Towards the resolution of the CGHS singularity in loop quantum gravity

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Outline

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- New observables

Introduction

Introduction and motivation

- This work is heavily based on [Gambini, Pullin, Olmedo (2013)] for the 3+1 spherically symmetric model. There are many very similar elements:
  - Kinematical Hilbert space is essentially the same.
  - Many of the important results of the physical sector (Hilbert space, properties of solutions, etc.) are very similar or the same.
  - The new observables in quantum regime are also the same.
- In many parts of this work, we are closely or identically following their line of arguments and reproducing their results but for the CGHS case.
- So one might ask is it really interesting or useful?

Introduction and motivation: why do this? I.

First, note: an important reason for these similarities is the way we cast CGHS [Gambini, Pullin, SR (2009)] in similar variables as in the 3+1 sph. sym. [Bojowald, Swiderski (2006)]. The Hamiltonians will become very similar in this formulation.

- So why are we doing this? What is useful / interesting?
- As soon as we heard about the results of [Gambini, Pullin, Olmedo (2013)], we thought that it could be applied to the CGHS due to their similarities in this formulation. So we wanted to see if it really works for the CGHS down to the details.
- Although one can guess that one gets the same main results for the CGHS, nevertheless it is not completely certain or convincing from the outset until one shows it explicitly.
- Maybe the central important reason: we are planning to follow this analysis for the CGHS with matter (full loop quantization, backreaction, etc.). So
    - this would be a good warm up for the case with matter,
    - will give us a better understanding of possible challenges and subtleties in the details.

Introduction and motivation: why do this? II.

- If one has a an interesting LQG result, better to have it for two models instead of one!
- CGHS is interesting on its own:
  - It has black hole, Hawking radiation etc., but relatively simple and classically solvable even in the presence of matter. One hopes to be able to proceed more in quantization compared to more complicated models.
  - It is a detailed studied system, massive previous work. There are many results in the literature that might somehow connect to the LQG analysis of the model.
  - Particularly the work of [Ashtekar, Pretorius, Ramazanoglu (2010)]:
    - interesting result about evaporation, backreaction, asymptotic properties of spacetime,
    - mean field approximation not loop quantization, semiclassical, numerical (not entirely analytical).
  - It would be nice to demonstrate this interesting result (singularity resolution) for this popular model. It adds to the variety of the results we have for the CGHS.
- Also there are some rather important differences which we mention along the way, which makes CGHS not exactly the same as the 3+1 sph. sym. So some of the results might be conceptually different from 3+1 sph. sym. and give us more insight.

The CGHS model

CGHS black hole

- CGHS: a 2D dilatonic model with a pure gravitational Lagrangian

$$S_{\text{g-CGHS}} = \int d^2x \sqrt{-|g|} e^{-2\phi} \left(R + 4g^{ab} \partial_a \phi \partial_b \phi + 4\lambda^2 \right) \tag{1}$$

  with ϕ the dilaton field and $4\lambda^2$ the cosmological constant.
- In conformal gauge and in double null coordinates $x^{\pm} = x^0 \pm x^1$:

$$g_{+-} = -\frac{1}{2} e^{2\rho}, \qquad g_{--} = g_{++} = 0 \tag{2}$$

  when no matter is present, the solution is:

