Future Directions of LQG:
Canonical Approach – Foundations

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Definition of QG

Minimal wishlist:

- QG: QFT methods applied to GR (geometry and matter)
- QFT on diff. mfd. $M \cong \mathbb{R} \times \sigma$
  - not on background (M, g_0)
  - in the continuuum, not some discretised theory
  - physical, Lorentzian signature
- fundamental, not only effective QFT, hence non-pert.
- Technically: Algebra of operators H driving dynamics densely represented on HS rep. $(\pi, \mathcal{H})$ of C(A)CR
- should reduce to matter QFT in CST (M, g) when fluctuations about g are small
- should repair classical GR singularities

What is LQG (not)?

- Attempt at concrete, math. rigorous implementation of above programme
- Tools: gauge covariant (Wilson loop) variables familiar from LQCD define C(A)CR, rep. theory of operators
- In contrast to LQCD: continuum theory, background indep. rep. rather than Fock rep. [Ashtekar, Baez, Fleischhack, Isham, Lewandowski, Marolf, Mourao, Okolow, Rovelli, Sahlmann, Smolin, TT, ..]

Status of (L)QG

- Applying perturbative QFT methods to QG dynamics yields 3 problems
  - A. UV divergences [Goroff, Marcus, Sagnotti 80's]
  - B. perturbation series likely to have zero radius of convergence
  - C. no miraculous cancellations: ∞ no of counter terms needed: not renormalisable, not predictive
- Performance of non-perturbative LQG dynamics (including matter [TT])
  - A. No UV divergences
  - B. No perturbation series
  - C. quantisation ambiguities: ∞ no of math. consistent q'ions, not predictive
- Message:
  Promising improvements, but LQG dynamics not yet in final shape

Problem and Solution Strategies

- Technical problem: quantum algebra of constraints C_I closes $[\hat{C}_I, \hat{C}_J] = i\, \hat{C}_K\, \hat{g}_{IJ}{}^K$, however
  - A. anomalous quantum structure "constants": $\{C_I, C_J\} = C_K\, f_{IJ}{}^K$ but $\hat{f}_{IJ}{}^K \neq \hat{g}_{IJ}{}^K$
  - B. $\hat{C}_I$ carries "memory" of diffeo cov. regulation scheme after regulator removal (ambiguities)
- Solution tracks (selection)
  - Avoid quantum constraints: reduced phase space q'ion (RPQ) using material reference systems [Dittrich, Domagala, Giesel, Husain, Kaminski, Kuchar, Lewandowski, Pawlowski, Rovelli, Smolin, TT,..]
  - Correct quantum structure "constants":
    - Quantum Ham. constr. $\sim$ Quantum Spat. Diffeo Constr. generated by "quantum vector field" (QVF) [Laddha, Tomlin, Varadarajan]
    - Hamiltonian operator renormalisation (HOR) and algebraic LQG [Bahr, Giesel, Lang, Liegener, TT]

Challenges and future tasks

TO DO:

- RPQ: proof of principle with scalar reference fields but phenomenologically viable? More realistic matter and/or geometric clocks [Bodendorfer, Duch, Giesel, Herzog, Kaminski, Lewandowski, Singh, Swiezewski,..]?
- QVF: promising results for PFT, ($U(1)^3$ truncation of) Euclidian vacuum GR but extension to matter, cosm. const., Lorentzian signature?
- HOR: Wilsonian renormalisation translated into canonical language, designed to remove ambiguities but also avoids anomalies?

Summary, selected recent results

- To turn LQG into a commonly accepted, predictive QG theory: quantum dynamics must be settled
- Why is this highly topical for the whole community? Recent examples:
  - Possible phenomenological relevance: Connection between LQG and LQC: Ambiguities have strong impact on fine details of bounce [Agullo, Assanioussi, Dapor, Kaminski, Liegener, Pawlowski,..]
  - Connection canonical/covariant (spin foam model SFM) approach: Are SFM amplitudes really the (rigging map) inner products between QEE solutions (correct measure) [Buffenoir, Henneaux, Noui, Roche]? Few results in LQC [Ashtekar, Campiglia, Henderson] or using coherent states [Han, Liu,..]
  - Semiclassical limit: Contact with QFT in CST? [Agullo, Ashtekar, Assanioussi, Dapor, Elizaga Navacues, Gambini, Lewandowski, Martin Benito, Mena Marugan, Pullin, Sahlmann, Schander, TT, ...]
  - Long range correlations in solutions of QEE [Smolin 90's]? Strong indication: existence of ∞ dim. kernel (not normalisable wrt spat. diff. inv. inner prod.) that displays propagation [TT, Varadarajan 21]
- Recent results required tedious, complex calculations, encourage modern computational (ML, QC) techniques, [Han, Liegener, Laflamme, Mielczarek, Rudnicki, Stottmeister..] c.f. SFM tensor network renormalisation [Bahr, Dittrich, Steinhaus, ...]