

A Planck Luminosity Bound in Quantum Gravity

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Motivation: Planck Luminosity

Four spacetime dimensions are special

Observation: in $D = 4$ spacetime dimensions, the Planck power (luminosity) is independent of \hbar

$$\mathcal{L}_P = \frac{m_P c^2}{t_P} = \frac{\hbar^{\frac{D-4}{D-2}} c^{\frac{2D+2}{D-2}}}{G^{\frac{2}{D-2}}}.$$

Imagine BH merger. Only in $D = 4$, can we have a formula

$$\mathcal{L}_{peak} = \mathcal{L}_P \times f(\text{scale-independent observables}).$$

In $D = 4$, peak luminosity is intensive.

Humanity has come close to observing such power

$$\mathcal{L}_P = \frac{c^5}{G} \approx 3,63 \times 10^{52} \text{W},$$
$$\mathcal{L}_{peak} \Big|_{\text{GW170729}} \approx 4 \times 10^{49} \text{W}.$$

Take the **quadrupole formula** (in $D = 4$) for the emitted power

$$\mathcal{L}_{\text{GW}} \sim \frac{G}{c^5} (\ddot{I})^2 \sim \frac{G}{c^5} (M\omega^3)^2 R^4$$

Virial theorem links average kinetic energy with potential energy

$$\bar{E}_{kin} = -\frac{1}{2}\bar{E}_{pot} \Rightarrow M\omega^2 R^2 \sim \frac{GM^2}{R}.$$

Emission can only happen before forming a black hole, $R \gtrsim 2GM/c^2$

$$\boxed{\mathcal{L}_{\text{GW}} \sim \frac{c^5}{G} \left(\frac{GM}{c^2 R} \right)^5 \lesssim \mathcal{L}_{\text{P}}.}$$

If there is such a bound, it can only exist in $D = 4$.

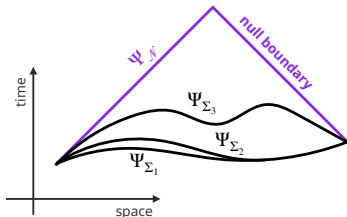
- Likely **invisible in S -matrix approach**, where we have a formal perturbative expansion wrt. $\ell_P = \sqrt{8\pi\hbar G/c^3}$.
- For same reason, **no such bound exists on classical radiative phase space** at null infinity either. Recall: $\mathcal{L} = \frac{c^5}{4\pi G} \oint_{S_2} d^2\Omega |\dot{\sigma}_{crit.}^{(0)}|^2$.
- Strategy: explore \mathcal{L}_P in non-perturbative quantum gravity.

This sounds strange. There is no \hbar . How does quantum theory enter?

- 1 **Analogy: Lifetime $\tau_{ij} = \Gamma_{ij}^{-1}$ of atomic transitions.** We can measure transition rates in terms of radiation field alone: $\Gamma \approx P/\Delta E$, where P and ΔE are the radiated power and emitted energy of a pulse of radiation with classical $T_{\mu\nu}^{rad.}$. No \hbar at this stage. But **atomic theory** and **energy conservation** tell us that ΔE can only come in discrete units. It is at this stage that \hbar enters.
- 2 **Here: LQG discreteness of spectra based on parity violating γ -action.** Holst term $e_\alpha \wedge e_\beta \wedge F^{\alpha\beta}$ allows to compactify certain directions on phase space (**edge mode variables**) without violating Lorentz invariance. Compactification leads to discretization in conjugate variables (area flux). Radiated power linked through **quasi-local conservation laws** to LQG quanta of geometry. LQG discreteness of area affects IR observables at infinity.

Quantum gravity in causal regions

- Initial data: three-metric h_{ab} and extrinsic curvature $\tilde{\pi}^{ab} \sim K_{ab} \sim \dot{h}_{ab}$.
- Constraints $\mathcal{H}[h, \tilde{\pi}] = 0$ and $\mathcal{H}_a[h, \tilde{\pi}] = 0$ generate gauge redundancies on phase space.
- Gauge redundancies: states on $\Sigma_1, \Sigma_2, \dots$ are gauge equivalent.
- **Basic idea:** Characterize the entire gauge equivalence class $[\Psi_{\Sigma_i}]$ by pushing the time-evolution (gauge) to its extreme.
- The boundary of the future Cauchy development of Σ_i is a null (light-like) boundary. **Quantize gravity at light-like boundary.** Problem simplifies. Less constraints.



