How do quantization ambiguities affect the spacetime across the central singularity?

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Based on A Corichi, PS arXiv:1506.08015,
N Dadhich, A Joe, PS arXiv:1505.05727,
A Joe, PS (To appear)

(and also on Ashtekar-Bojowald (06); Boehmer-Vandersloot (07))

Which quantization ambiguities?

Rigorous quantization of various isotropic, anisotropic and black hole spacetimes using techniques of LQG has been performed in the last decade. Different ways to construct loops and their area assignment yield different quantum Hamiltonian constraints.

Examples of these quantization ambiguities:

- Old quantization in LQC (Ashtekar, Bojowald, Date, Lewandowski, Pawlowski, PS, Vandersloot (01-06)): Loops considered with constant areas led to a quantum difference equation with equal steps in triads. Bounce occurs but drastic violations at small spacetime curvature for matter violating strong energy condition (recollapse instead of accelerated expansion!). Expansion scalar depends on fiducial cell. Inconsistent physics.
- Improved dynamics in LQC (Ashtekar, Pawlowski, PS (06)): Loops with areas which depend on the triad. Difference equation with a uniform grid in volume. No fiducial cell dependence. Consistent physics.

Are there not "unique quantizations"?

- Isotropic models: Only one way (improved dynamics) to consider loops such that following are all true (Corichi, PS (08))
  - Expansion scalar invariant under fiducial cell freedoms
  - Unambiguous curvature scale for singularity resolution
  - Recovery of GR at small spacetime curvature

- Bianchi-I models (Ashtekar, Chiou, Martin-Benito, Mena Marugan, Pawlowski, Vandersloot, Wilson-Ewing (08-12)): If spatial manifold $\mathbb{T}^3$ then two quantizations are allowed which satisfy above criterion. Only Ashtekar-Wilson-Ewing quantization allowed if spatial manifold is $\mathbb{R}^3$ (Corichi, PS (09)). Physics qualitatively similar in most situations on the other side of the bounce for both choices. But, special situations with drastic differences exist (eg. flat Kasner (PS(16)).

- Spatially curved models: Alternative quantization possible. Leads to two distinct bounces (Corichi, Karami (12)), in contrast to a distinct bounce in the improved dynamics quantization ((Ashtekar, Pawlowski, PS, Vandersloot (07)).

Schwarzschild interior/Kantowski-Sachs spacetime

Following the methods used for loop quantization of cosmological models, various authors have studied loop quantization of black hole spacetimes (Ashtekar, Bojowald (06); Modesto (06); Boehmer, Vandersloot (07); Campiglia, Gambini, Pullin, Olmedo, Rastgoo (07-15)). Singularity resolution indicated, but many details remain unaddressed.

Till recently, existing quantizations are:

Loops with 'constant areas': Based on old LQC. Non-singular quantum difference equation. Correct GR limit at small spacetime curvature. But scale at which bounce occurs is arbitrary due to fiducial length rescaling. As a result, second black hole's (or 'white hole') mass is arbitrary (Boehmer, Vandersloot (07)).

Loops with triad dependent areas (Boehmer, Vandersloot (07)): Analog of improved dynamics. Non-singular quantum difference equation. Independence from fiducial length freedom. No other consistent choice with triad dependent areas (Joe, PS (15)). Generic strong singularity resolution (Saini, PS (16)).

Unfortunately, Boehmer-Vandersloot prescription does not yield GR at small spacetime curvature. Coordinate singularity at the horizon creates "Planck scale" effects.

But it is still a consistent quantization for Kantowski-Sachs cosmology (no black hole interior). No problems with infra-red issues if no horizon. Indications in earlier works that spacetime after bounce retains quantum character and is "Nariai type." Exact nature now clarified (Dadhich, Joe, PS (15))

Recently an alternative loop quantization of the Schwarzschild interior (using constant areas of loops) which is free of fiducial cell issues, gives GR in infra-red scale and resolves the singularity found (Corichi, PS (15)) (Improvement of Ashtekar-Bojowald quantization).

Ignoring the horizon issue in Boehmer-Vandersloot prescription, two ways to loop quantize a Schwarzschild interior. Unlike quantization ambiguities in other spacetimes, we get very different properties of spacetime after the bounce. Resulting picture of bounce very different from LQC isotropic bounce.

