Coarse graining in spin foams [2007.01315]

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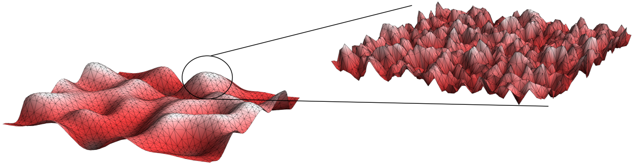
Renormalization: physics at different scales

- **Renormalization** universal tool in physics
  - "Zoom out" by relating theories at **different scales**



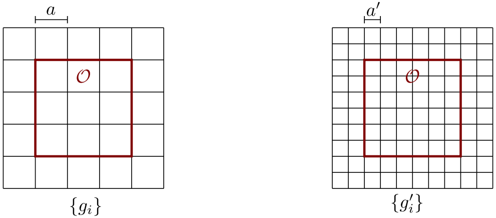
- LQG / spin foams are **bottom-up approaches**: ansatz for UV physics
- How can we connect to **observable physics**?

How can we define renormalization in a **background independent** setting?

Can it help us overcome **computational challenges**?

Renormalization in lattice gauge theory

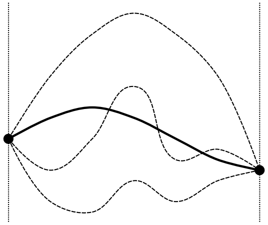
- **Renormalization group** formulated with respect to a **scale**
  - Lattice scale a in a lattice gauge theory
  - **Agnostic of physics** below scale a
- Observable $\mathcal{O}$ larger than a', a must agree for theories on a' and a

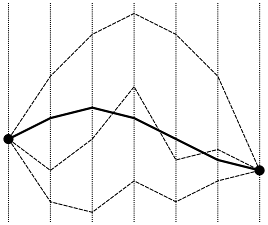


What is the interpretation in the **background independent setting**?

Outline

Spin foam gravity [Rovelli, Reisenberger, Barrett, Crane, Freidel, Livine, Krasnov, Perez, Speziale, Engle, Pereira, Kaminski...]



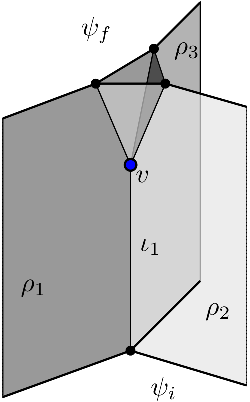


- **Path integral of geometries**
- Regulator: **Discretization** / 2-complex
  - Finitely many degrees of freedom
- **Quantum geometric** building blocks
  - (Constrained) **topological quantum field theory**
  - Discrete area spectrum
- Physical content: **Transition amplitudes**
  - Assign an amplitude $\mathcal{A} \sim e^{iS_{\text{EH}}}$ to each geometry
  - **Single building block** $\sim$ **discrete gravity** [Conrady, Freidel '08, Barrett, Dowdall, Fairbairn, Gomes, Hellmann '09, Kaminski, Kisielowski, Sahlmann '17, Liu, Han '18]
  - Quantum amplitudes (not Wick-rotated)

No reference to background geometry

Aim to implement **diffeomorphism symmetry**

Spin foam gravity - Basics

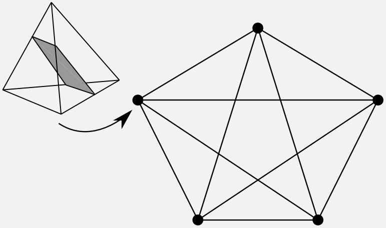


- Regulator: (dual) **2-complex** Δ^*
  - Vertices v, edges e, faces f
- Coloured with group theoretic data $\{\rho_f, \iota_e\}$
- Boundary graph $\sim$ **3D geometry**
  - **Polyhedra** $\sim$ intertwiner ι_e
  - **Area** of face $\sim$ representation ρ_f
- **Evolution**: bulk geometry
  - History interpolating between boundaries
- **Sum over all histories**
  - Sum over all ι and ρ
  - Assign amplitude to each history
- **Amplitude functionals**: $\mathcal{A}_b : \mathcal{H}_b \to \mathbb{C}$
  - From initial to final state: $\mathcal{H}_i \otimes \mathcal{H}_f^*$: $\langle \psi_f, \psi_i \rangle_{\mathcal{A}}$