Step 1: Parametrisation of Null Geometry

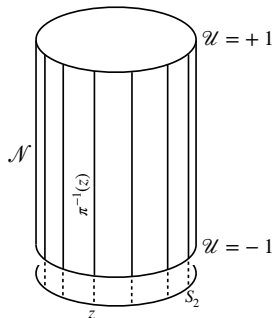
Signature (0++) metric.

$$\varphi_{\mathcal{N}}^* g_{ab} = q_{ab} = \delta_{ij} e^i_a e^j_b, \quad i, j = 1, 2.$$

Parametrisation of the co-dyad:

$$e^i = \Omega S^i_m e^m_{(o)}.$$

- **Conformal factor** Ω parametrizes the overall scale.
- **$SL(2, \mathbb{R})$ -Holonomy** S^i_m determines the shape degrees of freedom.
- **Fiducial background dyad** $e^j_{(o)}$, e.g.
 $(e^1_{(o)}, e^2_{(o)}) = (d\vartheta, \sin \vartheta d\varphi)$



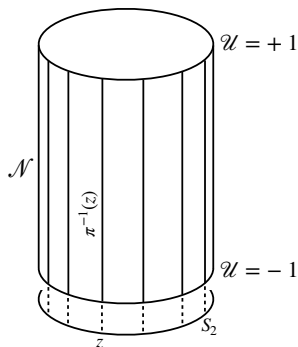
We consider a null strip \mathcal{N} with two corners. No unique clock along \mathcal{N} .
 Convenient choice

Boundary condition at $\partial\mathcal{N} = \mathcal{C}_+ \cup \mathcal{C}_-$,

$$\mathcal{U}(\partial\mathcal{N}, z, \bar{z}) = \pm 1,$$

Affinity proportional to expansion

$$\partial_{\mathcal{U}}^b \nabla_b \partial_{\mathcal{U}}^a = -\frac{1}{2} \left(\Omega^{-2} \frac{d}{d\mathcal{U}} \Omega^2 \right) \partial_{\mathcal{U}}^a$$



Comments:

- $\delta\mathcal{U} \neq 0$, but $\delta\mathcal{U}|_{\partial\mathcal{N}} = 0$.
- Teleological boundary condition: clock lands at the upper (lower) boundary assuming the value $\mathcal{U} = \pm 1$.

Upon fixing the direction of the null rays, we are left with two constraints.

Raychaudhuri equation: $G_{ab}\ell^a\ell^b = 0$

$$\frac{d^2}{d\mathcal{U}^2}\Omega^2 = -2\sigma\bar{\sigma}\Omega^2.$$

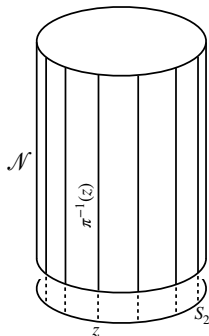
Transport equation for $SL(2, \mathbb{R})$ parallel propagator

$$\frac{d}{d\mathcal{U}}S = \left(\varphi J + (\sigma\bar{X} + \text{cc.})\right)S.$$

Note:

- $U(1)$ connection φ on \mathcal{N} .
- σ is the shear (free radiative data) on \mathcal{N} .
- The matrices J, X and \bar{X} are $SL(2, \mathbb{R})$ generators:

$$J^2 = -\mathbb{1}, \quad [J, X] = -2iX, \quad [X, \bar{X}] = iJ.$$



Step 2: Symplectic Structure (from Holst's Action)

In $D = 4$, there are **two Lorentz scalars** that we can build from the curvature tensor:

$$R[\omega, e] = R^{\alpha\beta}{}_{ab}[\omega]e_{\alpha}{}^a e_{\beta}{}^b,$$

$$R^*[\omega, e] = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu}R_{\alpha\beta ab}[\omega]e_{\mu}{}^a e_{\nu}{}^b \approx 0.$$

Note: If the torsionless equation is satisfied, R^* vanishes.