Outline:

- Preliminaries: Classical aspects
- Quantum constraint and physics from effective dynamics in constant area quantization
- Nature of spacetime after bounce using effective dynamics in Boehmer-Vandersloot approach (generalized to include matter)
- Conclusions

Classical aspects

Schwarzschild interior can be described by a vacuum Kantowski-Sachs spacetime. Spatial manifold: $\mathbb{R} \times \mathbb{S}^2$, with a fiducial metric:

$$\mathrm{d}s_o^2 = \mathrm{d}x^2 + r_o^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

To define symplectic structure, restrict the non-compact x coordinate by L_o. Fiducial volume of the cell $V_o = 4\pi r_o^2 L_o$.

Using the symmetries, the connection and triads become:

$$A_a^i\,\tau_i\,\mathrm{d}x^a = \bar{c}\,\tau_3\,\mathrm{d}x + \bar{b}\,r_o\,\tau_2\mathrm{d}\theta - \bar{b}\,r_o\,\tau_1\sin\theta\,\mathrm{d}\phi + \tau_3\cos\theta\,\mathrm{d}\phi$$

$$E_i^a\,\tau^i\frac{\partial}{\partial x^a} = \bar{p}_c\,r_o^2\,\tau_3\,\sin\theta\,\frac{\partial}{\partial x} + \bar{p}_b\,r_o\,\tau_2\,\sin\theta\,\frac{\partial}{\partial\theta} - \bar{p}_b\,r_o\,\tau_1\,\frac{\partial}{\partial\phi}$$

Symplectic structure in terms of $(\bar{b}, \bar{p}_b)$ and $(\bar{c}, \bar{p}_c)$ not invariant under the change of fiducial metric. Introduce

$$c = L_o\,\bar{c}, \quad p_c = r_o^2\,\bar{p}_c, \quad b = r_o\,\bar{b}, \quad p_b = r_o\,L_o\,\bar{p}_b$$

which satisfy $\{c, p_c\} = 2G\gamma, \quad \{b, p_b\} = G\gamma$

Classical aspects

Freedom to rescale fiducial length L_o still exists.
Under $L_o \to \alpha L_o$: $c \to \alpha c$, $p_b \to \alpha p_b$. p_c and b invariant.

The goal will be to choose the variables such that the quantization should be independent of the freedom to rescale L_o.

Spacetime metric:

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \frac{p_b^2}{|p_c| L_o^2}\, \mathrm{d}x^2 + |p_c| \left(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2\right)$$

where
$$\frac{p_b^2}{|p_c| L_o^2} = (2m/t - 1), \qquad |p_c| = t^2$$

Classical solutions:

$$b(t) = \pm\gamma \sqrt{(2m - t)/t}\ , \quad p_b(t) = L_o \sqrt{t(2m - t)}$$

$$c(t) = \mp\gamma \frac{m L_o}{t^2}, \qquad \text{and} \quad p_c(t) = \pm t^2$$

Singularity at $p_b = 0$ and $p_c = 0$. Horizon at $p_b = 0$, $p_c = 4m^2$.

Quantization

Holonomies of connections b and c generate an algebra of almost periodic functions with elements: $\exp(i(\mu b + \tau c)/2)$. Kinematical Hilbert space: $L^2(\mathbb{R}_{\mathrm{Bohr}}, \mathrm{d}\mu_b)$.

Action of triad operators:

$$\hat{p}_b \, |\mu, \tau\rangle = \frac{\gamma l_{\mathrm{Pl}}^2}{2} \, \mu \, |\mu, \tau\rangle, \quad \hat{p}_c \, |\mu, \tau\rangle = \gamma l_{\mathrm{Pl}}^2 \, \tau \, |\mu, \tau\rangle$$

