Natural dynamics for the cosmological constant

Lee Smolin
Perimeter Institute
ILQGS
March 19 2019

Joint work in progress with Stephon Alexander, Marina Cortes, Andrew Liddle, Joao Magueijo, and Robert Sims.

Very special thanks to Kirill Krasnov!!

Cosmologists are interested to study modified gravity theories in the IR in the hopes of understanding dark energy and, perhaps, dark matter.

But most candidates require new fields and new parameters. These reduce their testability and explanatory power.

Is there a principle which modifies gravity in a way that gives dynamics to the dark energy, but has no new parameters or fields?

Meanwhile, LQG theorists have learned that GR and quantum gravity are in important senses close to TQFT's. There are senses in which the low energy limit of QG is dominated by a TQFT. Λ plays an important role in these insights.

This suggests that any IR modification of gravity should be closely tied to topological field theories. Here is one way to do that:

Quasi topological principle: *Introduce only new terms in Λ that are topological when Λ is constant. ie Λ gets its dynamics from disrupting a topological symmetry.*

Quasi topological principle: introduce only new terms in Λ that are topological when Λ is constant. ie Λ gets its dynamics from disrupting a topological symmetry.

There are two topological invariants in 4d we might disrupt:

$$I^{G-B} = \int_{\mathcal{M}} \epsilon_{abcd} R^{ab} \wedge R^{cd} \qquad \rightarrow \qquad \int_{\mathcal{M}} f[\Lambda] \epsilon_{abcd} R^{ab} \wedge R^{cd}$$

$$I^{Pontryagin} = \int_{\mathcal{M}} R^{ab} \wedge R_{ab} \qquad \rightarrow \qquad \int_{\mathcal{M}} f[\Lambda] R^{ab} \wedge R_{ab}$$

They are both interesting. But is there a principle that fixes the functions f[Λ]?

There is a particular choice of f[Λ] that enhances what we might consider to be a symmetry on "ground states," ie solutions of maximal symmetry.

Quasi topological principle: introduce only new terms in Λ that are topological when Λ is constant. ie Λ gets its dynamics from disrupting a topological symmetry.

There are two topological invariants in 4d we might disrupt:

$$I^{G-B} = \int_{\mathcal{M}} \epsilon_{abcd} R^{ab} \wedge R^{cd} \qquad \rightarrow \qquad \int_{\mathcal{M}} f[\Lambda] \epsilon_{abcd} R^{ab} \wedge R^{cd}$$

$$I^{Pontryagin} = \int_{\mathcal{M}} R^{ab} \wedge R_{ab} \qquad \rightarrow \qquad \int_{\mathcal{M}} f[\Lambda] R^{ab} \wedge R_{ab}$$

In fact, each option leads to several theories.

We start with the simplest.

Theory one

The first step:

Recall the chiral Plebanski theory:

$$S^{\text{Pleb}} = \imath \int_{\mathcal{M}} \frac{1}{8\pi G} \left(\Sigma^{AB} \wedge R_{AB} - \frac{\Lambda}{6} \Sigma^{AB} \wedge \Sigma_{AB} - \frac{1}{2} \Phi_{ABCD} \Sigma^{AB} \wedge \Sigma^{CD} \right)$$
$$+ S^{\text{matter}}.$$

Eq's of motion:

$$0 = \frac{\delta S^{\text{Pleb}}}{\delta \Phi_{ABCD}} \rightarrow \qquad \Sigma^{(AB} \wedge \Sigma^{CD)} = 0 \,;$$

$$0 = \frac{\delta S^{\text{Pleb}}}{\delta \Sigma_{AB}} \rightarrow \qquad R_{AB} = \frac{\Lambda}{3} \Sigma_{AB} + \Phi_{ABCD} \Sigma^{CD} + 8\pi G \mathcal{T}_{AB} \,;$$

$$0 = \frac{\delta S^{\text{Pleb}}}{\delta A_{AB}} \rightarrow \qquad S^{AB} =: \mathcal{D}\Sigma^{AB} = 0$$

The Plebanski equations of motion (+c.c. equations)**:**

$$0 = \frac{\delta S}{\delta \Phi_{ABCD}} \to \Sigma^{(AB} \wedge \Sigma^{CD)} = 0$$

Implies there exists a frame field e $^{AA'}$, such that**:** $\quad \Sigma^{AB} = e^{A'A} \wedge e^{B}_{A'}$

$$0 = \frac{\delta S}{\delta \Sigma_{AB}} \to R_{AB} = \frac{\Lambda}{3}\Sigma_{AB} + \Phi_{ABCD}\Sigma^{CD} \qquad \textit{The Einstein eq's}$$

$$\boxed{\frac{\delta S}{\delta A_{AB}} = 0 \to S^{AB} =: \mathcal{D}\Sigma^{AB} = 0}$$

$$S^{AB} = T^{(AA'} \wedge e^{B)}_{A'} = 0 \qquad \text{So there is no torsion for pure GR}$$

Self-dual solutions and a partial duality symmetry

$$S^{\text{Pleb}} = \imath \int_{\mathcal{M}} \frac{1}{8\pi G} \left(\Sigma^{AB} \wedge R_{AB} - \frac{\Lambda}{6} \Sigma^{AB} \wedge \Sigma_{AB} - \frac{1}{2} \Phi_{ABCD} \Sigma^{AB} \wedge \Sigma^{CD} \right)$$
$$+ S^{\text{matter}} .$$

"Self-dual" solutions: Weyl=matter=0 $\boxed{R_{AB} = \frac{\Lambda}{3} \Sigma_{AB}}$

Suggests a duality symmetry:

$$\boxed{\frac{\Lambda}{3} \Sigma^{AB} \leftrightarrow R_{AB}}$$

This partial symmetry is enhanced if we add one new term:

$$S^{\text{Pleb}} = \imath \int_{\mathcal{M}} \frac{1}{8\pi G} \left(\Sigma^{AB} \wedge R_{AB} - \frac{\Lambda}{6} \Sigma^{AB} \wedge \Sigma_{AB} - \frac{1}{2} \Phi_{ABCD} \Sigma^{AB} \wedge \Sigma^{CD} - \frac{3}{2\Lambda} R^{AB} \wedge R_{AB} \right)$$
$$+ S^{\text{matter}} .$$

There is just one change to the Plebanski EoM:

$$0 = \frac{\delta S}{\delta A_{AB}} \to \mathcal{D} \wedge \Sigma^{AB} \equiv S^{AB} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{AB}$$

So now there is torsion proportional to dΛ:

$$S^{AB} =: \mathcal{D}\Sigma^{AB} = T^{AA"} \wedge e_{A'}^{\;B} = \frac{\delta S^{GB}}{\delta A_{AB}} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{AB}(\omega)\,.$$

When we evaluate it on self-dual solutions:

$$S^{ab} = \frac{3}{2\Lambda^2} d\Lambda \wedge R^{ab} = \frac{1}{2\Lambda} d\Lambda \wedge e^a \wedge e^b = 2T^{[a} \wedge e^{b]}$$

$$T^a = \frac{d\Lambda}{2\Lambda} \wedge e^a$$

This torsion is exactly what is needed to make the Einstein eq's with Weyl=matter=0 consistent for variable Λ. These are the generalized self-dual solutions.

$$R_{AB} = \frac{\Lambda}{3} \Sigma_{AB}$$

$$T^a = \frac{d\Lambda}{2\Lambda} \wedge e^a$$

Consistency of generalized self-dual solutions:

$$0 = \mathcal{D}R_{AB} = \frac{d\Lambda}{3}\Sigma_{AB} - \frac{\Lambda}{3}\frac{3d\Lambda}{\Lambda^2} \wedge R^{AB}(\omega) = \frac{d\Lambda}{\Lambda}[\frac{\Lambda}{3}\Sigma_{AB} - R_{AB}] = 0$$

$$T^a = \mathcal{D}e^a = de^a + A^a{}_b \wedge e^b$$

The connection is a 1-form: $A^{ab} = \omega^{ab}(e) + K^{ab}$

K^{ab} is the contortion 1-form, related to the torsion 2-form: $T^a = K^a{}_b \wedge e^b$

We also introduced the 3-form: $\mathcal{D}\Sigma^{ab} = S^{ab} = 2T^{[a} \wedge e^{b]}$

Which we found was: $S^{ab} = \dfrac{3}{2\Lambda^2} d\Lambda \wedge F^{ab} = \dfrac{1}{2\Lambda} d\Lambda \wedge e^a \wedge e^b = 2T^{[a} \wedge e^{b]}$

Thus, for self-dual solutions: $T^a = \dfrac{1}{\Lambda} d\Lambda \wedge e^a$

To compute the contortion trade for all Lorentz indices:

$$T_{abc} = e_a^\alpha e_b^\beta T_{\alpha\beta}^d \eta_{cd}, K_{abc} = e_a^\alpha K_{\alpha bc}$$

Use: $K_{abc} = \dfrac{1}{4}\left(T_{bac} + T_{acb} - T_{cba}\right)$

to find that **on self-dual solutions:** $\boxed{K_\alpha^{bc} = -\dfrac{1}{2\Lambda} e_\alpha^{[b} e^{c]\beta} \partial_\beta \Lambda}$

More on SD solutions

Self-dual solutions $\Lambda = constant$ **(CDJ)**

Pick an SU(2) connection, A^{AB}, such that F^{AB} satisfies $\mathcal{D}F^{AB} = 0$

$$F^{(AB} \wedge F^{CD)} = 0$$

Pick next a constant, Λ and *define:*

$$\Sigma^{AB} \equiv \frac{3}{\Lambda} F^{AB}$$

This satisfies: $\Sigma^{(AB} \wedge \Sigma^{CD)} = 0$ *and* $\mathcal{D}\Sigma^{A'B'} = 0$

ie Torsion vanishes.

so there exists a frame field e $^{AA'}$, such that*:*

$$\Sigma^{AB} = e^{A'A} \wedge e^{B}_{A'}$$

Example: de Sitter or AdS

Self-dual solutions $\Lambda = $ *variable*

Pick an SU(2) connection, A^{AB}, such that F^{AB} satisfies $\qquad \mathcal{D}F^{AB} = 0$

$$F^{(AB} \wedge F^{CD)} = 0$$

Pick next a *variable,* Λ and *define:*

$$\Sigma^{AB} \equiv \frac{3}{\Lambda} F^{AB} \qquad \textbf{(SD)}$$

This satisfies: $\Sigma^{(AB} \wedge \Sigma^{CD)} = 0$

But now there is torsion: and using (SD) it is what we need to let Λ be variable:

$$\boxed{\mathcal{D}\Sigma^{AB} \equiv S^{AB} = \mathcal{D}(\frac{3}{\Lambda(x)} F^{AB}) = -\frac{3}{\Lambda^2} d\Lambda \wedge F^{AB} = -\frac{1}{\Lambda} d\Lambda \wedge \Sigma^{AB}}$$

there still exists a frame field e ^{AA'}, such that*:* $\quad \Sigma^{AB} = e^{A'A} \wedge e^B_{A'}$

still, solves the Einstein eq with*:* $\quad \Phi_{ABCD} = 0$

$$F_{AB} = \frac{\Lambda}{3} \Sigma_{AB} + \Phi_{ABCD} \Sigma^{CD}$$

$$\Lambda \text{ also has eqs of motion:}$$

$$S^{\text{Pleb}} = \imath \int_{\mathcal{M}} \frac{1}{8\pi G} \left(\Sigma^{AB} \wedge R_{AB} - \frac{\Lambda}{6} \Sigma^{AB} \wedge \Sigma_{AB} - \frac{1}{2} \Phi_{ABCD} \Sigma^{AB} \wedge \Sigma^{CD} - \frac{3}{2\Lambda} R^{AB} \wedge R_{AB} \right)$$
$$+ S^{\text{matter}} \,.$$