Partition function and geometric interpretation

- **Amplitudes locally** assigned to building blocks

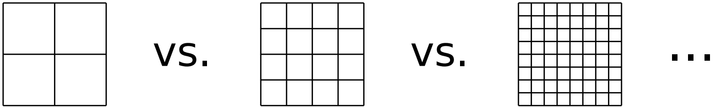
$$Z(\Delta^*) = \sum_{\rho_f, \iota_e} \prod_{f \in \Delta^*} \mathcal{A}_f(\rho_f) \prod_{e \in \Delta^*} \mathcal{A}_e(\iota_e) \prod_{v \in \Delta^*}$$



Quantum space-time as a **superposition of quantum geometric building blocks**

Open issues addressed by renormalization

- **Ambiguities** in definition of 4D models
- **Discretisation dependence / Continuum limit**
  - **Sum over 2-complexes** or **refining / coarse graining**?
  - **Diffeomorphism symmetry?** [Dittrich, Bahr '09, Bahr, S.St. '15]



Physics must be **independent** of choice of regulator!

- **Computational challenges**:
  - **Vertex amplitude:** numerical algorithm for EPRL/FK model [Dona, Fanizza, Sarno, Speziale '19, Dona, Gozzini, Sarno '20]
  - **Sum over configurations:** effective spin foam algorithm [Asanta, Dittrich, Haggard PRL '20, Asante, Dittrich, Padua-Argüelles '21]
  - **Observables**: MCMC on Lefshetz thimbles [Han, Huang, Liu, Qu, Wan '20]

Two complementary routes

- How can we ensure that **predictions** do not depend on **ambiguities**?

Summing: Group field theory [Oriti, Carrozza, Rivasseau, Ben Geloun, Gurau, Lahoche, Benedetti,...]

- Sum over all **triangulations** and **topologies**
- **Quantum field theory**: spin foam amplitudes as Feynman diagrams
- Building blocks $\sim$ **"atoms of space-time"**

Renormalizable as a (non-)perturbative QFT?

Refining / Coarse graining [Dittrich, Bahr, S.St., Delcamp, Martin-Benito, Mizera,...]

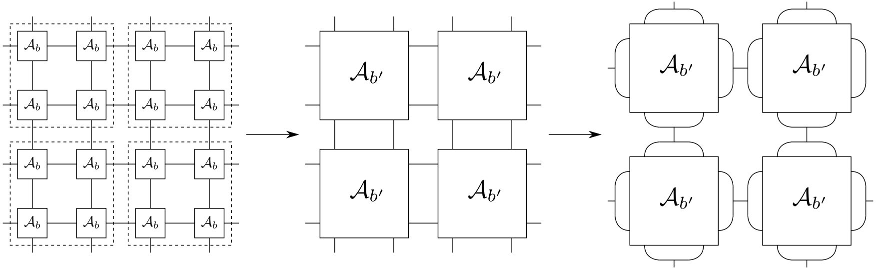
- Perform calculation for **fixed triangulation**
- **Lattice gauge theory**: 2-complex as a regulator
- **Relate theories** on different discretization?

Remove regulator in a non-trivial continuum / refinement limit?

Outline

Background independent RG [Dittrich, S.St. '13, Dittrich '14]

- **Amplitude functional** $\mathcal{A}_b : \mathcal{H}_b \to \mathbb{C}$
- Coarse graining involves **two main steps**:
  - **Summing** over **fine** degrees of freedom $\to \mathcal{A}_{b'}$ with finer boundary data
  - Define **embedding maps** $\iota_{bb'} : \mathcal{H}_b \to \mathcal{H}_{b'}$



Embedding Maps [Dittrich, S.St. '13, Dittrich '14]