Classical Hamiltonian constraint:

$$C_{\mathrm{Ham}} = -\int \mathrm{d}^3x \, e^{-1} \varepsilon_{ijk} E^{ai} E^{bj} (\gamma^{-2} \, F_{ab}^k - \Omega_{ab}^k)$$

$$\varepsilon_{ijk} e^{-1} E^{aj} E^{bk} = \sum_k \frac{{}^o\varepsilon^{abc} \, {}^o\omega_c^k}{2\pi\gamma G \delta_{(k)} \ell_{(k)}} \mathrm{Tr} \left(h_k^{(\delta_{(k)})} \, \{ (h_k^{(\delta_{(k)})})^{-1}, V \} \tau_i \right)$$

$$F_{ab}^k = -2 \lim_{Ar_\square \to 0} \mathrm{Tr} \left(\frac{h_{\square_{ij}}^{(\delta_{(i)}, \delta_{(j)})} - 1}{\delta_{(i)} \delta_{(j)} \ell_{(i)} \ell_{(j)}} \right) \tau^k \, \omega_a^i \, \omega_b^j$$

Holonomies considered over loops in $x-\theta$, $x-\phi$ and $\theta-\phi$ planes.

Loops with fixed area: The edge of the loop along x direction has length $\delta_c L_o$, and the edges along $\mathbb{S}^2$ have length $\delta_b r_o$. Quantum geometry fixes minimum area of loops ($\Delta = 4\sqrt{3}\pi\gamma l_{\rm Pl}^2$).

Minimum area of loop $\Box_{\theta-\phi}$: $(\delta_b r_o)^2 = \Delta$

Minimum area of the loops $\Box_{x-\theta}$ and $\Box_{x-\phi}$: $\delta_b r_o \delta_c L_o = \Delta$

Loops with triad dependent area:
Minimum area of loop $\Box_{\theta-\phi}$: $\delta_b^2 p_c = \Delta$

Minimum area of the loops $\Box_{x-\theta}$ and $\Box_{x-\phi}$: $\delta_b \delta_c p_b = \Delta$

In the fixed area case, quantization results in a quantum difference equation with fixed steps in triads. Non-singular evolution across the central singularity. Considering τ as a clock, and specifying the wavefunction at $\tau = 2n\delta_c$ and $\tau = 2(n-1)\delta_c$, wavefunction can be evolved across $\tau = 0$.

Quantum Hamiltonian constraint:

$$\begin{aligned}
\hat{C}\Psi(\mu,\tau) = \frac{1}{2\gamma^3\delta_b^2\delta_c l_{\rm Pl}^2}\Big[& \left(V_{\mu+\delta_b,\tau} - V_{\mu-\delta_b,\tau} + V_{\mu+3\delta_b,\tau+2\delta_c} - V_{\mu+\delta_b,\tau+2\delta_c}\right)\Psi(\mu+2\delta_b,\tau+2\delta_c) \\
& + \left(V_{\mu-\delta_b,\tau} - V_{\mu+\delta_b,\tau} + V_{\mu+\delta_b,\tau-2\delta_c} - V_{\mu+3\delta_b,\tau-2\delta_c}\right)\Psi(\mu+2\delta_b,\tau-2\delta_c) \\
& + \left(V_{\mu-\delta_b,\tau} - V_{\mu+\delta_b,\tau} + V_{\mu-3\delta_b,\tau-2\delta_c} - V_{\mu-\delta_b,\tau+2\delta_c}\right)\Psi(\mu-2\delta_b,\tau+2\delta_c) \\
& + \left(V_{\mu+\delta_b,\tau} - V_{\mu-\delta_b,\tau} + V_{\mu-\delta_b,\tau-2\delta_c} - V_{\mu-3\delta_b,\tau-2\delta_c}\right)\Psi(\mu-2\delta_b,\tau-2\delta_c) \\
& + (1/2)\ \Big[\left(V_{\mu,\tau+\delta_c} - V_{\mu,\tau-\delta_c} + V_{\mu+4\delta_b,\tau+\delta_c} - V_{\mu+4\delta_b,\tau-\delta_c}\right)\Psi(\mu+4\delta_b,\tau) \\
& + \left(V_{\mu,\tau+\delta_c} - V_{\mu,\tau-\delta_c} + V_{\mu-4\delta_b,\tau+\delta_c} - V_{\mu-4\delta_b,\tau-\delta_c}\right)\Psi(\mu-4\delta_b,\tau)\Big] \\
& + 2\,(1+2\,\gamma^2\delta_b^2)(V_{\mu,\tau-\delta_c} - V_{\mu,\tau+\delta_c})\Psi(\mu,\tau)\Big]
\end{aligned}$$

Reduces to the difference equation in Ashtekar-Bojowald quantization when $\delta_b = \delta_c$.

