Loop quantization of Kruskal spacetime

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ILQGS Black Hole Panel Discussion

Based on work with Abhay Ashtekar and Javier Olmedo

Loop quantization of Schwarzschild black holes

Interior of the Schwarzschild black hole corresponds to a Kantowski-Sachs vacuum spacetime. Loop quantization can be performed following techniques of LQC where quantum Riemannian geometry results in a big bounce.

Spatial metric is homogeneous but not isotropic. Presence of Weyl + spatial curvature makes quantization far more challenging. Physical implications studied by many groups:

(Ashtekar, Bodendorfer, Bojowald, Boehmer, Campiglia, Corichi, Han, Husain, Gambini, Li, Liu, Mele, Mena Marugán, Modesto, Münch, Navascués, Noui, Olmedo, Pullin, Rastgoo, Saini, PS, Perez, Wang, Wilson-Ewing)

Caveats:

Idealized picture of an eternal black hole.

Most of the results, including for dynamical situations, extracted assuming an effective spacetime description. In cosmological models this picture found to be quite accurate using HPC including in presence of Weyl curvature **(Diener, Joe, Megevand PS (17))**

Classical aspects

Spatial manifold: $\mathbb{R} \times \mathbb{S}^2$, with a fiducial metric:

$$\mathrm{d}s_o^2 = \mathrm{d}x^2 + r_o^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

Restrict the non-compact x coordinate by L_o. Fiducial volume of the fiducial cell $V_o = 4\pi r_o^2 L_o$.

With underlying symmetries, connection and triads become:

$$A_a^i\,\tau_i\,\mathrm{d}x^a \;=\; \bar{c}\,\tau_3\,\mathrm{d}x + \bar{b}\,r_o\,\tau_2\mathrm{d}\theta - \bar{b}\,r_o\,\tau_1\sin\theta\,\mathrm{d}\phi + \tau_3\cos\theta\,\mathrm{d}\phi$$

$$E_i^a\,\tau^i\,\frac{\partial}{\partial x^a} \;=\; \bar{p}_c\,r_o^2\,\tau_3\,\sin\theta\,\frac{\partial}{\partial x} + \bar{p}_b\,r_o\,\tau_2\,\sin\theta\,\frac{\partial}{\partial\theta} - \bar{p}_b\,r_o\,\tau_1\,\frac{\partial}{\partial\phi}$$

Introduce $c = L_o\,\bar{c},\;\; p_c = r_o^2\,\bar{p}_c,\;\; b = r_o\,\bar{b},\;\; p_b = r_o\,L_o\,\bar{p}_b$

Under $L_o \to \alpha L_o$: $c \to \alpha c$, $p_b \to \alpha p_b$, but p_c and b invariant.

Physics must be independent of this freedom. Curvature scale at the bounce should not depend on $\alpha \Rightarrow$ a consistent quantization.

Horizon at $p_b = b = 0$, singularity at $p_b = p_c = 0$.

Quantum aspects

Different quantizations result from different ways of expressing curvature in terms of holonomies, and how smallest loop area assigned using quantum geometry.

Quantum constraint consists of $\sin(\delta_b b)/\delta_b$ and $\sin(\delta_c c)/\delta_c$. δ_b is fractional length of each link of the plaquette on θ-ϕ 2-spheres, and δ_c is the fractional length of links in the x-direction for plaquettes in the θ-x and ϕ-x planes.

Departures from classical theory should occur only in the Planck regime, not when spacetime curvature is very small.

Many pitfalls even though models may seem non-singular:

Is spacetime curvature at singularity resolution universal? If not, how does it scale with mass of the black hole?

Do loop quantum effects distinguish phys. vs coord. singularity?

How symmetric or asymmetric is the bounce?

A prescription based on transition surface

Use δ_b and δ_c such that they are phase space functions which are constants along dynamical trajectories. Each solution is identified by a transition surface $\mathcal{T}$ which occurs at $T_{\mathcal{T}} = \frac{1}{2} \ln \left(\frac{\gamma L_o \delta_c}{8m} \right)$ and joins the black hole region with the white hole region.

(Ashtekar, Olmedo, PS (2018))

Consider plaquettes on the transition surface $\mathcal{T}$, where curvature invariants take largest values.