Eq's of motion:

$$0 = \frac{\delta S^{\text{Pleb}}}{\delta \Phi_{ABCD}} \rightarrow \qquad \Sigma^{(AB} \wedge \Sigma^{CD)} = 0 \,;$$

$$0 = \frac{\delta S^{\text{Pleb}}}{\delta \Sigma_{AB}} \rightarrow \qquad R_{AB} = \frac{\Lambda}{3} \Sigma_{AB} + \Phi_{ABCD} \Sigma^{CD} + 8\pi G \mathcal{T}_{AB} \,;$$

$$0 = \frac{\delta S^{\text{Pleb}}}{\delta A_{AB}} \rightarrow \qquad S^{AB} =: \mathcal{D}\Sigma^{AB} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{AB}(\omega) \,.$$

Plus one more:

$$\boxed{0 = \frac{\delta S^{\text{Pleb}}}{\delta \Lambda} \rightarrow \frac{\Lambda^2}{9} \Sigma_{AB} \wedge \Sigma^{AB} = R_{AB} \wedge R^{AB} \,.}$$

Solved identically on the generalized self-dual solutions!

$$\Sigma^{AB} \equiv \frac{3}{\Lambda} F^{AB}$$

Quasi topological dynamics of Λ

Integrate the new term by parts:

$$S^{CS} = -\frac{\imath}{16\pi G} \int_{\mathcal{M}} \frac{3}{\Lambda} (R^{AB} \wedge R_{AB} - R^{A'B'} \wedge R_{A'B'}) = -\frac{\imath}{16\pi G} \int_{\mathcal{M}} \frac{3}{\Lambda} d\mathcal{I}m(Y_{CS})$$

$$= -\frac{\imath}{8\pi G} \int_{\Sigma_{final}} \frac{3}{2\Lambda} \mathcal{I}mY_{CS} + \frac{\imath}{8\pi G} \int_{\Sigma_{initial}} \frac{3}{2\Lambda} \mathcal{I}mY_{CS}$$

Reproduces the Im part of the Chern-Simons invariant of the Ashtekar connection on initial and final surfaces

$$+ \frac{\imath}{16\pi G} \int_{\mathcal{M}} d(\frac{3}{\Lambda}) \mathcal{I}mY_{CS}$$

$$Y_{CS} = Tr\left(A \wedge dA + \frac{1}{3} A^3 \right)$$

$$\frac{\delta Y_{CS}}{\delta A_{AB}} = R^{AB}$$

Quasi topological dynamics of Λ

Integrate the new term by parts:

$$S^{CS} = -\frac{\imath}{16\pi G} \int_{\mathcal{M}} \frac{3}{\Lambda}(R^{AB} \wedge R_{AB} - R^{A'B'} \wedge R_{A'B'}) = -\frac{\imath}{16\pi G} \int_{\mathcal{M}} \frac{3}{\Lambda} d\mathcal{I}m(Y_{CS})$$

$$= -\frac{\imath}{8\pi G} \int_{\Sigma_{final}} \frac{3}{2\Lambda} \mathcal{I}mY_{CS} + \frac{\imath}{8\pi G} \int_{\Sigma_{initial}} \frac{3}{2\Lambda} \mathcal{I}mY_{CS}$$

Reproduces the Im part of the Chern-Simons invariant of the Ashtekar connection on initial and final surfaces

$$+ \frac{\imath}{16\pi G} \int_{\mathcal{M}} d(\frac{3}{\Lambda})\mathcal{I}mY_{CS}$$

Note that S_Σ is the right Hamilton-Jacobi function to enforce that, on the initial or final surface, the spacetime is deSitter.

$$S_\Sigma = \frac{\imath}{8\pi G} \int_\Sigma \frac{3}{2\Lambda} ImY_{CS}(A)$$

But, could the initial and final Λ's be different, as they appear to be in our universe? Does the new term suffice to make Λ variable, or even dynamical?

Quasi topological dynamics of Λ

Integrate the new term by parts:

$$S^{CS} = -\frac{\imath}{16\pi G} \int_{\mathcal{M}} \frac{3}{\Lambda}(R^{AB} \wedge R_{AB} - R^{A'B'} \wedge R_{A'B'}) = -\frac{\imath}{16\pi G} \int_{\mathcal{M}} \frac{3}{\Lambda} d\mathcal{I}m(Y_{CS})$$

$$= -\frac{\imath}{8\pi G} \int_{\Sigma_{final}} \frac{3}{2\Lambda}\mathcal{I}mY_{CS} + \frac{\imath}{8\pi G} \int_{\Sigma_{initial}} \frac{3}{2\Lambda}\mathcal{I}mY_{CS}$$

Reproduces the Im part of the Chern-Simons invariant of the Ashtekar connection on initial and final surfaces

$$+\frac{\imath}{16\pi G} \int_{\mathcal{M}} d(\frac{3}{\Lambda})\mathcal{I}mY_{CS}$$

The third term suggests Λ is a dynamical variable, conjugate to $\tau_{CS} = \mathcal{I}mY_{CS}(A)$

$$\{\Lambda(x), \tau_{CS}(y)\} = \frac{16\pi G\Lambda^2}{3}\delta^3(x,y)$$

Recall τ_{CS} was proposed as a measure of intrinsic time   (Smolin, Soo, 1994)

We consider the same theory in Palatini variables:

$$S = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon^{abcd} \left\{ e_a \wedge e_b \wedge R_{cd}(\omega) - 2\Lambda e_a \wedge e_b \wedge e_c \wedge e_d - \frac{3}{2\Lambda} R_{ab} \wedge R_{cd} \right\}$$

We see the new term is the Gauss-Bonnet invariant.

$$\tau_a \propto \frac{\delta S_M}{\delta e^a}$$

$$S^{ab} \equiv T^{[a} \wedge e^{b]} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{ab}$$

$$\epsilon_{abcd} e^b \wedge \left(R^{cd} - \frac{\Lambda}{3} e^c \wedge e^d \right) = \frac{\kappa}{3} \tau_a$$

$$\frac{\Lambda}{3} = \sqrt{\frac{\epsilon_{abcd} R^{ab} \wedge R^{cd}}{e^4}}$$

Consistency in Palatini variables:

$$S = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon^{abcd} \left\{ e_a \wedge e_b \wedge R_{cd}(\omega) - 2\Lambda e_a \wedge e_b \wedge e_c \wedge e_d - \frac{3}{2\Lambda} R_{ab} \wedge R_{cd} \right\}$$

$$\epsilon_{abcd} \, e^b \wedge \left(F^{cd} - \frac{\Lambda}{3} e^c \wedge e^d \right) = \frac{\kappa}{3} \tau_a \qquad \tau_a \propto \frac{\delta S_M}{\delta e^a}$$

$$D\tau_a = \frac{3}{\kappa} \epsilon_{abcd} \left(T^b \wedge F^{cd} - \Lambda T^b \wedge e^c \wedge e^d - \frac{d\Lambda}{3} \wedge e^b \wedge e^c \wedge e^d \right).$$

$$D\tau_a = \frac{3}{\kappa} \epsilon_{abcd} T^b \wedge \left(F^{cd} - \frac{\Lambda}{3} e^c \wedge e^d \right).$$

New non-linearities in the eq's of motion.

$$S^{AB} =: \mathcal{D}\Sigma^{AB} = T^{AA"} \wedge e_{A'}^{\ B} = \frac{\delta S^{GB}}{\delta A_{AB}} = -\frac{3}{2\Lambda^2} d\Lambda \wedge R^{AB}(\omega) \,.$$

But R^{AB} (ω) is a function of T^a:
$$\omega^{AB} = \tilde{\omega}(e)^{AB} + K^{AB}$$

Where K, the contorsion, is a linear function of T:
$$T^a = K^a_{\ b} \wedge e^b$$

So the curvature 2-form is a quadratic function of T:

$$R^{ab}(\omega) = R^{ab}(\tilde{\omega}(e) + K) = \tilde{R}^{ab}(\tilde{\omega}(e)) + \tilde{\mathcal{D}}K^{ab} + K^a_{\ c}K^{cb}$$

So we have to invert a quadratic equation for T:

$$S^{ab} = T^{[a} \wedge e^{b]} = -\frac{3d\Lambda}{2\Lambda^2} \wedge \left(\tilde{R}^{ab}(\tilde{\omega}(e)) + \tilde{\mathcal{D}}K^{ab} + K^a_{\ c} \wedge K^{cb} \right)$$

$$T^a = K^a_{\ b} \wedge e^b$$

$$\omega^{AB} = \tilde{\omega}(e)^{AB} + K^{AB}$$

So we have to invert a quadratic equation for T:

$$S^{ab} = T^{[a} \wedge e^{b]} = -\frac{3d\Lambda}{2\Lambda^2} \wedge \left(\tilde{R}^{ab}(\tilde{\omega}(e)) + \tilde{\mathcal{D}}K^{ab} + K^a_{\ c} \wedge K^{cb} \right)$$

Expand around a self-dual solution:

$$S^{ab} = T^{[a} \wedge e^{b]} = -\frac{d\Lambda}{2\Lambda} \wedge e^a \wedge e^b + \dots$$

$$T^a = -\frac{d\Lambda}{2\Lambda} \wedge e^a + \dots$$

New non-linearities in the Λ eq of motion. $\qquad \omega^{AB} = \tilde{\omega}(e)^{AB} + K^{AB}$

$$\frac{\Lambda}{3} = \sqrt{\frac{\epsilon_{abcd} R^{ab} \wedge R^{cd}}{e^4}}$$

But the curvature two form is itself a function of $d\Lambda/\Lambda$:

$$R^{ab}(\omega) = R^{ab}(\tilde{\omega}(e) + K[\frac{d\Lambda}{\Lambda}])$$

This time we have to invert an equation for Λ and $d\Lambda/\Lambda$

Again, we can expand around a self-dual solution.

What we know of solutions to first theory:

1) Torsion absorbs and protects derivatives of Lambda.

2) Generalized self-dual solutions.

3) ***Cosmological solutions: generalized FRW.***

Are there consistent vacuum solutions with varying Λ and nonzero Weyl curvature?

$$R_{AB} = \frac{\Lambda}{3}\Sigma_{AB} + \Phi_{ABCD}\Sigma^{CD}$$

$$\frac{\Lambda}{3} = \sqrt{\frac{\epsilon_{abcd}R^{ab} \wedge R^{cd}}{e^4}}$$

These together imply that Weyl curvature vanishes (Eulidean??) KK

$$\mathcal{D}R_{AB} = 0$$

$$\rightarrow [\mathcal{D} - 2\frac{d\Lambda}{\Lambda}]\Phi_{ABCD}\Sigma^{CD} = \frac{3d\Lambda}{\Lambda^2}\Phi_{ABCD}\Phi^{CDEF}\Sigma_{EF}$$

Does this imply that dΛ=0?

It seems theory one is unphysical. What are the options?

-Add or induce a kinetic energy for Λ.

-Go away from the special value for the coefficient
of the Gauss-Bonet term.

-Consider the theory based on the Pontryagin invariant.

Theory two:

Induce or add a Λ kinetic energy

Frees Λ and Weyl to both propagate independently.

Loosens constraints, cosmological and otherwise.

Λ *kinetic energy from torsion-squared*

Diffeomorphism invariance allows us to add the dimension two term:

$$S^{T^2} = \frac{\alpha}{8\pi G} \int_{\mathcal{M}} \sqrt{-g}\, g^{\alpha\beta} g^{\mu\nu} T^a_{\alpha\mu} T^b_{\beta\nu} \eta_{ab}$$

This might be induced by quantum corrections or a fermion condensate, or might simply be added, in which case we have a new parameter, α.