Various details of physical Hilbert space still unexplored.

Physics from effective Hamiltonian dynamics

Effective Hamiltonian:

$$H_{\text{eff}} = -\frac{N\text{sgn}(p_c)}{2G\gamma^2}\left[2\frac{\sin(\delta_c c)}{\delta_c}\frac{\sin(\delta_b b)}{\delta_b}\,|p_c|^{1/2} + \left(\frac{\sin^2(\delta_b b)}{\delta_b^2} + \gamma^2\right)\,p_b\,|p_c|^{-1/2}\right]$$

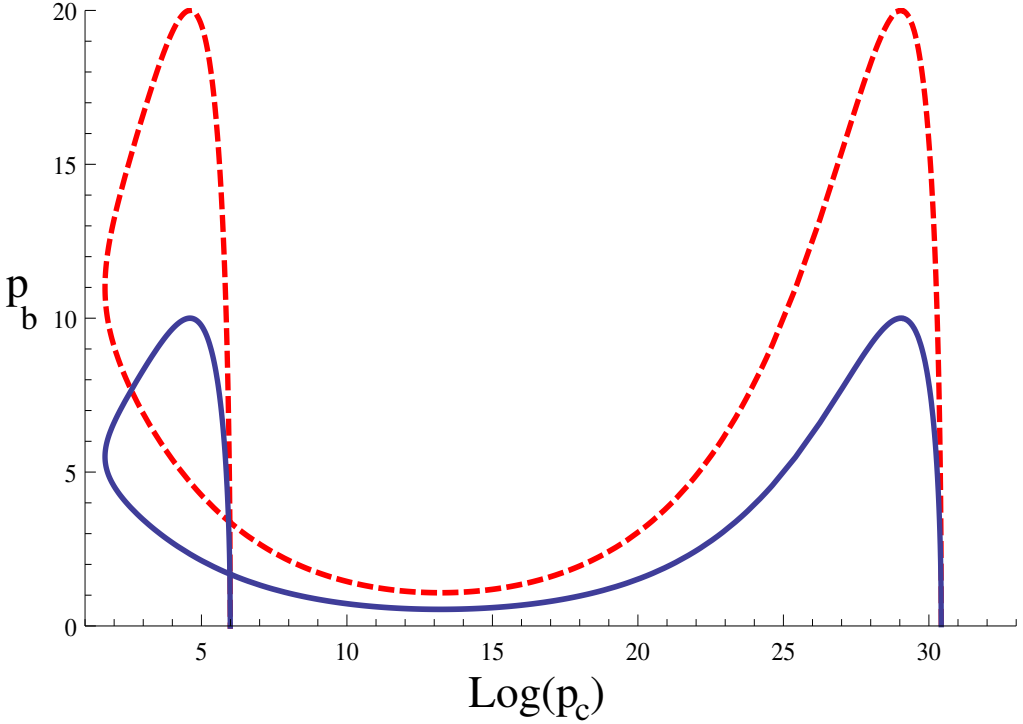
Modified Hamilton's equations yield a minimum allowed value of p_c: $p_{c\,(\text{min})} = \gamma\,\Delta^{1/2}\,m$. Central singularity replaced by a bounce.

Expansion and shear scalars are dynamically bounded

$$\theta = \frac{1}{\gamma p_c^{1/2}}\left[\frac{\sin(\delta_b b)}{\delta_b}(\cos(\delta_b b) + \cos(\delta_c c)) + \frac{\sin(\delta_c c)}{\delta_c}\cos(\delta_b b)\frac{p_c}{p_b}\right]$$

Unlike earlier fixed area quantizations, θ and σ^2 independent of fiducial length L_o.

Resulting physics leads to GR at infra-red scales, bounce near Planck scale and is independent of the fiducial length L_o.