- Key idea: **identify states** across Hilbert spaces
  - Represent **same transition** on different discretizations
  - **Partially ordered set** of boundaries and hence Hilbert spaces
- $\phi_b \in \mathcal{H}_b$ and $\psi_{b'} \in \mathcal{H}_{b'}$ with $b \prec b'$:

$$\phi_b \sim \psi_{b'} \quad \Longleftrightarrow \quad \iota_{b'b}(\phi_b) = \psi_{b'}$$

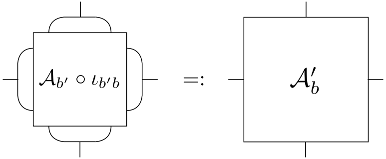
- **Inductive limit** Hilbert space $\mathcal{H} := \overline{\cup_b \mathcal{H}_b / \sim}$
  - **Cylindrical consistency**: $\iota_{b''b'} \circ \iota_{b'b} = \iota_{b''b}$ for $b \prec b' \prec b''$
  - Common refinements: embed b, b' into b'' with $b \prec b''$ and $b' \prec b''$
- **Prescription** how to add degrees of freedom
  - Ashtekar-Lewandowski vacuum [Ashtekar, Isham '92, Ashtekar, Lewandowski '95]
  - BF vacuum [Dittrich, Geiller '15, Bahr, Dittrich, Geiller '15]

Challenge to define **dynamical embedding maps**

Spin foam RG equations

- Compute **effective amplitude** $\mathcal{A}'_b$ with embedding maps:

$$\mathcal{A}'_b := \mathcal{A}_{b'} \circ \iota_{b'b}$$



$$Z = \sum_{\rho_{b'}} \prod_{b'} \mathcal{A}_{b'}(\rho_{b'}) \approx \sum_{\rho_b} \prod_b \left[\sum_{\rho_{b'}} \iota_{b'b}(\rho_{b'}, \rho_b) \mathcal{A}_{b'}(\rho_{b'}) \right] =: \sum_{\rho_b} \prod_b \mathcal{A}'_b(\rho_b)$$

Interpretation and advantages

- **Renormalization group flow** of **amplitudes** $\mathcal{A} \to \mathcal{A}' \to \mathcal{A}'' \to \ldots$ across different complexes Δ^*
- Uncover **phase diagram** / universality classes of dynamics
- Must hold for **all boundary states** $\to$ **all scales**!

Search for fixed point and **2nd order phase transition**

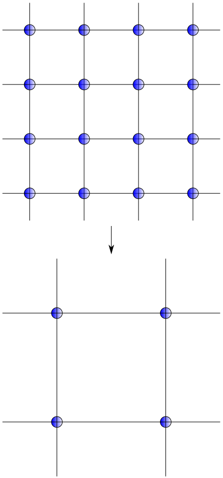
- **Prescription** for **efficient** (numerical) calculations
  - Evaluate Z in parts
  - "Use $\mathcal{A}$ for fine lattice, $\mathcal{A}'$ for coarse lattice."
  - **Approximations** / truncations necessary

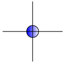
Study **coarse observables** on coarse discretisation.

Method numerically implemented via **Tensor Network Renormalization**

Outline

Tensor network renormalization [Levin, Nave '07, Gu, Wen '09]



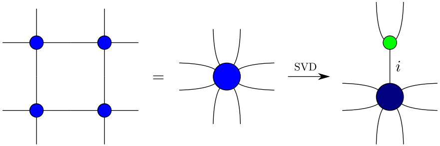
- Partition function as **tensor network**
  -  $\leftrightarrow$ local amplitude
  - Connected edges $\rightarrow$ sum over degrees of freedom
- **Local manipulations** of network:
  - Summation over fine degrees of freedom
  - Dynamical variable transformation and truncation
- **Numerical algorithm**
  - Direct summation of configurations
  - **Singular value decomposition**

Approximate same partition function as **coarse tensor** network

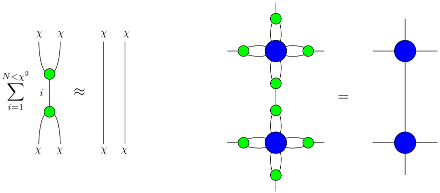
Renormalization group flow of **local** amplitudes