Expanding round a self dual solution, in powers of dΛ/Λ:

$$T^a = \frac{1}{\Lambda} d\Lambda \wedge e^a$$

This gives a standard kinetic energy term for λ = Ln Λ, near a self-dual solution.

$$S^{T^2} = \frac{3\alpha}{32\pi G} \int_{\mathcal{M}} \sqrt{-g}\, g^{\alpha\beta} \partial_\alpha \lambda \partial_\beta \lambda$$

Theory two: include a Λ kinetic energy from torsion squared

$$S^{two} = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon^{abcd} \left\{ e_a \wedge e_b \wedge R_{cd}(\omega) - 2\Lambda e_a \wedge e_b \wedge e_c \wedge e_d - \frac{3}{2\Lambda} R_{ab} \wedge R_{cd} \right\}$$

$$+\alpha\sqrt{-g}\, g^{\alpha\beta} g^{\mu\nu} T^a_{\alpha\mu} T^b_{\beta\nu} \eta_{ab}$$

This gives an effective dynamics for Λ, near a self-dual solution.

Effective dynamics for Λ

$$\tilde{\Box}\Lambda = \Sigma^{AB} \wedge \Sigma_{AB} \left[1 - (\frac{3}{\Lambda})^2 \frac{F^{AB} \wedge F_{AB}}{\Sigma^{AB} \wedge \Sigma_{AB}} \right]$$

Where

$$\tilde{\Box} = \frac{1}{\Lambda} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \frac{1}{\Lambda} \partial_\nu \right)$$

There are fixed points at

$$\partial_\mu \Lambda = 0; \qquad \frac{\Lambda}{3} = \sqrt{\frac{F^{AB} \wedge F_{AB}}{\Sigma^{AB} \wedge \Sigma_{AB}}}$$

Option two: go away from the special value.

$$S = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon^{abcd} \left\{ e_a \wedge e_b \wedge R_{cd}(\omega) - 2\Lambda e_a \wedge e_b \wedge e_c \wedge e_d - \frac{3\theta}{2\Lambda} R_{ab} \wedge R_{cd} \right\}$$

$$\frac{\Lambda}{3} = \sqrt{\frac{\theta \epsilon_{abcd} R^{ab} \wedge R^{cd}}{e^4}}$$

Weyl now propagates.

Λ *kinetic energy from torsion hidden in curvature*

On self-dual solutions to the A^{ab} *equations of motion,*

$$A^{ab} = \omega^{ab}(e) + K^{ab} \qquad\qquad K_\alpha^{bc} = -\frac{1}{2\Lambda} e_\alpha^{[b} e^{c]\beta} \partial_\beta \Lambda$$

$$R^{ab}(A) = \tilde{R}^{ab}[\omega(e)] + \mathcal{D}K^{ab} + K^a{}_c \wedge K^{bc}$$

The effective action has a new term in $(d\Lambda)^2$ from

$$S = \frac{1}{8\pi G} \int_\mathcal{M} -e e_a^\alpha e_b^\beta R_{\alpha\beta}{}^{ab}(A)$$

$$S^{new} = \frac{1}{8\pi G} \left(\int_\mathcal{M} \frac{3}{\Lambda^2} e g^{\alpha\beta} \partial_\alpha \Lambda \partial_\beta \Lambda + \int_{\partial\mathcal{M}} e^a \wedge e^b \wedge K_{ab} \right)$$

Λ *kinetic energy from torsion hidden in curvature-2*

In fact, there are, in the neighbourhood of a de Sitter background, contributions to a Λ kinetic energy, coming from both the Einstein and the Gauss-Bonet term. The result is

$$S^{eff}(e, \Lambda) = \int_{\mathcal{M}} \frac{1}{2}(1 - \theta)\frac{3\theta^2}{\Lambda^2}eg^{\alpha\beta}\partial_\alpha\Lambda\partial_\beta\Lambda + \ldots$$

Cosmological solutions of theory one: generalized FRW.

Cosmological solutions: generalized FRW.

FRW ansatz: $e^0 = dt \qquad e^i = a(t)dx^i$

Definition of torsion 2-form: $T^a \equiv \mathcal{D}e^a = de^a + \omega^a_{\ b} \wedge e^b$

Symmetries require of torsion: $T^0 = 0 \qquad T^i = -T(t)e^0 \wedge e^i$

$$\omega^i_{\ 0} = g(t)e^i = \left(\frac{\dot{a}}{a} + T \right) e^i \qquad \omega^i_{\ j} = 0,$$

Modified Hubble parameter: $g = \dfrac{\dot{a}}{a} + T$

Curvature components:

$$F^{0i} = \frac{1}{a}(ag(t))^{\cdot} e^0 \wedge e^i = \frac{1}{a}(\dot{a} + Ta)^{\cdot} e^0 \wedge e^i$$

$$F^{ij} = g^2(t)e^i \wedge e^j = \left(\frac{\dot{a}}{a} + T \right)^2 e^i \wedge e^j$$

Cosmological solutions: generalized FRW p2.

Perfect fluid:
$$\tau_0 = \rho(t)\epsilon_{ijk}e^i \wedge e^j \wedge e^k \qquad\qquad w = p/\rho$$

$$\tau_i = -p(t)\epsilon_{ijk}e^0 \wedge e^j \wedge e^k$$

Field equations reduced to FRW:

$$T = \frac{3\dot{\Lambda}}{2\Lambda^2}g^2 \qquad\qquad \text{δA: torsion equation}$$

$$g^2 = \left(\frac{\dot{a}}{a} + T\right)^2 = \frac{\Lambda + \kappa\rho}{3} \qquad\qquad \text{FRW equation}$$

$$g^2 + 2\frac{(ag)^\cdot}{a} = -\kappa p + \Lambda \qquad\qquad \text{Raychoudri equation}$$

$$g^2\frac{1}{a}(ag)^\cdot = \frac{\Lambda^2}{9} \qquad\qquad \Lambda \text{ equation of motion}$$

Solve for the torsion to find:

$$T = \frac{\dot{\Lambda}}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right)$$

$$w = p/\rho$$

$$g^2 = \left(\frac{\dot{a}}{a} + \frac{\dot{\Lambda}}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right)\right)^2 = \frac{\Lambda + \kappa\rho}{3}$$

$$\kappa = 8\pi G$$

$$\frac{(ag)^{\cdot}}{a} = \frac{1}{a}\left(\dot{a} + \frac{\dot{\Lambda}}{2\Lambda}a\left(1 + \frac{\kappa\rho}{\Lambda}\right)\right)^{\cdot} = \frac{\Lambda}{3} - \frac{\kappa}{6}(\rho + 3p)$$

$$(\Lambda + \kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho + 3p)\right) = \Lambda^2.$$

From which we deduce, by the usual way, the conservation eq:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -T(\rho + 3p) + \frac{2\Lambda T - \dot{\Lambda}}{\kappa}.$$

Using the field equations, the RHS=0:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0.$$

So matter is conserved via the torsion free connection. The role of torsion is just to account for the non-conservation of the Lambda energy-momentum.

We use this to simplify the FRW equations:

$$\left(\frac{\dot{a}}{a} + T\right)^2 = \frac{\Lambda + \kappa\rho}{3}$$

$$T = \frac{\dot{\Lambda}}{2\Lambda}\left(1 + \frac{\kappa\rho}{\Lambda}\right)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

$$(\Lambda + \kappa\rho)\left(\Lambda - \frac{\kappa}{2}(\rho + 3p)\right) = \Lambda^2.$$

$$w = p/\rho$$

$$\kappa = 8\pi G$$

$$\rho_\Lambda = \Lambda/\kappa$$

We discover Λ just tracks matter.

$$\Lambda = \kappa\rho\frac{1 + 3w}{1 - 3w} \qquad \Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho + \rho_\Lambda} = \frac{1 + 3w}{2}$$

We can say this different ways.

$$\frac{\dot{\Lambda}}{\Lambda} = \frac{\dot{\rho}}{\rho} = -3(1 + w)\frac{\dot{a}}{a}. \qquad T = \frac{\dot{\Lambda}}{\Lambda}\frac{1}{1 + 3w} = -3\frac{1 + w}{1 + 3w}\frac{\dot{a}}{a},$$

The effect of Λ is to renormalize Newton's constant:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\bar{\kappa}\rho}{3} \qquad \bar{\kappa} = \frac{\kappa}{2}\frac{(1+3w)^2}{1-3w}.$$

$$w = p/\rho$$

$$\kappa = 8\pi G$$

$$a \propto t^{\frac{2}{3(1+w)}},$$

$$\rho_\Lambda = \Lambda/\kappa$$

Pure radiation (w=1/3) plus Λ is forbidden:

$$\Lambda = \kappa\rho\frac{1+3w}{1-3w} \qquad \Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho+\rho_\Lambda} = \frac{1+3w}{2}$$

Two ways out: add Λ kinetic energy or go away from θ=1.

With θ away from one :

$$\frac{\kappa \rho}{\Lambda} = \frac{1}{2} \left[\frac{1 - 3w}{1 + 3w} + \sqrt{\left(\frac{1 - 3w}{1 + 3w}\right)^2 + \frac{8(\theta - 1)}{\theta(1 + 3w)}} \right].$$

$$w = p/\rho$$

$$\kappa = 8\pi G$$

$$\rho_\Lambda = \Lambda/\kappa$$

Now we consider pure radiation (w=1/3) and any θ, and find a renormalized Newton's constant.

$$\bar{\kappa} = \kappa \frac{1 + \sqrt{\frac{\theta}{\theta - 1}}}{\left(1 - 2\theta\sqrt{\frac{\theta - 1}{\theta}}\right)^2}.$$

The usual BBN constraint on ΔG gives a constraint on θ

$$-0.1 < \frac{\Delta G}{G} < 0.14 \qquad \Longrightarrow \qquad 1.83 < \theta < 1.92$$

Theory three:

We use the Pontryagin invariant instead of the Gauss-Bonnet invariant.

Theory three:

We use the Pontryagin invariant instead of the Gauss-Bonnet invariant.

$$S = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon^{abcd} \left\{ e_a \wedge e_b \wedge R_{cd}(A) - 2\Lambda e_a \wedge e_b \wedge e^c \wedge e^d \right\} + \frac{3}{2\Lambda} R^{ab} \wedge R_{ab}$$

The Pontryagin density is parity odd and vanishes on FRW spacetimes.

It couples to matter fields through an anomaly, in the conservation of the chiral current.

$$\mathcal{D}_\mu \mathcal{J}^\mu = \frac{3}{16\pi^2} R^{ab} \wedge R_{ab}$$

So, the Λ Eom ties Λ to an anomaly, and hence to the L-R matter creation rates. Might this explain why Λ is presently small?