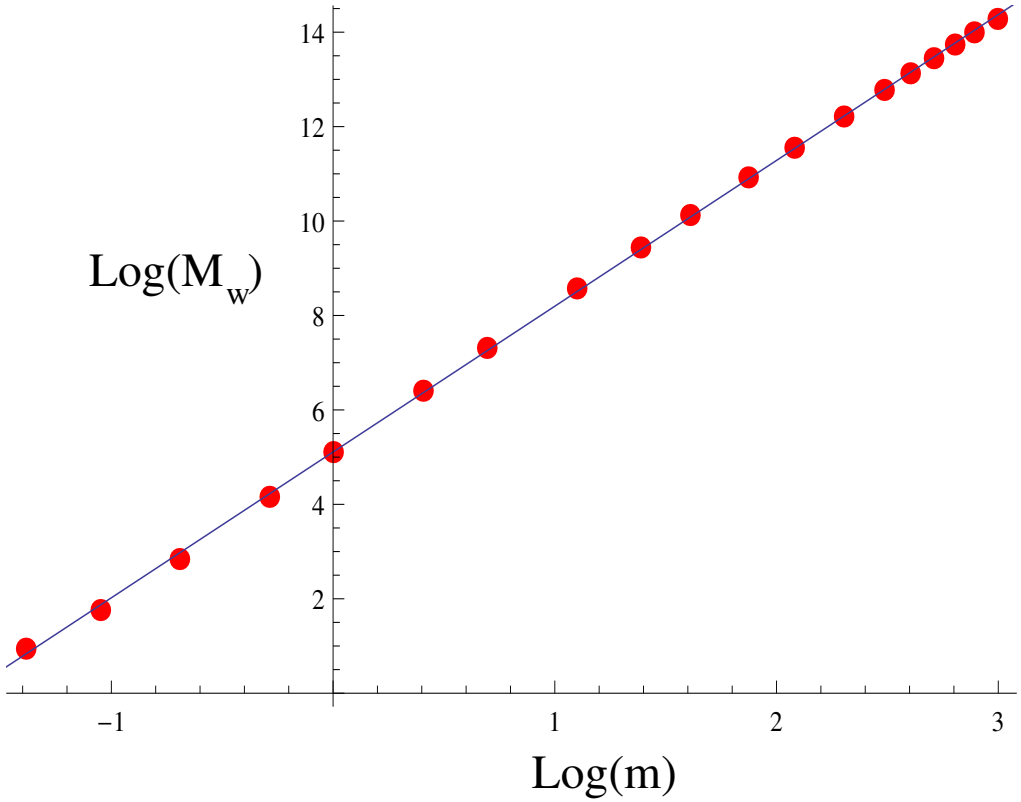


(Different curves correspond to different L_o)

Mass of the black hole determined by the value of p_c in the when $p_b = 0$ in the small spacetime curvature regime.

Mass of the second black hole independent of the fiducial length L_o. Feature missing in the previous quantizations.

The final white hole mass is approximately a cubic power of the initial black hole mass.



Bounce turns out to be highly asymmetric (unlike in the isotropic models in LQC).

Boehmer-Vandersloot prescription in generalized setting: Physics from effective dynamics

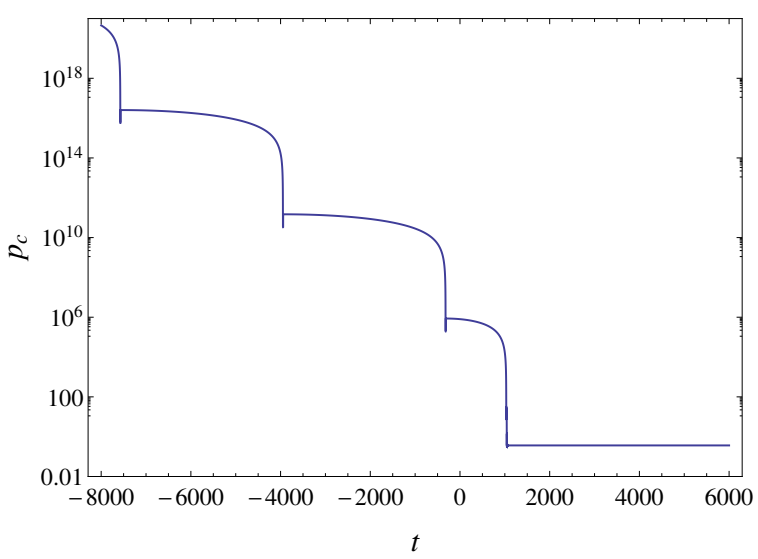
Effective Hamiltonian constraint:

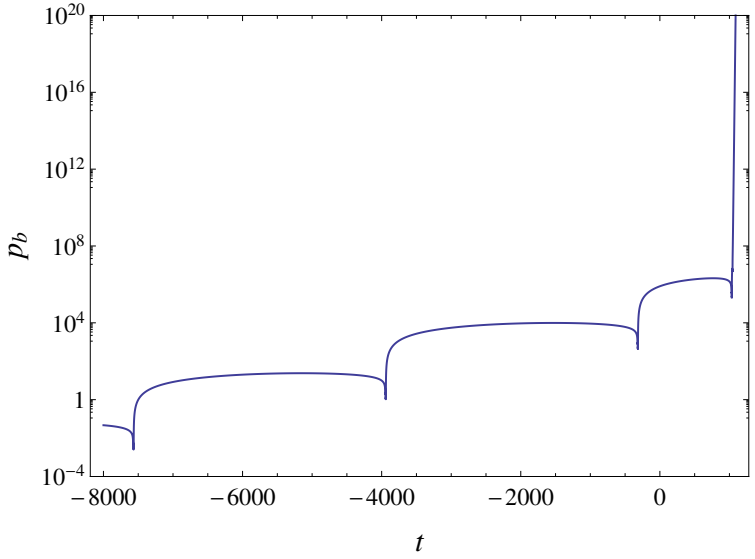
$$\mathcal{H}_{\text{eff}} = -\frac{p_b\sqrt{p_c}}{2G\gamma^2\Delta}\left(2\sin(c\delta_c)\sin(b\delta_b) + \sin^2(b\delta_b) + \frac{\gamma\Delta^2}{p_c}\right) + V\rho = 0$$

where $\delta_b = \sqrt{\Delta/p_c}$, $\delta_c = \sqrt{\Delta p_c}/p_b$

Expansion and shear scalars turn out to be universally bounded in LQC for arbitrary matter for this metric (Joe, PS (2014))

Classical singularity is resolved and there are multiple bounces. The triad component p_b increases exponentially in the forward evolution, and decreases through various cycles in the backward evolution. p_c approaches a constant value asymptotically in the forward evolution, and increases through various cycles in the backward evolution.