Dynamical embedding maps

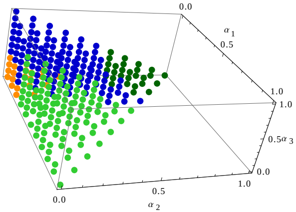
- Embedding maps via **singular value decomposition**

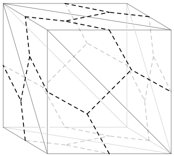


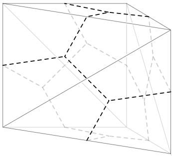
- Define and truncate **effective degrees of freedom**
  - Singular values $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_\chi \geq 0$ indicate relevance



Properties of tensor networks

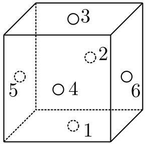


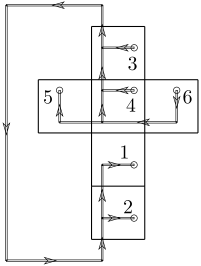




- Efficient for **oscillating amplitudes**
  - Direct sum over fine degrees of freedom
  - **Opposite to Monte Carlo** simulations
- Ideal to map out **phase diagram**
  - Attractive fixed point characterizes phase
- Require **finite dimensional** Hilbert space
  - Linear algebra operations
  - Numerical costs grow with complexity
- Previous work:
  - 2D analogue models for $\mathrm{SU}(2)_k$ [Dittrich, Martin-Benito, S.St. '13, S.St. '15, Cameron, Dittrich, Schnetter, S.St. '16]
  - Decorated tensor networks for **3D lattice gauge theories** ($\mathbb{Z}_2$ and S_3) [Dittrich, Mizera, S.St. '14, Delcamp, Dittrich '16]

Decorated TNW with Fusion Charges





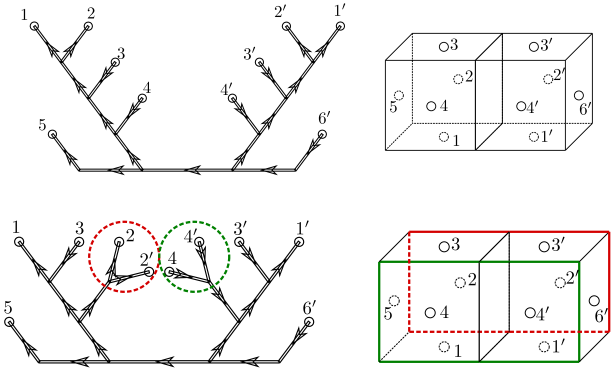
- Lattice gauge theories **challenge** TNW
  - Redundancy due to gauge d.o.f
  - Non-local assignment of variables
- **Efficient representation** of boundary data
  - Avoid **redundant / gauge** d.o.f.
  - Stable under coarse graining

Fusion basis [Dittrich, Geiller '16, Delcamp, Dittrich, Riello '16]

- **Diagonalize commuting** Ribbon operators
  - Choice encoded in fusion tree
- Measure **magnetic** and **electric** excitations
- Adapt choice of fusion tree to **coarse observables**.

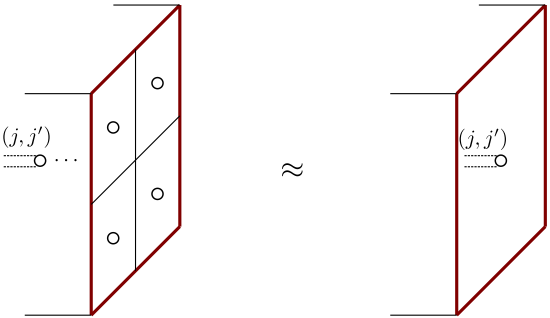
Sketch of algorithm

- **Glue cubes** together → define **effective punctures**
  - **Transform fusion** basis: **diagonal** in **coarse observables**
  - Coarse graining map defined from **singular value decomposition**

