$

A=slowly varying prefactor different in the two schemes, p.r.= parity-related. The q-corr. are

1. $\hbar$ corrections $\rightsquigarrow$ quantum gravity corrections
2. $\gamma\hbar$ corrections $\rightsquigarrow$ quantum geometry γ-corrections comp. single term $e^{\frac{i}{\hbar} S_{EH}(g_{\mu\nu})}$

$e^{\frac{i}{\hbar} S_{\text{Regge}}}$ is quantum gravity path integral with Holst-Palatini action, and on-shell connection

Einstein eqns must be obtained studying full eom (spin variations)

Get extra eom in both schemes, but now they reveal difference

1. $\delta_{j_f} S = 0 \quad \rightsquigarrow \quad \Theta_f = 0$ flatness problem (Mamone-Rovelli, Bonzom)
2. $\delta_{j_f} S' = 0 \quad \rightsquigarrow \quad \Theta^*_f = 0$ automatically satisfied, torsionless

Suggests postulate 2. (instead of 1.) to get rid of the flatness problem.

first restrict to Regge geometries, then vary the action wrt spins

$\Downarrow$

Regge equations $\sim$ Ricci flatness **correct!**

But what is the physical origin of $\gamma \to 0$? After all, γ is a fixed parameter...

Not clear to me, but recall are using truncated effective theory and not in the continuum regime.

Could check if γ **runs to zero** (?) under coarse-graining, whatever it means

The spinfoam n-point functions (Rovelli et al.)

Semiclassical n-point function over the background coherent state $|\Psi_{a_0,\theta_0}\rangle$, schem.

$$|\Psi_{a_0,\theta_0}\rangle = \sum_j \exp\Big(-\frac{\gamma\alpha(j-j_0)^2}{j_0}\Big)\exp(i\gamma j\theta_0)|j,\vec{n}\rangle$$

codes background geometry: first real factor=<u>intrinsic</u>, second phase factor=<u>extrinsic</u>

$$\begin{aligned} &\text{0-point function:} && \langle W|\Psi_{a_0,\theta_0}\rangle \\ &\text{1-point function:} && \langle W|G|\Psi_{a_0,\theta_0}\rangle \\ &\text{2-point function:} && \langle W|GG|\Psi_{a_0,\theta_0}\rangle \\ &\vdots \end{aligned}$$

Connected n-point functions, e.g. connected 2-point function

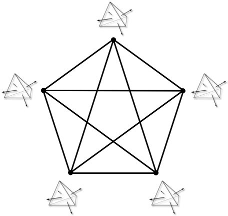
$$D^{(2)}_{12} = \frac{\langle W|G_1G_2|\Psi_{a_0,\theta_0}\rangle}{\langle W|\Psi_{a_0,\theta_0}\rangle} - \frac{\langle W|G_1|\Psi_{a_0,\theta_0}\rangle}{\langle W|\Psi_{a_0,\theta_0}\rangle}\frac{\langle W|G_2|\Psi_{a_0,\theta_0}\rangle}{\langle W|\Psi_{a_0,\theta_0}\rangle}$$

analogous to standard perturbative QFT

$$D^{(2)}_{\mu\nu\rho\sigma}(x,y) = \langle 0|h_{\mu\nu}(x)h_{\rho\sigma}(y)|0\rangle$$

Graviton propagator (Bianchi-CP-Magliaro)

$$D_{12}^{(2)} = a_0^3 (R_{12}^{(2)} + \gamma X_{12} + \gamma^2 Y_{12}) + \mathcal{O}(\hbar a_0^2)$$



- Scales as a_0^3 as expected for correlations of objects with dimensions of area square
- No component is suppressed, solves Barrett-Crane difficulties
- Matches with standard QFT and Regge calculus, up to quantum geometry γ-corrections

Most interesting feature:

- The γ corrections are parity-breaking ! $\gamma X + \gamma^2 Y = e^{i\frac{2\pi}{3}}(\gamma PX + \gamma^2 PY)$

$$\boxed{L.H. \neq R.H.} \rightsquigarrow \text{chiral gravitons?}$$

Potentially important prediction, must be studied further.

To be done

(homeworks)

- Devised a new semiclassical expansions: the flipped limit
- Have shown how classical geometry emerges from constructive interference in path integral
- Besides the 'good' $e^{iS/\hbar}$ term, parity-related terms and degenerate contribution: show these don't affect semiclassical observables (in known simple examples it is the case!)
- How does the amplitude behave in the limit of large 2-complexes?
- Do the semiclassical results for the truncated theory still hold?
- Check the robustness of graviton propagator structure, compute next orders
- Include matter in the analysis

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I know how to do it!