Note: $\quad G\Lambda \approx (\Delta m_\nu)^4$

$\Delta m_\nu \approx 3 \times 10^{-3}$

$$\frac{\Lambda}{3} = \left(\frac{16\pi^2}{3}\right)^{\frac{1}{2}} \sqrt{\frac{\nabla_\mu J_5^\mu}{\sqrt{-g}}}$$

Theory four:

We use the Pontryagin invariant and give Λ a kinetic energy

$$S = \frac{1}{8\pi G} \int_{\mathcal{M}} \epsilon^{abcd} \left\{ e_a \wedge e_b \wedge R_{cd}(A) - 2\Lambda e_a \wedge e_b \wedge e^c \wedge e^d \right\} + \frac{3}{2\Lambda} R^{ab} \wedge R_{ab}$$

$$+\alpha \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} T^a_{\alpha\mu} T^b_{\beta\nu} \eta_{ab}$$

FRW reduction

Effective dynamics for Λ

$$S_{\text{eff}} = -\frac{1}{8\pi G} \int d^4 x e \left(\Lambda + \frac{b}{\Lambda} R\tilde{R} + \frac{c}{\Lambda^2} g^{\mu\nu} \partial_\mu \Lambda \partial_\nu \Lambda \right),$$

$$\ddot{\Lambda} - \left(\frac{2}{\eta} + \frac{\dot{\Lambda}}{\Lambda} \right) \dot{\Lambda} + \frac{1}{2cH^2\eta^2} \left(\Lambda^2 - bR\tilde{R} \right) = 0$$

Potential for Λ $\qquad V(\Lambda) = \frac{\Lambda}{6c} \left(\Lambda^2 - 3bR\tilde{R} \right)$

change to ϕ=*log* Λ *to make kinetic energy canonical:*

$$\phi = \Lambda/M_p^2 = \rho \exp[\varphi]$$

To find potential for ϕ=*log* Λ

$$V(\phi) = \frac{1}{\tilde{c}} \left(e^\phi + \tilde{b} e^{-\phi} \right)$$

Effective Potential For Lambda

Robert Sims et al: work in progress.

potential for $\phi = log\ \Lambda$
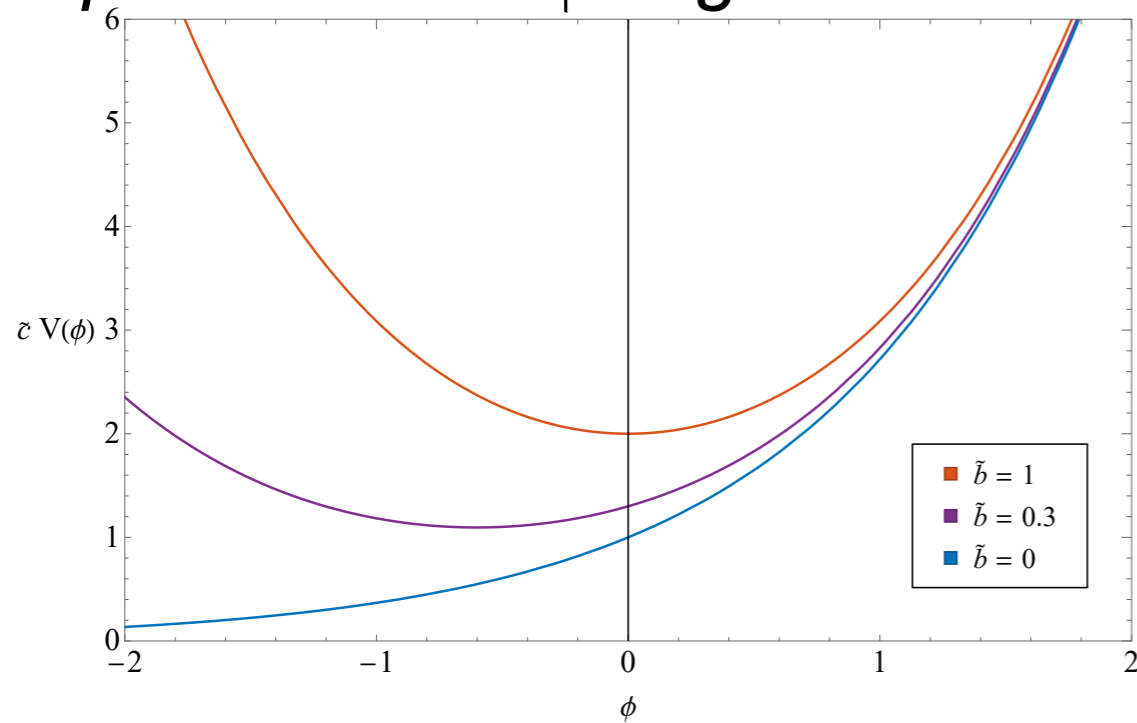
(b) Effective potential for field ϕ with various $\tilde{b}$ values. For $\tilde{b} = a(\eta)^{-n}$ for positive n (as we expect), propagating time forward results in smaller $\tilde{b}$, thus the minima for $V(\phi)$ becomes more negative.
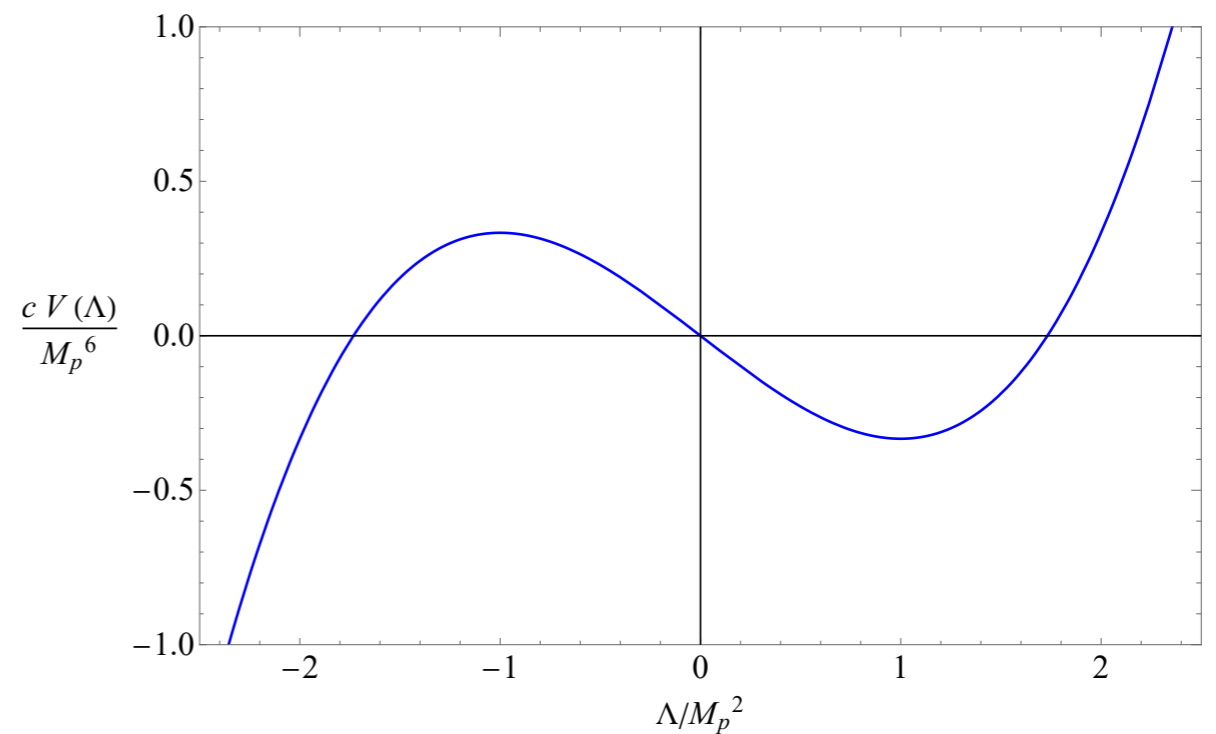
(a) Effective potential for the field Λ with $bR\tilde{R} = M_p^4$.

45

Numeric Solutions

Dimensionless
Variables

$$\lambda'' + \left(3\frac{a'}{a} - \frac{\lambda'}{\lambda}\right)\lambda' + \frac{1}{\tilde{c}}\left(\lambda^2 - \tilde{b}\right) = 0,$$

$$\left(\frac{a'}{a}\right)^2 = \frac{\tilde{\rho} + \lambda + \frac{1}{2}\tilde{c}\left(\lambda'/\lambda\right)^2}{3m^2/M_p^2},$$

$$\frac{a''}{a} = \frac{-\frac{1}{2}\tilde{\rho}(1 + 3\omega) + \lambda - \tilde{c}\left(\lambda'/\lambda\right)^2}{3m^2/M_p^2},$$

$$\lambda = \Lambda/M_p^2$$
$$\tilde{\rho} = \rho/M_p^4$$
$$\tilde{b} = bR\tilde{R}/M_p^4$$
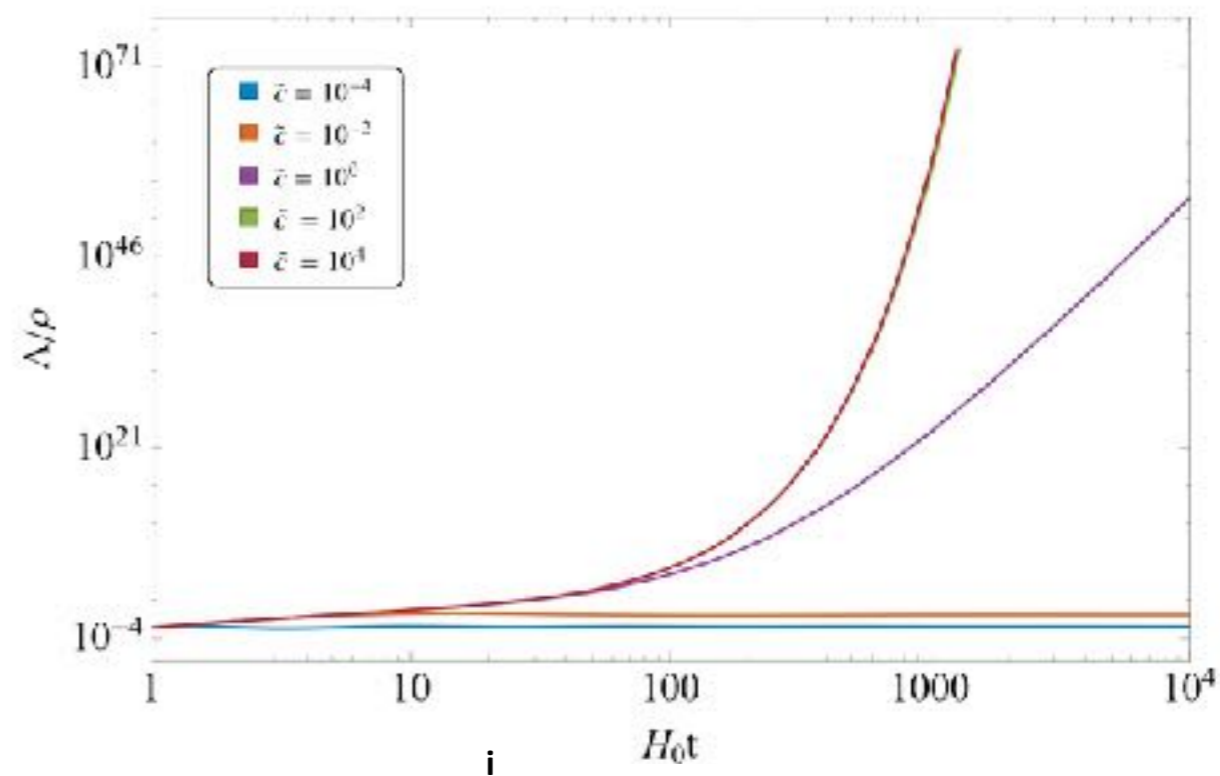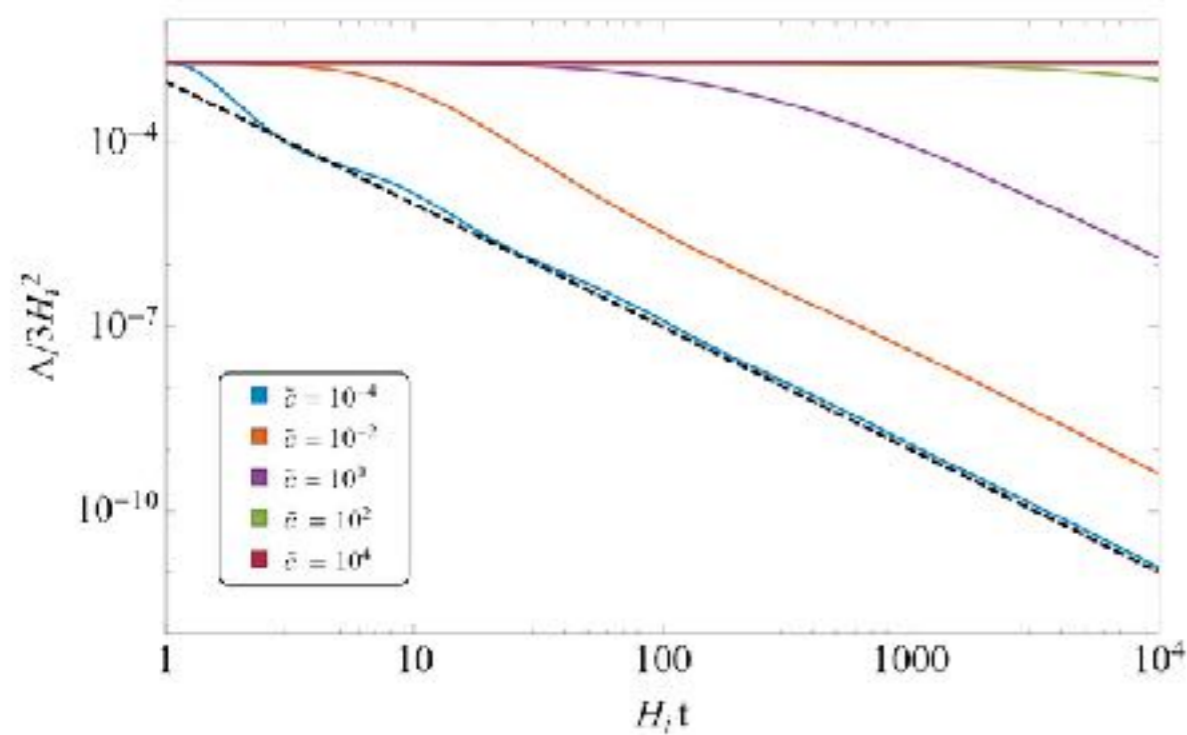$$\tilde{c} = 2c(m/M_p)^2$$