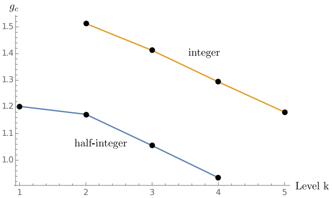


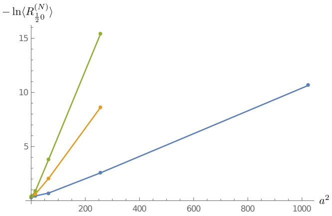
Measure coarse observables (approximately)

- Expectation value of diagonalized Ribbon operators only depend on **coarsest labels**
  - Use these labels to define **effective punctures**



Fusion basis results





- **Fusion basis** for 3D $\mathrm{SU}(2)_\mathrm{k}$ **lattice gauge theory** [Dittrich, Geiller '16, Cunningham, Dittrich, S.St. '20]
  - Phase transition: **confining** and **deconfining** phase
  - **Critical** g_c **drops** as we increase k.
  - Expectation values of Wilson loops

Hint at **no deconfining phase** for **continuous group** $\mathrm{SU}(2)$.

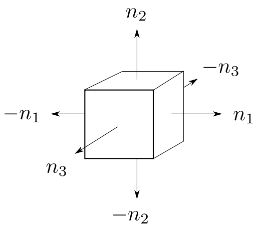
- Future projects:
  - Abelian 3D lattice gauge theory
  - **Cosmological constant** in 3D [Dittrich, S.St. w.i.p.]
  - **Further optimizations** (smaller building blocks, matrix multiplications, GPUs?)

More work necessary to apply TNW to 4D spin foams

Outline

Towards a simplified model [Bahr, S.St. '15]

- **Strategy**: study a **subset** of the full spin foam path integral
- **Quantum cuboids**: 4D Riemannian EPRL model [Engle, Pereira, Rovelli, Livine '08] on hypercubic 2-complex [Lewandowski, Kaminski, Kisielowski '09']



- **Restrict shape** of intertwiner
  - **Coherent** $\mathrm{SU}(2)$-intertwiner [Livine-Speziale '07]

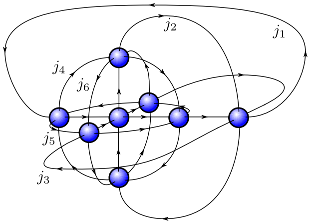
$$|\iota_{j_1,j_2,j_3}\rangle = \int_{\mathrm{SU}(2)} dg\; g \triangleright \bigotimes_{i=1}^{3} |j_i, e_i\rangle \otimes |j_i, -e_i\rangle$$

  - Peaked on the shape of a cuboid
  - e_i normal unit vectors in $\mathbb{R}^3$.

Drastic restrictions on spins and intertwiners

Use **asymptotic expansion** of full amplitude.

Semi-classical spin foam amplitudes [Bahr, S.St. '15]



- Vertex amplitude $\mathcal{A}_v$:

$$\mathcal{A}_v \sim \int_{\mathsf{SU}(2)^8} \prod_a dg_a \, e^{\sum_{ab} 2 j_{ab} \ln(\langle -\vec{n}_{ab} | g_a^{-1} g_b | \vec{n}_{ba} \rangle)}$$

- Face amplitude $\mathcal{A}_f$:

$$\mathcal{A}_f \sim (2j+1)^{2\alpha}$$

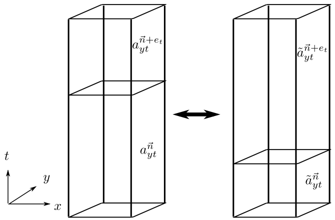
- **Stationary phase approximation**:

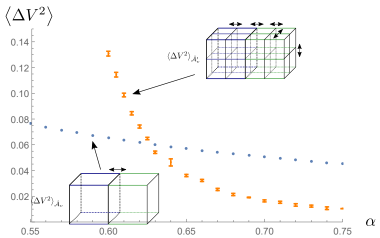
$$\hat{\mathcal{A}}_v(j_1, j_2, j_3, j_4, j_5, j_6) \sim j_i^{2\alpha} \left(\frac{1}{\sqrt{D}} + \frac{1}{\sqrt{D^*}} \right)^2$$

Flat space-time: discrete gravity / Regge action vanishes.