$$e^{-2\rho} = e^{-2\phi} = \frac{M}{\lambda} - \lambda^2 x^+ x^- \tag{3}$$

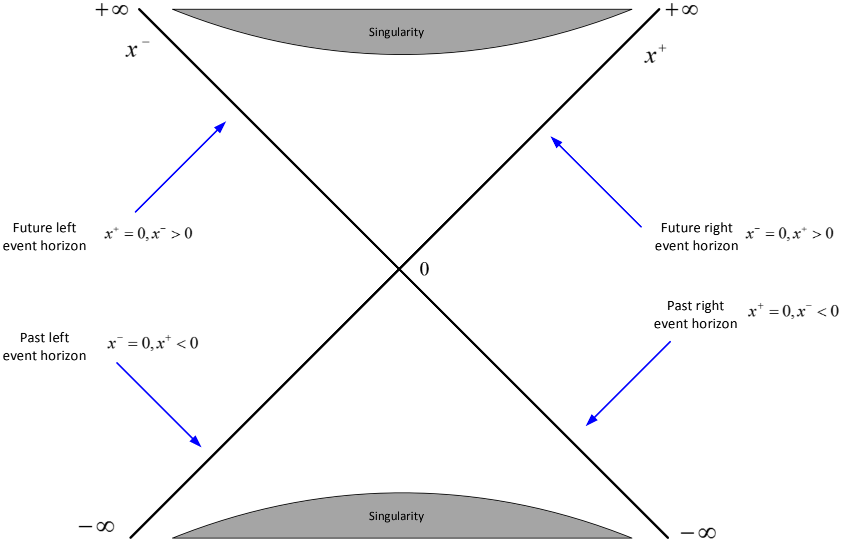
  with M the ADM (or Bondi) mass.
- Scalar curvature

$$R = \frac{4M\lambda}{\frac{M}{\lambda} - \lambda^2 x^+ x^-} \tag{4}$$

  corresponds to a black hole of mass M at

$$x^+ x^- = \frac{M}{\lambda^3}. \tag{5}$$

Kruskal diagram of the vacuum CGHS black hole



Very similar to the Kruskal diagram of the Schwarzschild.

CGHS vs. 3+1 spherically symmetric model,

Classical formulation

from a generic 2D dilatonic model

The 1+1 generic dilatonic model

- The most general diffeomorphism invariant action yielding second order differential equations for the metric g and a scalar (dilaton) field Φ [Klosch, Strobl (97)]

$$S_{1+1} = \int d^2x \sqrt{-|g|} \left\{ Y(\Phi)R + \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi + V(\Phi) \right\} \tag{6}$$

- $Y(\Phi)$ the non-minimal coupling term, $V(\Phi)$ the dilaton potential, the dilaton kinetic term can be removed via a conformal transformation.
- Both CGHS and 3+1 sph. sym. can be cast in this form and look similar

$$S_{\text{CGHS}} = \int d^2x \sqrt{-|g|} \left(\frac{1}{8} \Phi^2 R + \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi - \frac{1}{8} \Phi^2 \Lambda \right) \tag{7}$$

$$S_{\text{spher}} = \int d^2x \sqrt{-|g|} \left(\frac{1}{4} \Phi^2 R + \frac{1}{2} g^{ab} \partial_a \Phi \partial_b \Phi + \frac{1}{2} \right) \tag{8}$$

- As a result, as we will see, their Hamiltonian will also be very similar and the singularity method applied to the 3+1 sph. sym. [Gambini, Pullin, Olmedo (2013)] can be extended to the CGHS.

A subtle difference

- Although CGHS and 3+1 sph. sym. can be cast in this form and look similar

$$S_{\text{CGHS}} = \int d^2x\sqrt{-|g|}\left(\frac{1}{8}\Phi^2 R + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi - \frac{1}{8}\Phi^2\Lambda\right) \tag{9}$$

$$S_{\text{spher}} = \int d^2x\sqrt{-|g|}\left(\frac{1}{4}\Phi^2 R + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi + \frac{1}{2}\right) \tag{10}$$

  but

  - Φ in 3+1 sph. sym. is just a part of the metric in $ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \Phi^2(d\theta^2 + \sin^2(\theta)d\phi^2)$.
  - Φ in CGHS is truly a distinct degree of freedom not being present in the CGHS metric. It is truly a scalar field non-minimally coupled to gravity.

- Due to this difference, one should take extra care in quantization, representation of operators on the Hilbert space and interpreting the results.