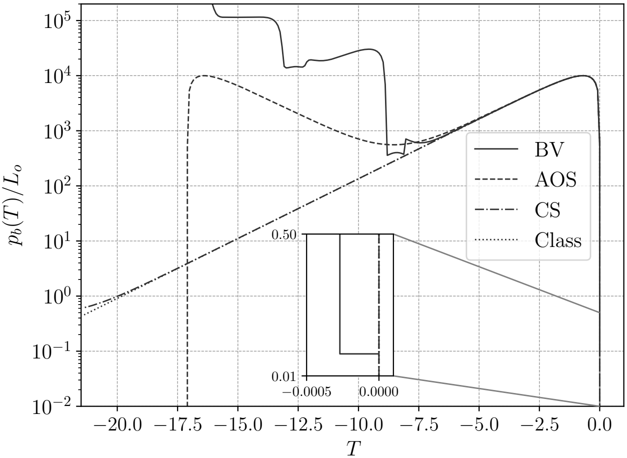
Physical fractional area of the anullus around equators computed at $\mathcal{T}$ gives $2\pi\, \delta_c \delta_b\, |p_b| = \Delta$

Physical fractional area of 2-sphere at $\mathcal{T}$ yields $4\pi\, \delta_b^2\, p_c = \Delta$

For BH masses much larger than Planck mass:

$$\delta_b = \left(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m} \right)^{1/3}, \qquad L_o \delta_c = \frac{1}{2} \left(\frac{\gamma \Delta^2}{4\pi^2 m} \right)^{1/3}.$$

Comparison of various prescriptions



In AOS approach, a symmetric bounce occurs with BH mass = WH mass, unlike in Corichi-Singh approach.

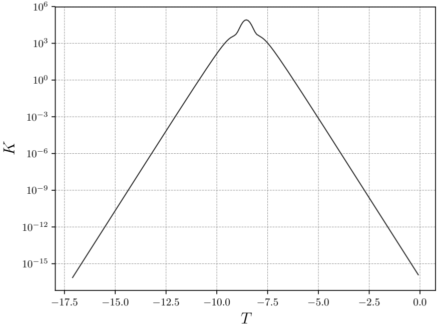
In Boehmer-Vandersloot approach a WH horizon does not emerge after bounce and there is a big departure from GR near BH horizon.

Curvature invariants

Unlike other attempts, curvature invariants are bounded above, and have a maximum value at the transition surface that is independent of BH mass:

$$C_{abcd}C^{abcd}\,|_{\mathcal{T}} = \frac{1024\pi^2}{3\gamma^4\Delta^2} + \mathcal{O}\Big(\big(\frac{\Delta}{m^2}\big)^{\frac{1}{3}}\,\ln\frac{m^2}{\Delta}\Big)$$

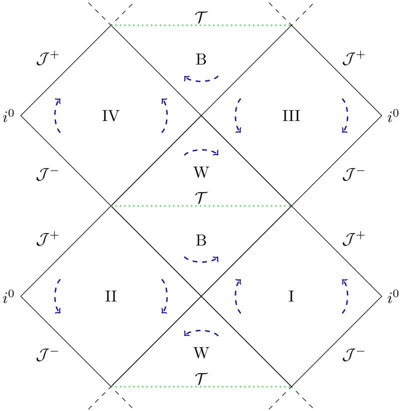
$$K\,|_{\mathcal{T}} = \frac{768\pi^2}{\gamma^4\Delta^2} + \mathcal{O}\Big(\big(\frac{\Delta}{m^2}\big)^{\frac{1}{3}}\,\ln\frac{m^2}{\Delta}\Big)$$



A complete spacetime picture

We can obtain an extension to the exterior using time-like hypersurfaces with a 3-metric with $(- + +)$ signature.

Connection and triads are $SU(1,1)$ valued. Following strategy for interior, effective Hamiltonian for exterior can be obtained. δ_b and δ_c fixed using interior. Detailed asymptotic properties studied.



Summary/open issues

- Loop quantization of Schwarzschild spacetime has been a tricky problem. Ideas which have been quite successful in LQC have to be used with caution.
- AOS prescription, guided by effective dynamics, provides a consistent picture of physics. Passes various tests for viability.
- There exist an infinite number of trapped, anti-trapped and asymptotic regions, with consecutive asymptotic regions having identical ADM mass. (Contrast with BV scheme).
- Curvature scalars have an upper bound at the transition surface which is independent of mass for macroscopic black holes. (Contrast with CS scheme).
- GR is recovered in low curvature regimes.
- What are the lessons for quantization prescriptions in general, in particular for dynamical collapse scenarios?