Model: **superpostion of hypercuboidal, flat lattices**.

Proof of principle [Bahr, S.St. PRL '16, PRD '17]





- Abelian **sub-class of diffeomorphisms**
  - Moving entire hyperplanes
  - Different subdivisions of flat space-time
- **Symmetry broken** – α dependent
- Compute **observables** on different foams

$$\langle \mathcal{O} \rangle_{\alpha}^{\text{fine}} \approx \langle \mathcal{O} \rangle_{\alpha'}^{\text{coarse}}$$

- Use this to define **renormalization group flow** $\alpha \to \alpha'$
  - Flow from “fine” (UV) to “coarse” (IR)

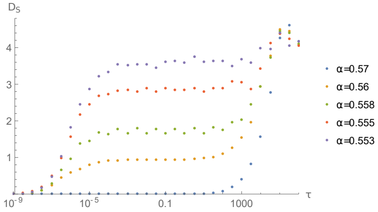
Indication for a **UV-attractive fixed point**

and restored **diffeomorphism symmetry**

Restricted spin foam models as test cases

- More **general shapes**: frusta [Bahr, Klöser, Rabuffo '17, Bahr, Rabuffo, S.St. '18]
- Beyond renormalization ideal **test case**
  - Study **observables**
  - Test new methods and develop new models

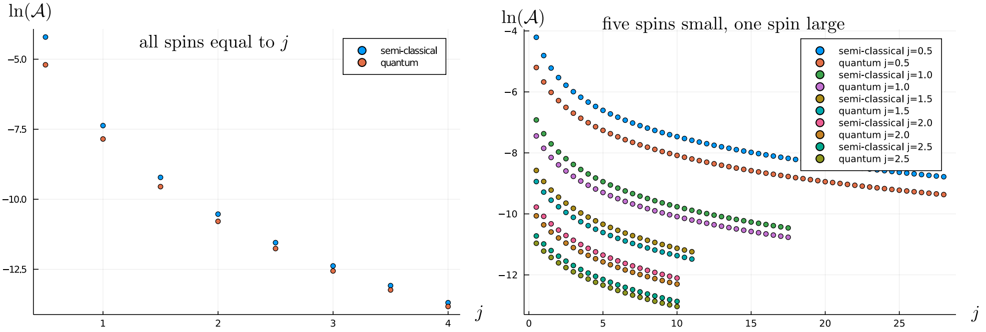
- **Diffusion process** on space-time
  - **Spectral dimension** D_S of restricted spin foam [S.St., Thürigen '18]
  - $D_S \leq 4$ due to **superposition of geometries**
  - Scaling behaviour of amplitude



- New projects:
  - **Lorentzian spin foams** (timelike polyhedra and faces) [Simao, S.St. w.i.p.]
  - Reexamine RG of cuboids beyond observables [S.St. w.i.p.]

Cuboids in the quantum regime [Allen, Girelli, S.St. w.i.p.]

- When is the leading order **asymptotic expansion** a **good approximation**?



Numerical costs much higher than for 4-simplices!

Cuboids in the quantum regime [Allen, Girelli, S.St. w.i.p.]

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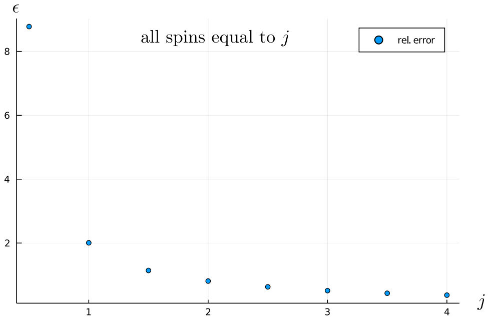
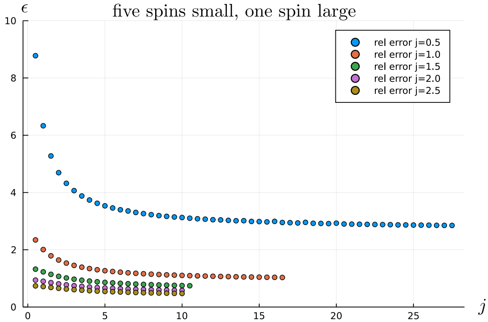
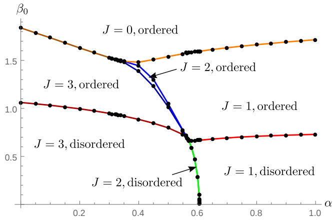




Numerical costs much higher than for 4-simplices!