CGHS vs. 3+1 sph. sym.: conformal transformation

- To get to the 3+1 variables suggested by [Bojowald, Swiderski (2006)]: use a conformal transformation to remove the dilaton kinetic term.
- For the similar variables in CGHS: we choose not to do a conformal transformation [Gambini, Pullin, SR (2009)] $\Rightarrow$ variables are direct-geometric $\Rightarrow$ no need to take extra care at the end; direct interpretation of variables.

CGHS vs. 3+1 sph. sym.: canonical variables I.

3+1 sph. sym.:

- Variables: $\{{}^*X^I, \omega_1\}$
- Momenta:

$$P_I = \frac{\partial L}{\partial {}^*\dot{X}^I} = 2\sqrt{q}n_I, \qquad (11)$$

$$P_\omega = \frac{\partial L}{\partial \dot{\omega}_1} = 2Y(\Phi). \qquad (12)$$

CGHS:

- Variables: $\{{}^*X^I, \omega_1, \Phi\}$
- Momenta:

$$P_I = \frac{\partial L}{\partial {}^*\dot{X}^I} = 2\sqrt{q}n_I, \qquad (13)$$

$$P_\omega = \frac{\partial L}{\partial \dot{\omega}_1} = 2Y(\Phi), \qquad (14)$$

$$P_\Phi = \frac{\partial L}{\partial \dot{\Phi}} = \frac{\sqrt{q}}{N}\left(N^1\Phi' - \dot{\Phi}\right). \qquad (15)$$

- No conformal transformation in CGHS $\Rightarrow$ presence of dilaton kinetic term (and thus $\dot{\Phi}$) in the Lagrangian $\Rightarrow$ (14) is a new primary constraint

$$\mu = P_\omega - 2Y(\Phi) \approx 0. \qquad (16)$$

CGHS vs. 3+1 sph. sym.: canonical transformations

3+1 sph. sym.: new variables are $\{(K_x, E^x), (K_\varphi, E^\varphi), (Q_\eta, \eta)\}$

Using the det. of spatial metric q:

$$|P|^2 = P_1^2 - P_2^2 = 4q = \frac{4\,(E^\varphi)^2}{(E^x)^{\frac{1}{2}}} \tag{17}$$

and thus

$$P_\omega = E^x, \tag{18}$$

$$|P| = \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}}, \tag{19}$$

$$P_1 = \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} \cosh(\eta), \tag{20}$$

$$P_2 = \frac{2E^\varphi}{(E^x)^{\frac{1}{4}}} \sinh(\eta) \tag{21}$$

CGHS: new variables are $\{(K_x, E^x), (K_\varphi, E^\varphi), (Q_\eta, \eta), (\Phi, P_\Phi)\}$

Using the det. of spatial metric q:

$$|P|^2 = P_1^2 - P_2^2 = 4q = 4\,(E^\varphi)^2 \tag{22}$$

and thus

$$P_\omega = E^x, \tag{23}$$

$$|P| = 2E^\varphi, \tag{24}$$

$$P_1 = 2\cosh(\eta)E^\varphi, \tag{25}$$

$$P_2 = 2\sinh(\eta)E^\varphi, \tag{26}$$

CGHS vs. 3+1 sph. sym.: canonical variables II.

After these transformations we have the following pairs

3+1 sph. sym.:

- Canonical pairs:

$$\{(K_x, E^x), (K_\varphi, E^\varphi)\} \tag{27}$$

- Total H (with a gauge fixing):

$$H = N\mathcal{H} + N^1\mathcal{D} \tag{28}$$

CGHS:

- Canonical pairs:

$$\{(K_x, E^x), (K_\varphi, E^\varphi), (\Phi, P_\Phi)\} \tag{29}$$

- Total H (with a gauge fixing):

$$H = N\mathcal{H} + N^1\mathcal{D} + B\mu \tag{30}$$

- Due to this gauge fixing:

$$K_x = \omega_1 \tag{31}$$

CGHS without conformal trans.: a 2nd class system I.