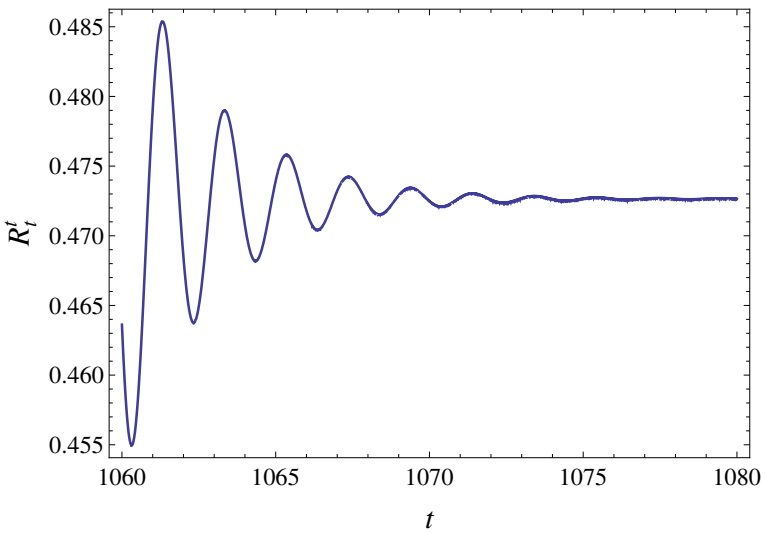


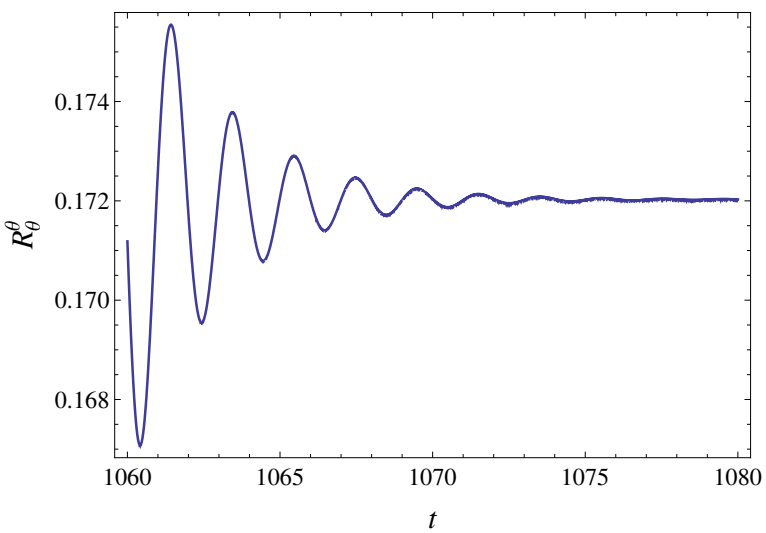


For vacuum and perfect fluids, the asymptotic value of p_c is a universal value. Only for $\pm\Lambda$, the asymptotic value of p_c varies.

Spacetime with $p_c = \mathrm{const}$, and p_b growing exponentially is not a solution of the classical Kantowski-Sachs Hamiltonian constraint!

Spacetime does not become classical on one side of the temporal evolution for generic initial condition and arbitrary matter.





The Ricci components in the asymptotic approach: $R_t^t = R_x^x = \alpha^2$ (where α is a constant), and $R_\theta^\theta = R_\phi^\phi = p_c^{-1}$. In general, $R_t^t \neq R_\theta^\theta$. The signs of Ricci components independent of the choice of the matter in Kantowski-Sachs spacetime.

The spacetime curvature is Planckian in the asymptotic regime! Spacetime after the bounce retains its quantum nature.

What is the nature of the effective spacetimes after the bounce?

The metric in the asymptotic regime is

$$ds^2 = -dt^2 + \cosh^2(\alpha t) + p_c(d\theta^2 + \sin^2\theta d\phi^2)$$

Solution of the Einstein's field equations! The "Nariai type" spacetime metric can be interpreted to be that of the 'charged' classical Nariai spacetime with $\Lambda_{\rm eff} > 0$ and $Q^2_{\rm eff} > 0$

(Dadhich, Joe, PS (15))

Is it possible to obtain the exact Nariai spacetime after the bounce?

Yes, but only for the case of $\Lambda > 0$ in the Kantowski-Sachs model. For a fine tuned case with $\rho_\Lambda \approx 0.31\rho_{\rm Pl}$, the Ricci components become equal. Unstable solution.