Matter in spin foams

- How to **incorporate matter** in spin foam quantum gravity?
  - Matter on top of spin foam [Oriti, Pfeiffer '03, Speziale '07, Bianchi, Han, Rovelli, Wieland, Magliaro, Perini '13]
  - Unification scenarios [Crane '00, Smolin '09]
  - Massless scalar field [Lewandowski, Sahlmann '15, Kisielowski, Lewandowski '18]
- **Dynamics** of combined system?



- Toy model: **2D Ising model** coupled to an **analogue spin foam** [S.St. '15]
  - **"Minimal coupling"**: Ising coupling dependent on spin foam
  - **Renormalize** both systems (with tensor networks)
  - Phase diagram: combined phases of matter and gravity

Geometric phase transitions due to **matter coupling**

Does **quantum gravity** affect the **matter sector**? [Dona, Eichhorn, Percacci '14]

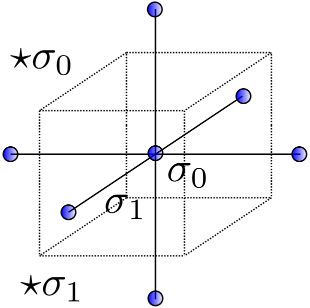
Massive scalar field coupled to a **restricted 4D spin foam**

Scalar field on an irregular lattice [Ali, S.St. w.i.p.]

- Scalar field action in terms of **forms**:

$$S = \frac{1}{2}\int d\phi \wedge *d\phi + \frac{M_0^2}{2}\int \phi \wedge *\phi \;\rightarrow\; S = \frac{1}{2}\sum_{\sigma_1}\langle d\phi|d\phi\rangle + \frac{M_0^2}{2}\sum_{\sigma_0}\langle\phi|\phi\rangle\;.$$

  - **Discrete exterior calculus** [Desbrun, Hirani, Leok, Marsden '05, Arnold, Falk, Winther '09, McDonal, Miller '10, Sorkin '75, Calcagni, Oriti, Thürigen '12]
- **Scalar field action:** $S = \frac{1}{2}\phi(\sigma_0^i)\,K_{ij}\,\phi(\sigma_0^j)$ [Hamber, Williams '93]:



$$K_{ij} = \begin{cases} \sum_{\sigma_1 \supset \sigma_0^i} \frac{V_{\star\sigma_1}}{V_{\sigma_1}} + M_0^2\, V_{\star\sigma_0} & i = j \\ -\sum_{\sigma_1 \supset \sigma_0^i} \frac{V_{\star\sigma_1}}{V_{\sigma_1}} & i \neq j \end{cases}$$

- Notation:
  - $V_{\star\sigma_d}$: $(4-d)$-volume dual to vertex σ_d
  - V_{σ_1}: length of edge σ_1.

Define **massive scalar field coupled to cuboid spin foam.**

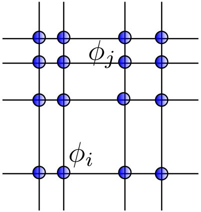
Scalar field coupled to cuboid spin foams [Ali, S.St. w.i.p.]