- The preservation of μ leads to a new constraint α

$$\dot{\mu} \approx 0 \Rightarrow \alpha = K_\varphi + \frac{1}{2}\frac{P_\Phi \Phi}{E^\varphi} \approx 0 \tag{32}$$

- These two are second class $\{\mu, \alpha\} \not\approx 0 \Rightarrow$ solve them

$$\mu = 0 \Rightarrow \Phi = 2\sqrt{E^x}, \tag{33}$$
$$\alpha = 0 \Rightarrow P_\Phi = -\frac{K_\varphi E^\varphi}{\sqrt{E^x}}. \tag{34}$$

+ introduce the Dirac brackets:

$$\{K_x(x), E^x(y)\}_D = \{K_\varphi(x), E^\varphi(y)\}_D = \{f(x), P_f(y)\}_D = \delta(x-y), \tag{35}$$
$$\{K_x(x), K_\varphi(y)\}_D = \frac{K_\varphi}{E^x}\delta(x-y), \tag{36}$$
$$\{K_x, E^\varphi\}_D = -\frac{E^\varphi}{E^x}\delta(x-y), \tag{37}$$
$$\{E^x, K_\varphi\}_D = \{E^x, E^\varphi\}_D = \{f, \text{any}\}_D = \{P_f, \text{any}\}_D = 0, \tag{38}$$

CGHS without conformal trans.: a 2nd class system II.

- The total Hamiltonian is now

$$H = N\mathcal{H} + N^1\mathcal{D} \tag{39}$$

- The canonical pairs present in the Hamiltonian are now

$$\{(K_x, E^x), (K_\varphi, E^\varphi)\} \tag{40}$$

- Also with a simple redefinition

$$U_x = K_x + \frac{E^\varphi K_\varphi}{E^x} \tag{41}$$

  the Dirac brackets can be cast into the “standard” form:

$$\{U_x(x), E^x(y)\}_D = \{K_\varphi(x), E^\varphi(y)\}_D = \{f(x), P_f(y)\}_D = \delta(x-y) \tag{42}$$

  with the rest of them being zero.

Lie algebra among constraints

- A rescaling of shift
$$\overline{N}^1 = N^1 + \frac{NK_\varphi}{E^{x\prime}} \tag{43}$$
followed by a rescaling of lapse
$$\overline{N} = N\frac{E^\varphi E^x}{E^{x\prime}} \tag{44}$$
leads to a total derivative $\mathcal{H}$:
$$H = \overline{N}\left[\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{E^{x\prime 2}}{E^{\varphi 2}E^x} - 2E^x\lambda^2 - \frac{1}{2}\frac{K_\varphi^2}{E^x}\right)\right] + \overline{N}^1\left[-U_x E^{x\prime} + f' P_f + E^\varphi K_\varphi'\right] \tag{45}$$
- This means
$$\{\mathcal{H}(x), \mathcal{H}(y)\}_D = 0 \tag{46}$$
- Now we have a Lie algebra
$$\{\mathcal{D}, \mathcal{D}\}_D = \kappa_1 \mathcal{D}, \qquad \{\mathcal{D}, \mathcal{H}\}_D = \kappa_2 \mathcal{H}, \qquad \{\mathcal{H}, \mathcal{H}\}_D = 0 \tag{47}$$
and the Dirac quantization can be pursued.

Preparing $\mathcal{H}$ for quantization

- Integrating the Hamiltonian constraint by parts, renaming $\overline{N}' \to N$ and rescaling by $N \to 2NE^{\varphi}\left(E^{x}\right)^{2}$ we will get an $\mathcal{H}$ suitable for our purposes (especially operator ordering and representation):

$$\mathcal{H}(N) = NE^{x}\left[4\left(E^{x}\right)^{2}E^{\varphi}\lambda^{2} + K_{\varphi}^{2}E^{\varphi} - 4GME^{\varphi}E^{x} - \frac{\left(E^{x\prime}\right)^{2}}{E^{\varphi}}\right] \tag{48}$$