Set mass scale
$$m = H_i$$

$$\rho = \rho_0 a^{-3(1+\omega)} \qquad \text{(Conserved independently from } \Lambda\text{)}$$

Ansatz for Pontryagin
density

$$\tilde{b}(t) = \left(10^{-6}\tilde{\rho}_0^2\right)/t^4$$

Initial Conditions:

Λ_0 at twice fixed point value
$^{\rho_0}/_{\Lambda_0} = 10^4$, with $\omega = 0$

Robert Sims

Numeric Solutions

Robert Sims
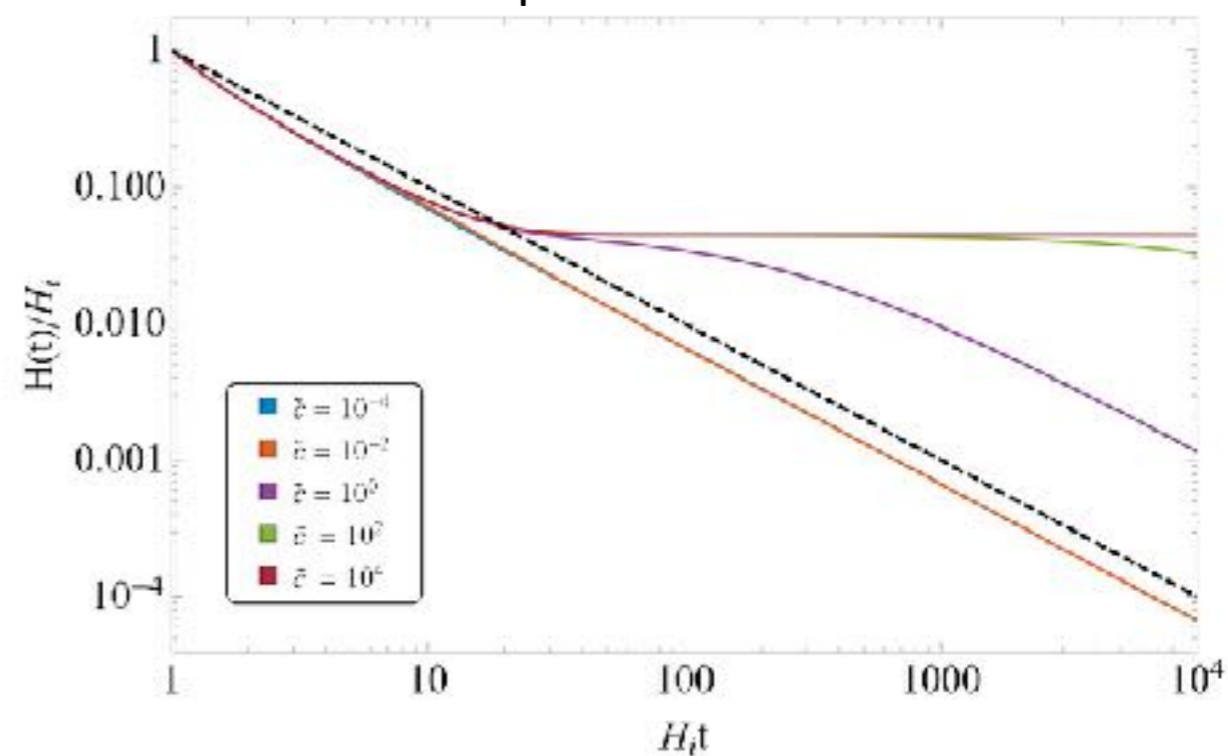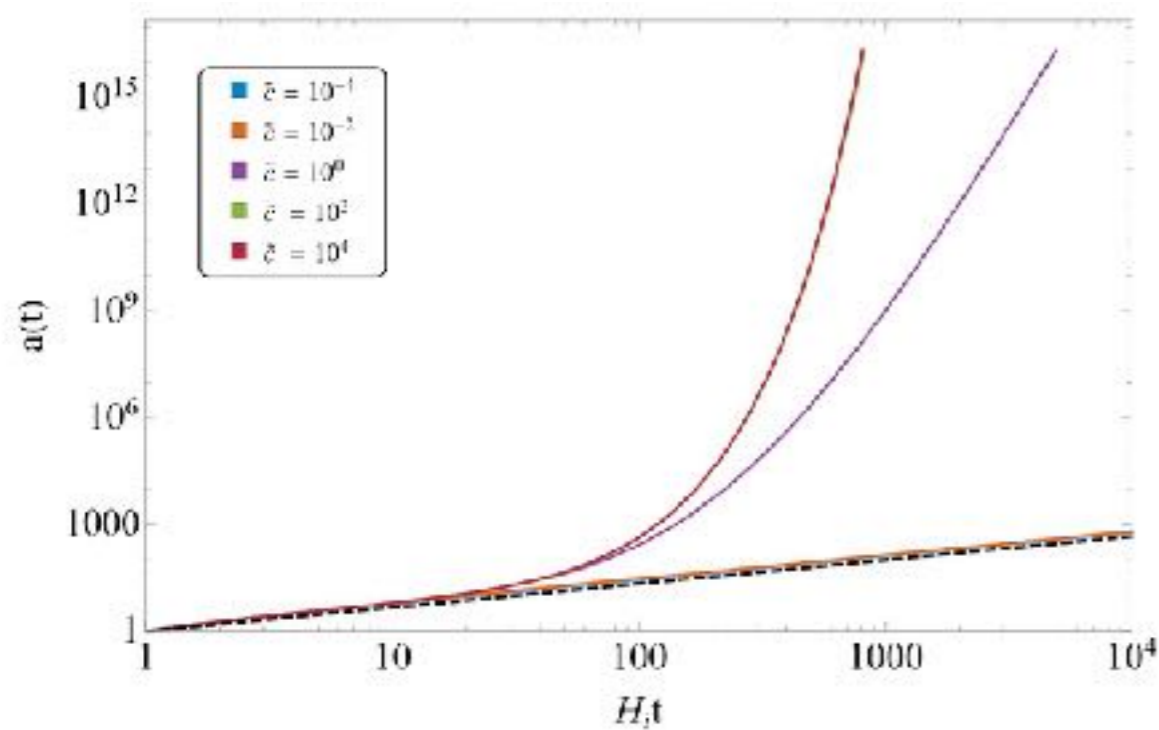
Typical behaviours seen, depending on initial conditions:

- *Sign of Λ never changes.*

- *Λ goes to time dependent fixed point, which takes it into 0.*

- *Λ first shows damped oscillations around fixed point.*

- *Or Λ freezes out, leading to Λ domination.*

Tentative conclusions:

Two new extensions of GR which are diffeo invariant, have one less parameter than GR, which each allow Λ to vary or evolve dynamically.

dS spacetime is enhanced to a space of generalized-self-dual spacetimes, with variable Λ, which are consistent because a torsion arises from the eq's of motion, proportional to dΛ.

With Weyl=matter=0, Λ is free to vary, because its field eq is redundant. Enlarged self dual sector. When matter is turned on, Λ tracks its density.

Theory one (θ=1, vacuum, no Λ-self-energy), appears to have no propagating modes.

Terms in torsion-squared, may be induced by going away from θ=1 or introduced by hand; these yield a Λ kinetic energy when examined near a self-dual spacetime.

FRW reductions have been studied. Without the Λ kinetic energy these are highly constrained to have Λ stuck in fixed points where it follows the matter density; when the Λ gains an independent kinetic energy, it becomes free to oscillate around or travel between fixed points.

The Pontryagin inv based theory predicts a relation between Λ and the gravitational chiral anomaly, possibly explaining the small value of Λ.

$$\frac{\Lambda}{3} = \left(\frac{16\pi^2}{3} \right)^{\frac{1}{2}} \sqrt{\frac{\nabla_\mu J_5^\mu}{\sqrt{-g}}}$$

Much to do:

- _Study theory away from $\theta=1$ and or with Λ kinetic energy._

- _Λ appears to clump around matter: Dark matter?_

- _Black holes?_

- _Linearization?   with Λ K.E. are there scalar waves? Coupled to what?_

- _Perturbation theory in $d\Lambda/\Lambda$?_

- _Better understanding of new non-linearities from solving A and Λ eqs._

- _Λ a function only of time in a preferred splicing?_

Thank you

Slides for discussion:

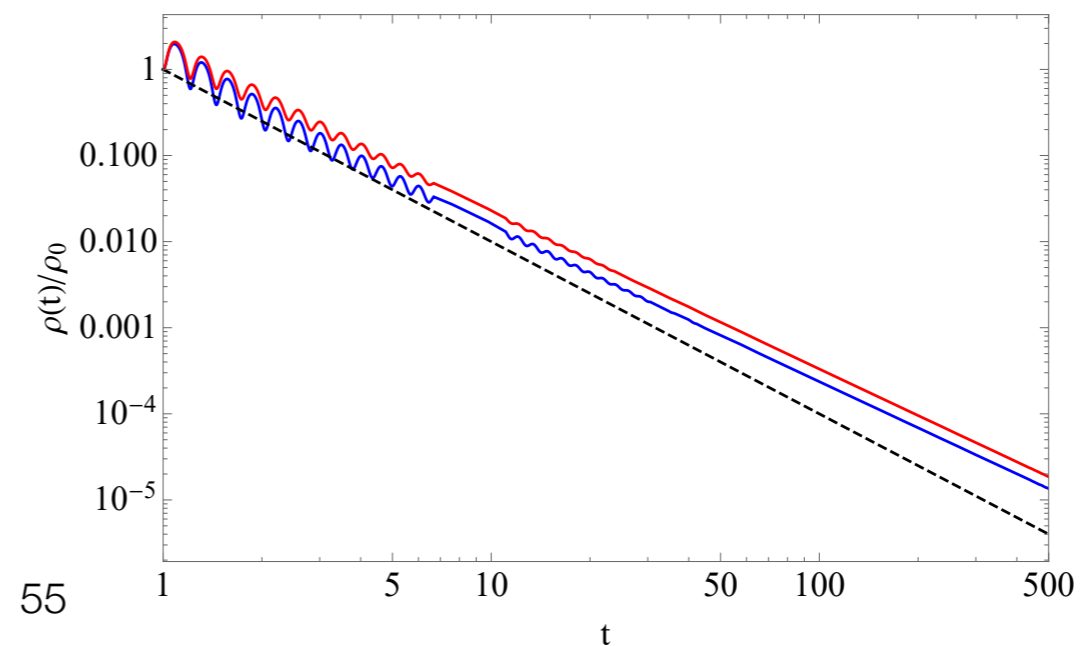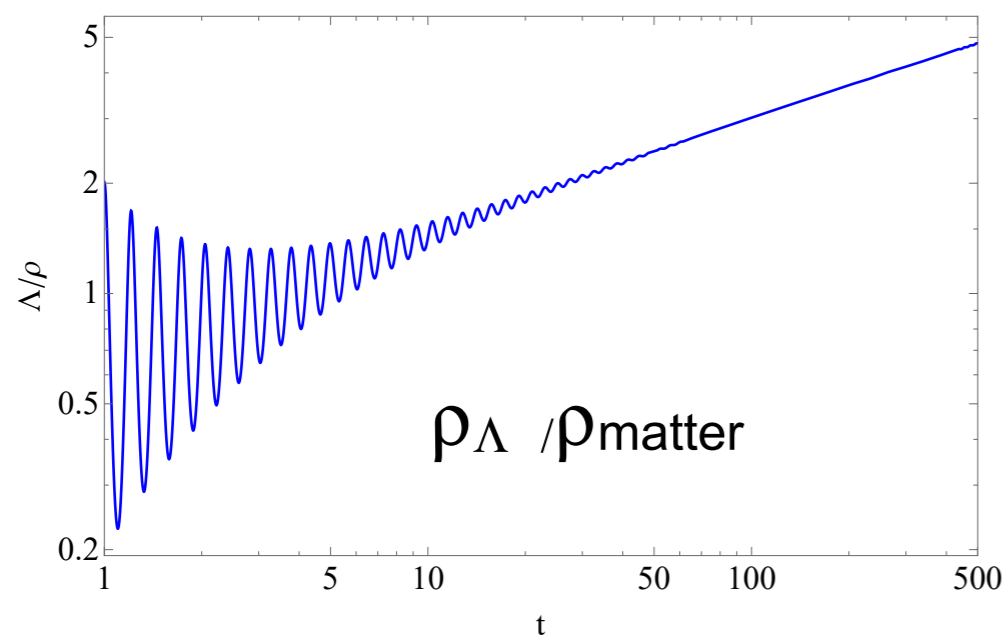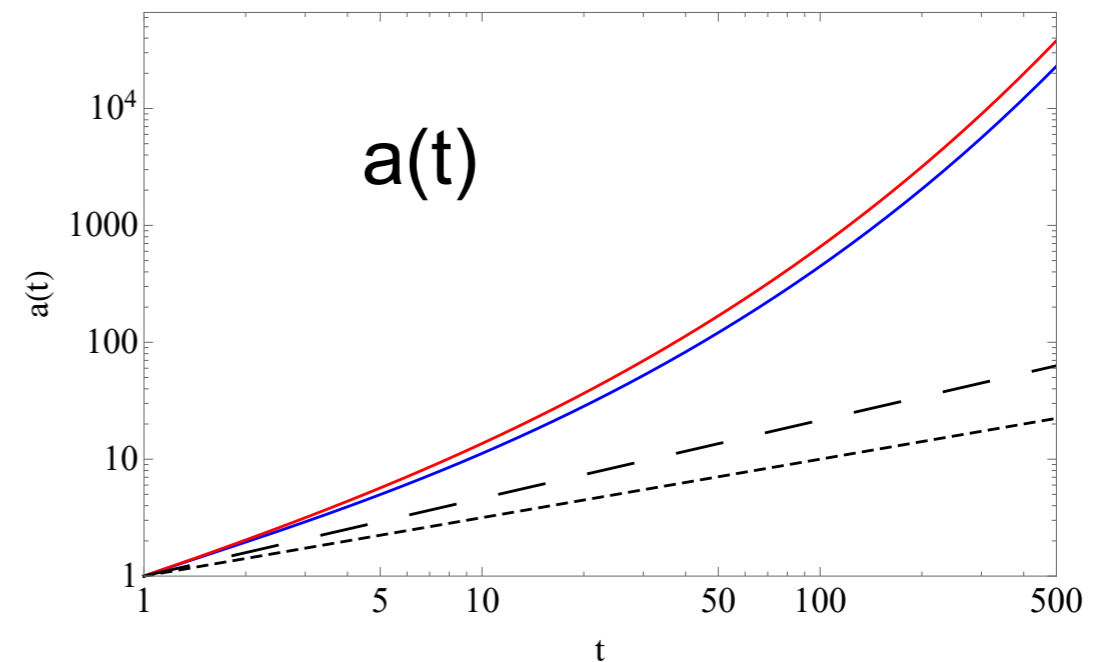**Coupled numerical evolution** *Robert Sims, in progress.*

$$\ddot{\Lambda} + \left(3\frac{\dot{a}}{a} - \frac{\dot{\Lambda}}{\Lambda} \right) \dot{\Lambda} + \frac{1}{2c} \left(\Lambda^2 - bR\tilde{R} \right) = 0$$

$$bR\tilde{R} = t^{-3}$$

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_p^2} \left(\rho + \Lambda \right)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho(1 + \omega) = -M_p^2\dot{\Lambda}.$$

Initial condition Λ_0 is at twice the fixed point.
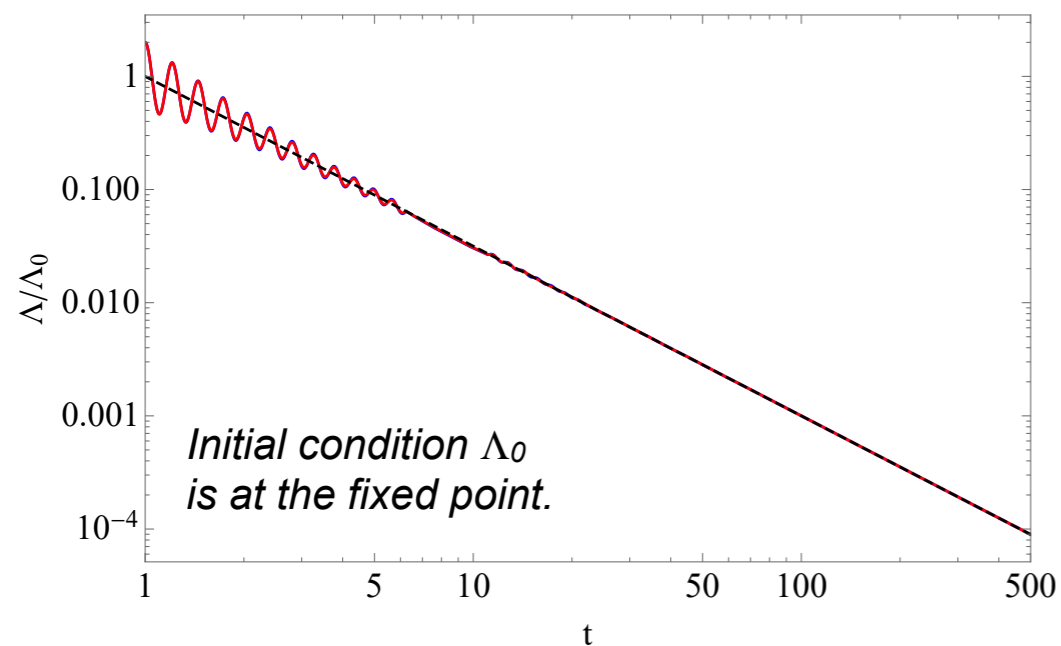
Coupled numerical evolution *Robert Sims, in progress.*

$$\ddot{\Lambda} + \left(3\frac{\dot{a}}{a} - \frac{\dot{\Lambda}}{\Lambda}\right)\dot{\Lambda} + \frac{1}{2c}\left(\Lambda^2 - bR\tilde{R}\right) = 0 \qquad bR\tilde{R} = t^{-3} \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}\left(\rho + \Lambda\right)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+\omega) = -M_p^2\dot{\Lambda}.$$

Λ oscillates around and tracks fixed point (black line)

*Initial condition Λ_0
is at the fixed point.*

$\rho_\Lambda / \rho_{matter}$

55

Check of Λ-variable solutions in Palatini

$$R_a = \frac{1}{6}\epsilon_{abcd}e^b \wedge R^{cd}$$

Einstein eq's in terms of 3-forms:

$$R_a - \frac{1}{2}e_a R = -\Lambda e_a \quad \text{(E)}$$

$$e_a = \frac{1}{6}\epsilon_{abcd}e^b \wedge e^c \wedge e^d$$

This is solved by, with variable Λ:

$$R^{ab} = \frac{\Lambda}{3}e^a \wedge e^b \quad \text{(SD)}$$

When the torsion is defined by:

$$\mathcal{D}e^a \equiv T^a = \frac{1}{2\Lambda}d\Lambda \wedge e^a \quad \text{(T)}$$

To show this, take covariant curl of both sides of (SD):

D LHS =0, D RHS =0 using the definition of torsion (T)

To show consistency, take curl again and use (T) again.

MORE ON CHERN-SIMONS TIME

Thermality of the exact quantum theory on $\Sigma = S^3$

Recall:
- The KMS condition. Thermal states are periodic in imaginary time.
- The natural time coordinate is:

$$T_{CS} = \mathcal{I}m \int Y_{CS}(A)$$

- The Euclidean continuation has A_a real

Hence the natural Euclidean time coordinate is

$$T_{ECS} = \int Y_{CS}(A)$$

But this is a periodic coordinate on the configuration space.

Under large gauge transformations:

$$\int Y_{CS}(A) \rightarrow \int Y_{CS}(A) + 8\pi^2 n$$

Hence there is a dimensionless temperature.

$$T_{dimless} = \frac{1}{8\pi^2}$$

Hence, the whole quantum theory of gravity with $\Sigma = S^3$ is thermal!

$$\mathcal{T}_{dimless} = \frac{1}{8\pi^2}$$

To connect this with the deSitter temperature we scale on a trajectory corresponding to an S³ slicing of dS:

The relation between the two time coordinates is given by

$$\frac{\partial T_{CS}}{\partial t} = \int_{S^3} N\{T_{CS}(A), \mathcal{H}\} = 4\pi\sqrt{\frac{\Lambda}{3}}$$

This leads to the dimensional Gibbons-Hawking temperature:

$$\mathcal{T}_{dS} = \frac{1}{2\pi}\sqrt{\frac{\Lambda}{3}}$$

Note: this does not just say that QFT on dS is thermal. It says quantum gravity with a positive CC is intrinsically thermal.

The Lorentzian Chern-Simons time in the homogeneous case:

$$A_{ai} = \imath \delta_{ai} \dot{a} = \imath \delta ai H a$$

$$T_{CS} = \int_{S^3} \mathcal{Im} Tr A^3 = H^3 a^3$$

This is the number of co-moving volumes in an Horizon volume.