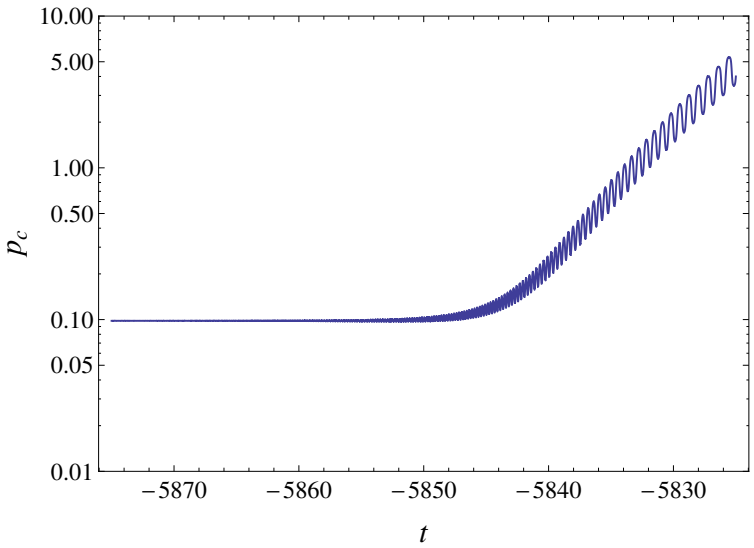
Higher genus black hole interior

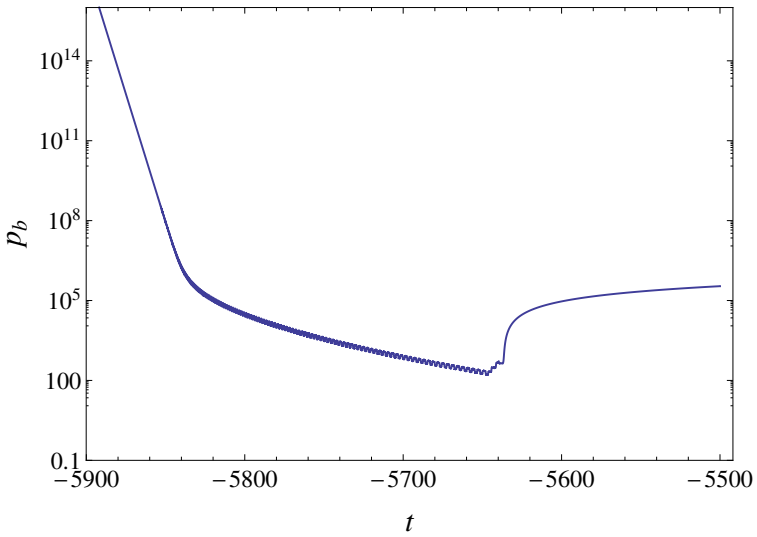
Motivated by Boehmer-Vandersloot prescription for Schwarzschild interior case, higher genus black holes with $-\Lambda$ studied using interior as Bianchi-III LRS spacetime **(Brannlund, Kloster, DeBenedictis (09))**

Effective Hamiltonian constraint:

$$\mathcal{H}_{\text{eff}} = -\frac{p_b\sqrt{p_c}}{2G\gamma^2\Delta}\left(2\sin(c\delta_c)\sin(b\delta_b) + \sin^2(b\delta_b)\frac{\gamma\Delta^2}{p_c}\right) + V\rho_\Lambda = 0$$

Behavior similar to Schwarzschild interior after bounce. For $\rho_\Lambda < 0$, p_c approaches a constant value asymptotically after the singularity is resolved.





In the asymptotic regime where p_c is a constant, $R^t_t = R^x_x = \alpha^2 > 0$, and $R^\theta_\theta = R^\phi_\phi = -p_c^{-1} < 0$.

As in the Kantowsi-Sachs model, this is not an allowed solution of the classical Hamiltonian constraint of the Bianchi-III LRS model. The spacetime curvature is in the Planck regime.

It was earlier thought that the resulting spacetime is "Nariai type" (Brannlund et al (09)). However, the spacetime is anti Bertotti-Robinson with an effective negative cosmological constant (Dadhich, Joe, PS (15))

For a fine tuned case, with $\rho_\Lambda = -0.113\rho_{\rm Pl}$, $R^t_t = R^\theta_\theta$. This can be interpreted as a $\Lambda_{\rm eff} = 0$, anti Bertotti-Robinson solution.

For Boehmer-Vandersloot prescription, spacetime after the bounce settles in to a 'charged' Nariai or an anti-Bertotti-Robinson spacetime. Spacetime very different from the black hole interior one started with. No second black hole solution.

Conclusions

- Details of the singularity resolution in loop quantization of black hole spacetimes still being explored. Many issues still unaddressed.
- Two loop quantizations of the interior spacetimes known which are free from fiducial cell issues. Unlike other examples with similar quantization ambiguities, these bring out a very different picture of the spacetime once the central singularity is resolved.
- In the prescription where loops have fixed area, bounce results in a second black hole with mass proportional to a cubic power of the initial black hole mass. Highly asymmetric bounce. Quantization independent of various issues of previous quantizations. Improved dynamics not needed.
- In the prescription where loops have area dependent on triads (improved dynamics analog), after the bounce spacetime can be interpreted as a 'charged' Nariai or anti-Bertotti-Robinson spacetime. No 'white hole' spacetime.

Conclusions (contd.)

- These quantizations provide explicit examples of drastic changes in properties of spacetime due to quantization ambiguities. Spacetimes after the bounce not even qualitatively similar.
- Bounce in black hole interiors seems quite different from what we learnt in isotropic models in LQC.
- Care needed to implement improved dynamics for such spacetimes. (Problems with Boehmer-Vandersloot approach).
- Worthwhile to explore alternative consistent quantizations (if any).
- Quantization with loops with fixed areas does not necessarily leads to inconsistent physics, and minimum loop areas which are triad dependent not necessarily free of all problems. For any given choice, a rigorous understanding of consistency conditions and detailed physics very important.