- The term GM appears after imposing boundary condition for the lapse.
- To see the difference, compare this with the one for the 3+1 sph. sym.

$$\mathcal{H}(N) = NE^{x}\left[E^{\varphi} + K_{\varphi}^{2}E^{\varphi} - \frac{2GME^{\varphi}}{\sqrt{E^{x}}} - \frac{\left(E^{x\prime}\right)^{2}}{4E^{\varphi}}\right] \tag{49}$$

Quantization

Kinematical Hilbert space: states (CGHS & 3+1)

- Let's take the following polymerization $K_\varphi \to \sin(\rho K_\varphi)/\rho$.
- The space of Cyl consists of

$$\langle U_x, K_\varphi | g, \vec{k}, \vec{\mu} \rangle = \prod_{e_j \in g} \exp\left(\frac{i}{2} k_j \int_{e_j} dx\, U_x(x)\right) \prod_{v_j \in g} \exp\left(\frac{i}{2} \mu_j K_\varphi(v_j)\right). \quad (50)$$

with $k_j \in \mathbb{Z}$ edge color, and $\mu_j \in \mathbb{R}$ being vertex color $\Rightarrow$ point holonomies are quasi-periodic function $\Rightarrow$ nonseparable Hilbert space.

- Hilbert space of the point holonomies is $L^2(\mathbb{R}_{\text{Bohr}}, d\mu_{\text{Bohr}})$; of the normal holonomies is space of square summable functions ℓ^2; of the global degree of freedom M is $\mathcal{H}^M_{\text{kin}} = L^2(\mathbb{R}, dM)$; the whole space is:

$$\mathcal{H}_{\text{kin}} = \mathcal{H}^M_{\text{kin}} \otimes \left[\bigotimes_{j=1}^{V} \ell^2_j \otimes L^2_j(\mathbb{R}_{\text{Bohr}}, d\mu_{\text{Bohr}})\right] \quad (51)$$

- The basis set of this space is $\{|g, \vec{k}, \vec{\mu}, M\rangle\}$.

Kinematical Hilbert space: inner product (CGHS & 3+1)

- A spin network defined on g can be regarded as a spin network with support on a larger graph $\bar{g} \supset g$ by assigning trivial labels to the edges and vertices which are not in g.
- For any two graphs g and g', take $\bar{g} = g \cup g'$. Then the inner product is

$$
\begin{aligned}
\langle g, \vec{k}, \vec{\mu}, M | g', \vec{k}', \vec{\mu}', M' \rangle =& \delta(M - M') \prod_{\text{edges}} \delta_{k_j, k'_j} \prod_{\text{vertices}} \delta_{\mu_j, \mu'_j} \\
=& \delta(M - M') \delta_{\vec{k}, \vec{k}'} \delta_{\vec{\mu}, \vec{\mu}'}
\end{aligned} \tag{52}
$$

Representing the operators (CGHS & 3+1)

- Due to the canonical relations, we have the following representation

$$\hat{E}_\varphi|g,\vec{k},\vec{\mu},M\rangle =\ell_{\rm Pl}^2\sum_{v_j\in g}\delta(x-x_j)\mu_j|g,\vec{k},\vec{\mu},M\rangle \tag{53}$$

$$\hat{E}_x|g,\vec{k},\vec{\mu},M\rangle =\ell_{\rm Pl}^2 k_j|g,\vec{k},\vec{\mu},M\rangle \tag{54}$$

$$\hat{M}|g,\vec{k},\vec{\mu},M\rangle =M|g,\vec{k},\vec{\mu},M\rangle \tag{55}$$

- $\hat{M}$ corresponds to the Dirac observable on the boundary associated to the mass of the black hole.
- The operator associated to the holonomy is

$$\hat{N}^\varphi_{n\rho}(x)|g,\vec{k},\vec{\mu},M\rangle = |g,\vec{k},\vec{\mu}'_{\pm n\rho},M\rangle, \qquad n\in\mathbb{N} \tag{56}$$

where the new vector $\vec{\mu}'_{\pm n\rho}$ either has just the same components as $\vec{\mu}$ up to $\mu_j\to\mu_j\pm n\rho$ if x coincides with a vertex of the graph located at $x(v_j)$, or it will be $\vec{\mu}$ with a new component $\{\ldots,\mu_j,\pm n\rho,\mu_{j+1},\ldots\}$ with $x(v_j)<x<x(v_{j+1})$.