T_{CS} <1 "comoving volume is within the horizon"

T_{CS} >1 " comoving volume is outside the horizon"

Basics of ashtekar variables and de Sitter

The Ashtekar variables are complex coordinates for real, Lorentzian spacetimes:

$$A_{ai} = 3\mathrm{d} \text{ spin connection}_{ai} + \frac{\imath}{\sqrt{q}} K_{ab} E_i^b$$

$$q q^{ab} = E^{ai} E_i^b \qquad K_{ab} \approx \dot{q}_{ab}$$

$$\{A_a^i(x), E_j^b(y)\} = \imath G \delta_a^b \delta_j^i \delta^3(y, x)$$

$$I^{GR} = \int dt \int_\Sigma \imath E^{ai} \dot{A}_{ai} - N\mathcal{H} - N^a H_a - w_i \mathcal{G}^i$$

Constraints generate gauge transformations:

Gauss's law for $SU(2)$: $\mathcal{G}^i = \mathcal{D}_a E^{ai}$

Diffeomorphism constraint $\mathcal{H}_a = E^b_i F^i_{ab}$

All constraints are cubic!

Hamiltonian constraint: $\mathcal{H} = \epsilon_{ijk} E^{ai} E^{bj} (F^k_{ab} + \frac{\Lambda}{3} \epsilon_{abc} E^{ck})$

Equations of motion:

$$\dot{A}_{ai} = \{A_{ai}, \int N\mathcal{H}\} = N\imath G \epsilon_{ijk} E^{bj} (2 F^k_{ab} + \Lambda \epsilon_{abc} E^{ck})$$

$$\dot{E}^{ai} = \{E^{ai}, \int N\mathcal{H}\} = \imath G \epsilon^{ijk} \mathcal{D}_b (N E^a_j E^b_k)$$

Self-dual solutions:

$$F^i_{ab} = -\frac{\Lambda}{3} \epsilon_{abc} E^{ci}$$

All eom are quadratic!

Explicit deSitter solution:

$$F_{ab}^i = -\frac{\Lambda}{3}\epsilon_{abc}E^{ci}$$

deSitter spacetime is (was) the unique lorentzian self-dual solution:

We make the spatially flat ansatz:

$$A_{ai} \approx \text{3d spin connection}_{ai} + i\dot{e}_{ai}$$

$$A_{ai} = i\sqrt{\Lambda/3}f(t)\delta_{ai} \quad \mapsto \quad F_{abi} = -f^2(t)\frac{\Lambda}{3}\epsilon_{abi}$$

The self-dual condition implies:

$$E^{ai} = f^2\delta^{ai} \quad \mapsto \quad e_{ai} = f\delta_{ai}$$

To fix the solution fix the lapse N

$$N \approx det(e)^{-1} = f^{-3}$$

The equations of motion give:

$$\dot{f} = \sqrt{\Lambda/3}Nf^4 = \sqrt{\Lambda/3}f$$

This gives the dS metric:

$$ds^2 = -dt^2 + e^{2\sqrt{\Lambda/3}t}(dx^a)^2$$

Hamilton-Jacobi, deSitter and Chern-Simons theory

Let us solve the constraints with a Hamilton-Jacobi function S(A).

The momenta are given by
$$E^{ai} = -\frac{\delta S(A)}{\delta A_{ai}}$$

To get deSitter we impose the self-dual condition:

$$F^i_{ab} = -\frac{\Lambda}{3}\epsilon_{abc}E^{ci} = \frac{\Lambda}{3}\epsilon_{abc}\frac{\delta S(A)}{\delta A_{ci}}$$

This has the unique solution:

$$S_{CS} = \frac{2}{3\Lambda}\int Y_{CS}$$

Chern-Simons invariant:

$$Y_{CS} = Tr(A \wedge dA + \tfrac{2}{3}A^3)$$

$$\frac{\delta \int Y_{cs}}{\delta A_{ai}} = 2\epsilon^{abc}F^i_{bc}$$

The Kodama State

Hence the H-J function for dS is:

$$S_{CS} = \frac{2}{3\Lambda} \int Y_{CS}$$

This suggests as an ansatz the state:

$$\Psi_K(A) = \mathcal{N} e^{\frac{3}{2\lambda} \int Y_{CS}}$$

Here we are using the connection representation:

$$< A|\Psi >= \Psi(A) \qquad E^{ai} = -\hbar G \frac{\delta}{\delta A_{ai}}$$

In fact, with a certain choice of operator ordering,
this is an exact solution to the quantum constraints:

The Kodama State

$$\Psi_K(A) = \mathcal{N} e^{\frac{3}{2\lambda} \int Y_{cs}}$$

$$S_{CS} = \frac{2}{3\Lambda} \int Y_{CS}$$

Its transform to the spin network representation is exact:

$$\Psi[\Gamma] = \int dA \; T[\Gamma, A] e^{\frac{\kappa}{4\pi} S_{CS}(A)}$$

for A Euclidean, this is the Kauffman bracket or Jone's polynomial of the network.

—> Requires framed spin networks labeled with SU_q (2) reps.

—> The level, k, is related to Λ:

$$k = \frac{6\pi}{\hbar G \Lambda}$$

QM

A new uncertainty relation.

Another approach is to define a preferred slicing, and
define Λ and the Chern-Simons time as a function of the slices.

$$T_{CS} = \mathcal{I}m \int_{\Sigma} Y_{CS}(A)$$

Then the new term in the action is

$$S^{new} = \frac{3}{16\pi\hbar G} \int dt \frac{\dot{\Lambda}}{\Lambda^2} \mathcal{I}m \int_{\Sigma} Y_{CS}(A)$$

This implies a new Poisson bracket and uncertainty relation.

$$\{\Lambda, \int_{\Sigma} \mathcal{I}m Y_{CS}(A)\} = \frac{16\pi G\Lambda^2}{3}$$

$$\Delta\Lambda\Delta\tau_{CS} \geq \frac{8\pi\hbar G}{3}\langle\hat{\Lambda}^2\rangle.$$

Waiting/storage fOR formulas

$$S^{ab} = T^{[a} \wedge e^{b]} = -\frac{3d\Lambda}{2\Lambda^2} \wedge \left(\tilde{R}^{ab}(\tilde{\omega}(e)) + \tilde{\mathcal{D}}K^{ab} + K^a_{\ c} \wedge K^{cb} \right)$$

$$S^{ab} = T^{[a} \wedge e^{b]} = -\frac{d\Lambda}{2\Lambda} \wedge e^a \wedge e^b + \ldots$$

Effective dynamics for Λ

Effective equation of motion for Λ

$$S_{\text{eff}} = -\frac{1}{8\pi G} \int d^4 x e \left(\Lambda + \frac{b}{\Lambda} R\tilde{R} + \frac{c}{\Lambda^2} g^{\mu\nu} \partial_\mu \Lambda \partial_\nu \Lambda \right),$$

FRW solutions

$$ds^2 = a^2(\eta) \left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j \right]$$

$$\ddot{\Lambda} + \left(2\mathcal{H} - \frac{\dot{\Lambda}}{\Lambda} \right) \dot{\Lambda} - (\delta^{ij} + h^{ij}) \left(\partial_i \partial_j \Lambda - \frac{1}{\Lambda} \partial_i \Lambda \partial_j \Lambda \right) = -\frac{a^2}{2c} \left(\Lambda^2 - bR\tilde{R} \right)$$

deSitter solutions

$$\ddot{\Lambda} - \left(\frac{2}{\eta} + \frac{\dot{\Lambda}}{\Lambda} \right) \dot{\Lambda} + \frac{1}{2cH^2\eta^2} \left(\Lambda^2 - bR\tilde{R} \right) = 0.$$
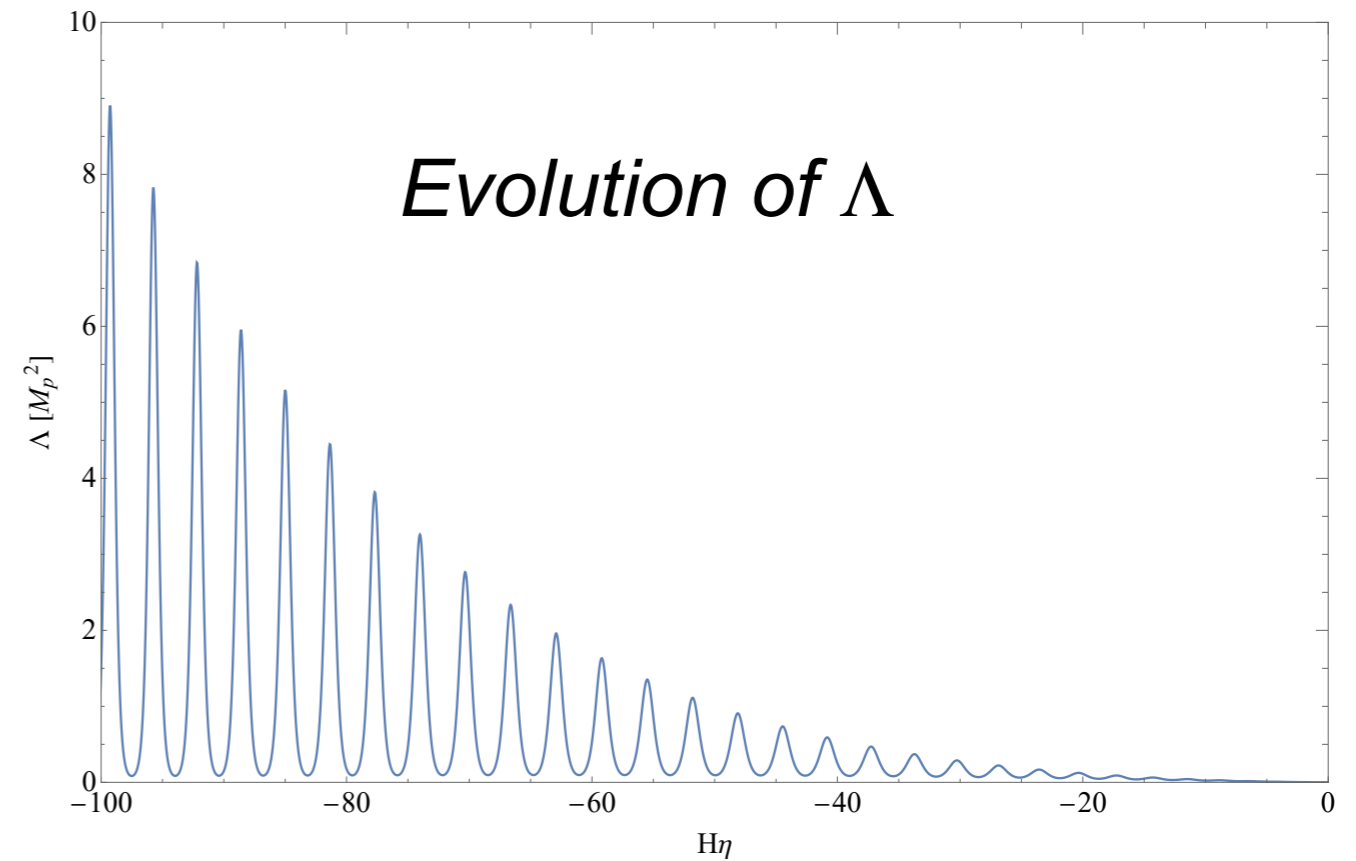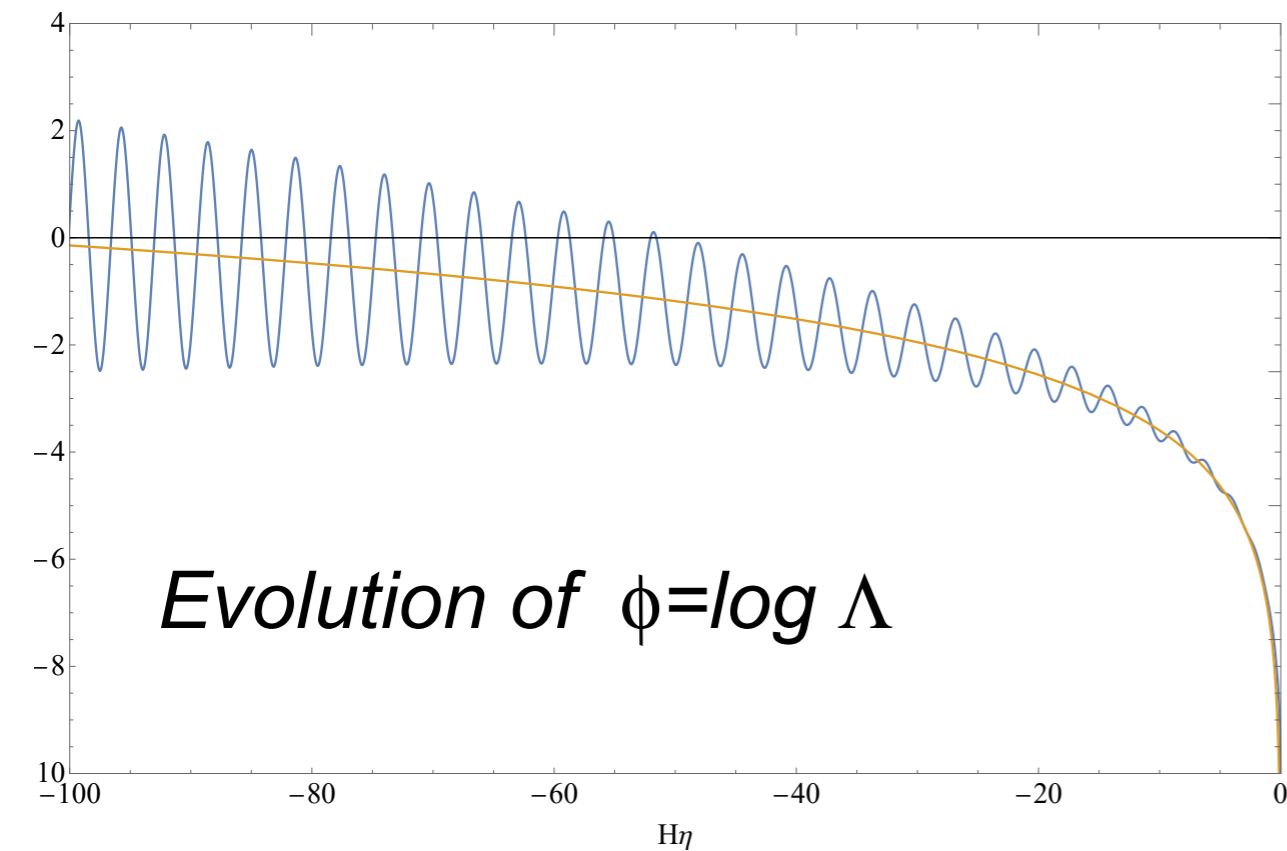
time dependent potential