- **Partition function** of combined system reads:

$$Z = \int \prod_{\sigma_1} dl_{\sigma_1} \prod_{\sigma_0} d\phi(\sigma_0) \prod_{\sigma_4} \hat{\mathcal{A}}_{\sigma_4}(\alpha, \{l_{\sigma_1}\}) e^{-\frac{1}{2}\phi(\sigma_0^i)\, K_{ij}(M_0)\, \phi(\sigma_0^j)}$$

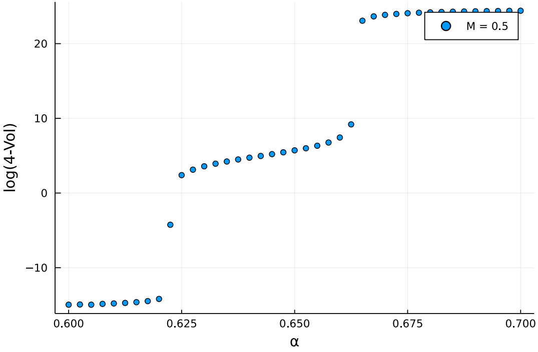
- **Observables** of combined system:
  - **Geometric** obervables: edge length, volume
  - **(Non-local) correlator** $\langle \phi\phi \rangle$ of scalar field [Ambjørn, Görlich, Jurkiewicz, Loll '12]

$$\langle \phi\phi(R) \rangle = \int \mathcal{D}[g_{\mu\nu}]\, e^{iS_{\text{EH}}} \int d^4x \sqrt{-g(x)} \int d^4y \sqrt{-g(y)}\, \langle \phi(x)\phi(y) \rangle^{[g_{\mu\nu}]}_{\text{matter}}\, \delta(R - d_{g_{\mu\nu}}(x,y))$$



Massive, free scalar field coupled to
a **superposition of hypercubodial lattices**
weighted by **spin foam amplitudes**.

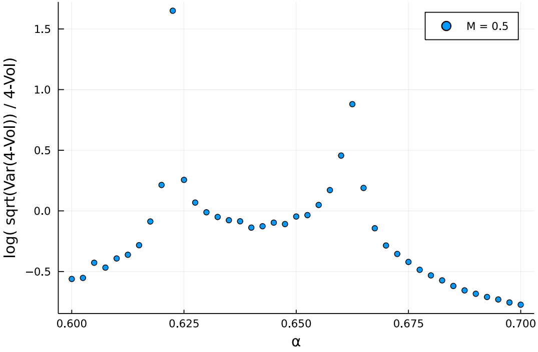
Sneak peak: Volume expectation value [Ali, S.St. w.i.p.]



Infrared divergencies of spin foams lessened by **matter**?

Phase transition of **matter-gravity system**?

Sneak peak: Volume expectation value [Ali, S.St. w.i.p.]



Infrared divergencies of spin foams lessened by **matter**?

Phase transition of **matter-gravity system**?

Summary

- Brief introduction to **spin foam models**
  - Defined on cellular complex $\Delta^* \sim$ regulator
  - **Quantum amplitudes** for **quantum geometric building blocks**
- **Background independent renormalization** via coarse graining
  - Relate transitions on different boundaries via embedding maps
  - Road towards efficient calculations (coarse observables)
- **Tensor network renormalization**
  - **Fusion basis** for 3D lattice gauge theory (for SU$(2)_k$)
  - Deconfing and confining phase for finite k
- **Restricted, semi-classical path integral**
  - Indications for **UV-attractive fixed point**
  - Vertex amplitude in **quantum regime**
  - **Massive scalar field** coupled to restricted spin foam

Coarse graining is **essential** to turn spin foams into a **computational framework**.

Outlook

- Recent **encouraging progress** in spin foam models
  - **Numerical algorithm** to compute **vertex amplitude** [Dona, Fanizza, Sarno, Speziale '19, Dona, Gozzini, Sarno '20]
  - Emerging consensus on **flatness problem** between numerics and asymptotic formula [Engle, Kaminski, Oliveira '21]
  - **Effective spin foam model** [Asanta, Dittrich, Haggard PRL '20, Asante, Dittrich, Padua-Argüelles '21]
  - **Lefshetz thimbles** and MCMC for observables [Han, Huang, Liu, Qu, Wan '20]
- Use these **new tools** in coarse graining spin foams.
- **Explore Lorentzian sector** of the theory
- Study **matter** coupled to spin foams
- **Use spin foam amplitudes as embedding maps** [Dittrich, S.St. '14]
  - Under evolution, spin foam adds / removes degrees of freedom $\rightarrow$ **dynamical embedding map**

Thank you for your attention!