Representing the operators (CGHS vs. 3+1)

- Note the difference in the interpretation of the E^x although they are represented the same in both models

$$\hat{E}_x|g,\vec{k},\vec{\mu},M\rangle = \ell_{\rm Pl}^2 k_j|g,\vec{k},\vec{\mu},M\rangle \tag{57}$$

  - In the CGHS case, classically E^x is actually the (square) of the dilaton field and should have info about that, while in the 3+1 sph. sym. it is the triad component and have information about the metric.
  - Care should be taken in interpreting and applying the seemingly same results, especially in cases involving E^x.

Hamiltonian constraint operator I.

- Adopt the following form and ordering for $\mathcal{H}$:

$$\hat{\mathcal{H}}(N) = \int dx N(x) \hat{E}^x \left\{ \hat{\Theta} + 4\left(1 - \frac{G\hat{M}}{\hat{E}^x \lambda^2}\right) \hat{E}^\varphi \lambda^2 (\hat{E}^x)^2 - \widehat{\left[\frac{1}{\hat{E}^\varphi}\right]} \left[(\hat{E}^x)'\right]^2 \right\} \tag{58}$$

- $\hat{\Theta}(x)$ acting on the kinematical states as [Martin-Benito, Mena Marugan, Olmedo, Pawlowski]

$$\hat{\Theta}(x)|g, \vec{k}, \vec{\mu}, M\rangle = \sum_{v_j \in g} \delta(x - x(v_j)) \hat{\Omega}^2_\varphi(v_j)|g, \vec{k}, \vec{\mu}, M\rangle, \tag{59}$$

is defined by means of the non-diagonal operator

$$\hat{\Omega}_\varphi(v_j) = \frac{1}{4i\rho} |\hat{E}^\varphi|^{1/4} \left[\widehat{\mathrm{sgn}(E^\varphi)} \left(\hat{N}^\varphi_{2\rho} - \hat{N}^\varphi_{-2\rho}\right) + \left(\hat{N}^\varphi_{2\rho} - \hat{N}^\varphi_{-2\rho}\right) \widehat{\mathrm{sgn}(E^\varphi)}\right] |\hat{E}^\varphi|^{1/4} \Big|_{v_j}, \tag{60}$$

where

$$|\hat{E}^\varphi|^{1/4}(v_j)|g, \vec{k}, \vec{\mu}, M\rangle = \ell_{\mathrm{Pl}}^{1/2} |\mu_j|^{1/4} |g, \vec{k}, \vec{\mu}, M\rangle, \tag{61}$$

$$\widehat{\mathrm{sgn}(E^\varphi(v_j))}|g, \vec{k}, \vec{\mu}, M\rangle = \mathrm{sgn}(\mu_j)|g, \vec{k}, \vec{\mu}, M\rangle, \tag{62}$$

have been constructed by means of the spectral decomposition of $\hat{E}^\varphi$ on the kinematical Hilbert space.

Hamiltonian constraint operator II.