$$V(\phi) = \frac{\phi}{3\tilde{c}} \left(\phi^2 - 3\tilde{b}(x) \right).$$

$$\phi = \Lambda/M_p^2 = \rho \exp[\varphi]$$

$$\tilde{b}(x) = bR\tilde{R}/M_p^4$$

Assume falloff: $R\tilde{R} \propto a^{-n}$

Evolution of $\phi = \log \Lambda$

Evolution of Λ

Note: Λ **does not change sign**

$$\Lambda \lesssim H^2 \exp\left[-\mathcal{O}(10^{-1})H^2\left(\frac{B}{\sqrt{\delta}}\right)\right]$$

Varying the initial H_0 *Robert Sims, in progress.*

$$\phi'' + \left(3\frac{a'}{a} - \frac{\phi'}{\phi}\right)\phi' + \frac{1}{\tilde{c}}\left(\frac{\phi^2 - \tilde{b}}{\tilde{\rho}_0^2}\right) = 0$$

$$\phi = \Lambda/M_p^2$$

$$\left(\frac{a'}{a}\right)^2 = \frac{\tilde{\rho} + \phi}{\tilde{\rho}_0}$$

$$\tilde{c} = 2c/9\left(M_p/H_0\right)^2$$

$$\tilde{\rho} = \rho/M_p^4$$

$$\tilde{\rho}' + 3\frac{a'}{a}\tilde{\rho}(1 + \omega) = -\phi'$$

$$x = mt$$

$$\tilde{b}(x) = bR\tilde{R}/M_p^4$$

Initial conditions:

Λ_0 =twice fixed point
H_0 fixed by fixing c-tilde
ρ_0 is then found by solving the Friemann eq.

Varying the initial H_0 _Robert Sims, in progress._

$$\phi'' + \left(3\frac{a'}{a} - \frac{\phi'}{\phi}\right)\phi' + \frac{1}{\tilde{c}}\left(\frac{\phi^2 - \tilde{b}}{\tilde{\rho}_0^2}\right) = 0$$

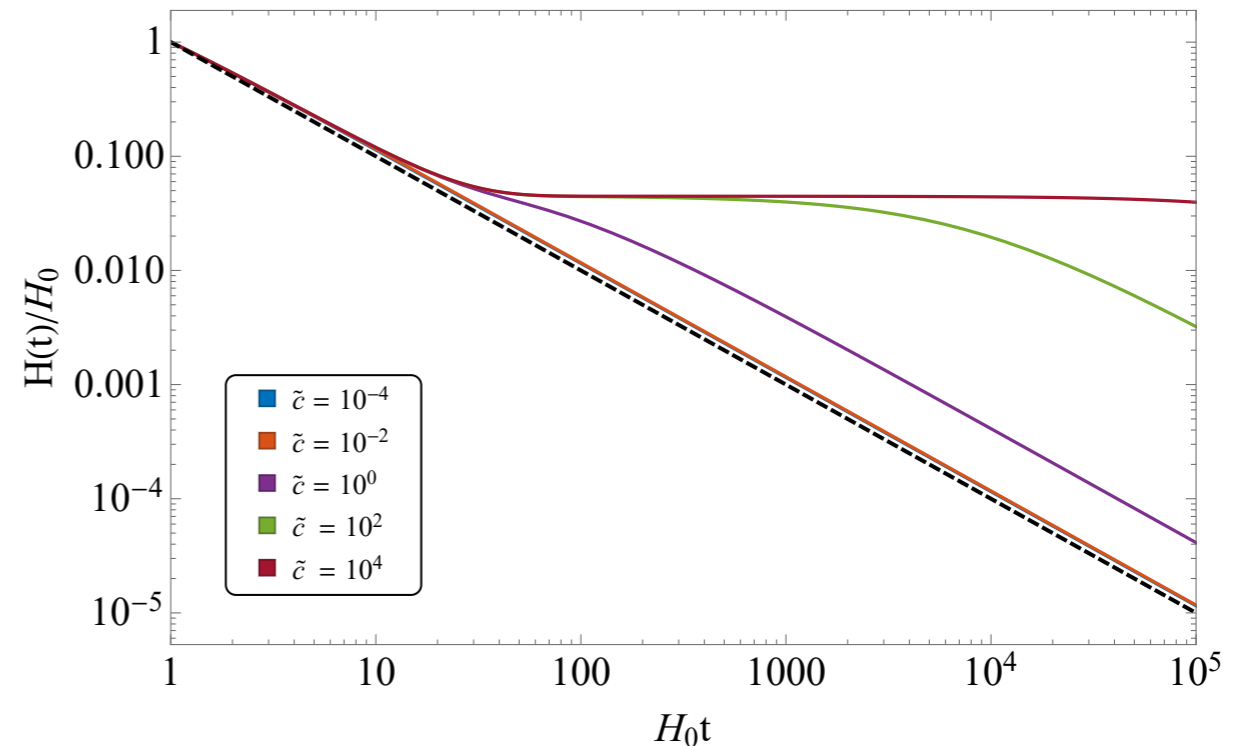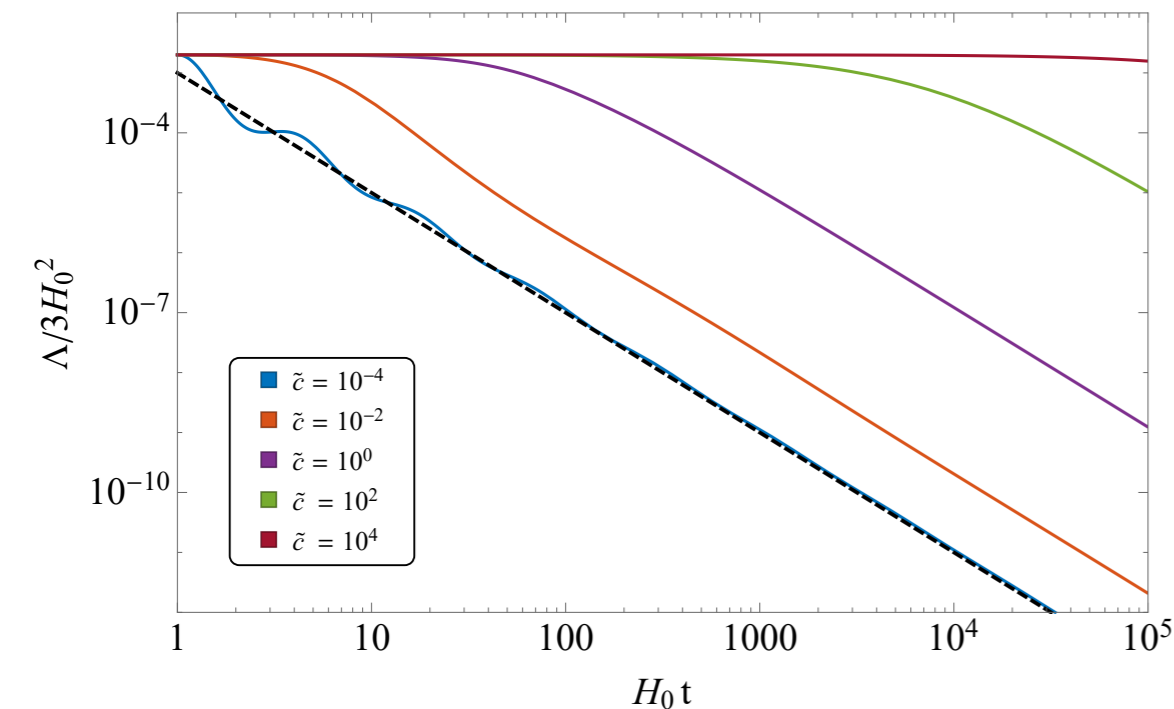$$\phi = \Lambda/M_p^2$$

$$\tilde{c} = 2c/9\left(M_p/H_0\right)^2$$

$$\left(\frac{a'}{a}\right)^2 = \frac{1}{3}\left(\frac{M_p^2}{m^2}\right)(\tilde{\rho} + \phi)$$

$$\tilde{\rho} = \rho/M_p^4$$

$$x = mt$$

$$\tilde{\rho}' + 3\frac{a'}{a}\tilde{\rho}(1+\omega) = -\phi'$$

$$\tilde{b}(x) = bR\tilde{R}/M_p^4$$

Varying the initial H_0 Robert Sims, in progress.

$$\tilde{c} = 2c/9 \left(M_p/H_0\right)^2$$