- Also for the operator $\widehat{\left[1/\hat{E}^{\varphi}\right]}$, we use the Thiemann's trick

$$\frac{\mathrm{sgn}(E^{\varphi})}{\sqrt{|E^{\varphi}|}} = \frac{2}{G}\{K_{\varphi}, \sqrt{E^{\varphi}}\} \tag{63}$$

to get

$$\widehat{\left[\frac{1}{\hat{E}^{\varphi}}\right]}|g,\vec{k},\vec{\mu},M\rangle = \sum_{v_j \in g} \delta(x - x(v_j)) \frac{\mathrm{sgn}(\mu_j)}{\ell_{\mathrm{Pl}}^2 \rho^2} (|\mu_j + \rho|^{1/2} - |\mu_j - \rho|^{1/2})^2 |g,\vec{k},\vec{\mu},M\rangle \tag{64}$$

Solving Hamiltonian constraint

- Acting the Hamiltonian constraint on $|g, \vec{k}, \vec{\mu}, M\rangle$ yields

$$\hat{\mathcal{H}}(N)|g, \vec{k}, \vec{\mu}, M\rangle = \sum_{v_j \in g} N(x_j)(\ell_{\mathrm{Pl}}^4 k_j) \Big[f_0(\mu_j, k_j, M)|g, \vec{k}, \vec{\mu}, M\rangle - f_+(\mu_j)|g, \vec{k}, \vec{\mu}_{+4\rho_j}, M\rangle - f_-(\mu_j)|g, \vec{k}, \vec{\mu}_{-4\rho_j}, M\rangle \Big] \tag{65}$$

- It can be seen that the RHS above vanishes for $k_j = 0$.
- Also the functions $f_\pm$ and f_0 are such that the RHS of above is vanishing for $\mu_j = 0$.
- Above equation slightly different in 3+1 sph. sym. (details of the forms of the f's for example) but the above two point are true there too.
- Let's call $\{|g, 0\rangle\}$ the set of states with $\mu_j = 0$ and/or $k_j = 0$.

Singularity resolution

- The states $\{|g,0\rangle\}$ have finite norm. This together with group averaging allows us to decouple them from the physical Hilbert space.
- Thus we are left only with the orthogonal complement of $\{|g,0\rangle\}$ for the physical Hilbert space.
- The volume operator for the CGHS model is

$$\hat{\mathcal{V}}|g,\vec{k},\vec{\mu},M\rangle \propto \sum_{v_j \in g} \mu_j |g,\vec{k},\vec{\mu},M\rangle \tag{66}$$

- Thus since the physical Hilbert space does not contain any state with $\mu_j = 0$ and/or $k_j = 0$, there is no singularity state present at the quantum level.

Other properties of the solutions I. (CGHS & 3+1)

- The states annihilated by $\hat{\mathcal{H}}(N)$, assuming they belong to Cyl*, can be written as

$$(\Psi_g| = \int_0^\infty dM \sum_{\vec{k}} \sum_{\vec{\mu}} \langle g, \vec{k}, \vec{\mu}, M| \psi(M) \chi(\vec{k}) \phi(\vec{k}, \vec{\mu}, M) \tag{67}$$

with $\psi(M)$ being the analog of Kuchař mass function and $\chi(\vec{k})$ an arbitrary function of finite norm.

- The action of the $\hat{\mathcal{H}}(N)$ on these, $\sum_{v_j \in g} (\Psi_g | \hat{H}(N_j)^\dagger = 0$, gives a set of difference equations, one for each v_j,

$$-f_+(\mu_j - 4\rho)\phi_j(\mu_j - 4) - f_-(\mu_j + 4\rho)\phi_j(\mu_j + 4) f_0(k_j, k_{j-1}, \mu_j, M)\phi_j(\mu_j) = 0, \tag{68}$$

where there is a natural decomposition

$$\phi(\vec{k}, \vec{\mu}, M) = \prod_{j=1}^{V} \phi_j(k_j, k_{j-1}, \mu_j, M) = \prod_{j=1}^{V} \phi_j(\mu_j) \tag{69}$$

- This action does not mix different graphs g: The subspace associated with a given graph g is preserved by the action of the scalar constraint such that no new vertices are created.

Other properties of the solutions II. (CGHS & 3+1)

- Looking at (65), we note that under the action of $\hat{\mathcal{H}}(N)$:
  - The color of edges $\{k_j\}$ in preserved.
  - The color of the vertices, is mixed by means of a difference operator of step 4ρ in the labels μ_j.
- The Hamiltonian constraint only mixes states with support in lattices of the labels μ_j, with $j = 1, 2, \ldots, V$, of step 4ρ.
- The functions $f_{\pm}(\mu_j)$ vanish in the intervals $[0, \mp 2\rho]$ respectively $\Rightarrow$ different orientations of the labels μ_j are decoupled.
- So, the solution states belong to the subspaces with support on the semilattices $\mu_j = \epsilon_j \pm 4n\rho$, with $n \in \mathbb{N}$ and $\epsilon_j \in (0, 4\rho]$:
  - The constraint only relates states belonging to separable subspaces of the kinematical one.

Work in progress: physical Hilbert space

- To complete the analysis of this part, one should apply group averaging to the aforementioned states to find the solution to the constraint,

$$(\Psi_g^C| = \int_{-\infty}^{\infty} d\lambda_1 \cdots \int_{-\infty}^{\infty} d\lambda_V \; e^{i\sum_j \lambda_j \hat{C}_j^\dagger} \int_0^{\infty} dM \times \sum_{\vec{k}} \sum_{\vec{\mu}} \langle g, \vec{k}, \vec{\mu}, M | \psi(M) \chi(\vec{k}) \phi(\vec{k}; \vec{\mu}; M) \tag{70}$$

with $\hat{C}_j^\dagger$ being the difference operator in $\hat{\mathcal{H}}(N_j) = N_j \hat{C}_j$

- It should also be equipped with an inner product that is induced by the averaging process.

Work in progress: Diff. group averaging, inner product, etc.

- To find the physical sector of the model we should apply group averaging with respect to the diffeomorphism too. This process:
  - picks out the diffeomorphism invariant states,
  - induces a natural inner product on the resulting vector space of these states.
- Since there is only one dimensional spatial manifold:
  - diffeomorphism constraint averages states on the the only (radial) direction,
  - the resulting space of diffeomorphism invariant states is given by linear combinations of spin networks with vertices in all possible positions along the radial line.
  - The corresponding $\mathcal{H}_{\text{Diff}}$ has basis states, characterized by the diffeomorphism class of graphs $[g]$, and each state in a given class is characterized by colorings of edges and vertices (s-knot states).

New observables

- Due to the symmetry group, the order of the position of the vertices of the diffeomorphism invariant states is preserved: they can not pass each other.
- The coloring of the edges k_j and and the order of vertices is also preserved.
- So we can identify two new strictly quantum observables in the bulk:
  - An observable $\hat{N}_v$ corresponding to the fixed number N_v of vertices,

$$\hat{N}_v \Psi_{\text{phys}} = N_v \Psi_{\text{phys}} \tag{71}$$

  - An observable $\hat{O}$ associated to the order of the vertices in the graph such that

$$\hat{O}(z)\Psi_{\text{phys}} = \ell_{\text{Pl}}^2 k_{\text{Int}(zN_v)} \Psi_{\text{phys}}, \qquad z \in [0,1] \tag{72}$$

  with $\text{Int}(zN_v)$ being the integer part of zN_v.

Summary

- As a result of the variable chosen for the CGHS, its Hamiltonian can be written in a very similar form to the 3+1 sph. sym. This allows us to extend the [Gambini, Pullin, Olmedo (2013)] method to the vacuum CGHS to
  - resolve the singularity
  - obtain new observables
- Many of the aspects of this analysis is similar or identical to the 3+1 sph. sym. case. Although there are some mathematical and conceptual issues that should be understood better.
- There are also some important differences, e.g. E^x in CGHS is classically the dilaton field and not related to the metric while in 3+1 sph. sym. case it is the triad.
- One can try to apply the same method to the CGHS with matter. This will be more complicated for several reasons, among them the form of the Hamiltonian constraint that can not be integrated by part when matter is present. As a result its representation and ordering will be more involved.