In the first-order formalism, there are *two coupling constants* at linear order in the curvature,

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} d^4v \left[R - \frac{1}{\gamma} R^* \right] + \text{boundary terms.}$$

G is Newton's constant, γ is the Barbero-Immirzi parameter.

The Barbero-Immirzi parameter is akin to the θ -angle in QCD.

Symplectic structure $\Theta = p \, dq$ determines Heisenberg relations $[\hat{q}, \hat{p}] = i\hbar$. Starting from the γ -action, we obtain the symplectic structure on \mathcal{N} .

$$\begin{aligned}\Theta_{\mathcal{N}} = & -\frac{1}{16\pi\gamma G} \int_{\partial\mathcal{N}} d^2v_o \, \Omega^2 \, \text{Tr} (J \, dS S^{-1})_+ \\ & - \frac{1}{8\pi G} \int_{\mathcal{N}} d\mathcal{U} \wedge d^2v_o \, \Omega^2 \, \text{Tr} ((\sigma_I \bar{X} + \bar{\sigma}_I X) \mathbb{D} S_I S_I^{-1})_+ \\ & - \frac{1}{8\pi G} \int_{\mathcal{N}} d\mathcal{U} \wedge d^2v_o \, d\mathcal{U} \left(\frac{d^2}{d\mathcal{U}^2} \Omega^2 + 2\sigma_I \bar{\sigma}_I \Omega^2 \right).\end{aligned}$$

Key Observations:

- Local $SL(2, \mathbb{R})$ symplectic structure on \mathcal{N} .
- **First line:** initial data for the Raychaudhuri equation.
- **Second line:** radiative data. Akin to gauged $SL(2, \mathbb{R})/U(1)$ sigma model.
- **Third line:** Clock variable is conjugate to the constraint.

Starting from the γ -action, we obtain the symplectic structure on \mathcal{N} .

$$\begin{aligned} \Theta_{\mathcal{N}} = & -\frac{1}{16\pi\gamma G} \int_{\partial\mathcal{N}} d^2v_o \Omega^2 \operatorname{Tr} (J dS S^{-1}) + \\ & -\frac{1}{8\pi G} \int_{\mathcal{N}} d\mathcal{U} \wedge d^2v_o \Omega^2 \operatorname{Tr} ((\sigma_I \bar{X} + \bar{\sigma}_I X) \mathbb{D} S_I S_I^{-1}) + \\ & -\frac{1}{8\pi G} \int_{\mathcal{N}} d\mathcal{U} \wedge d^2v_o d\mathcal{U} \left(\frac{d^2}{d\mathcal{U}^2} \Omega^2 + 2\sigma_I \bar{\sigma}_I \Omega^2 \right). \end{aligned}$$

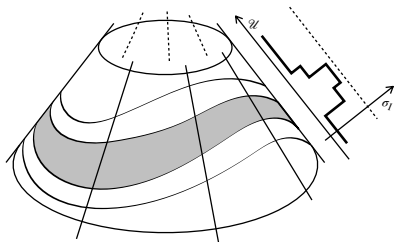
Technical remarks:

- Interaction picture: $S(\mathcal{U}) = \exp(\int^{\mathcal{U}} d\mathcal{U}' \varphi(\mathcal{U}') J) S_I = \underbrace{e^{J\Delta}}_{U(1)} S_I$.
- Dressed field space derivative: $\mathbb{D} = d - d\mathcal{U} \frac{d}{d\mathcal{U}}$ [Carrozza, Hoehn, Riello, Gomes, Freidel, Ciambelli, Kabel, Brukner, ...]
- **Barbero-Immirzi parameter** only appears in corner term.
- Looks innocent but highly non-linear constraint between shear σ_I and conformal factor $\Omega = \Omega[\sigma_I, \Omega_{\pm}]$.

Truncation to Impulsive Null Initial Data

Split the shear profile (free radiative data) into sequence of pulses.

- Each pulse represents a **quasi-local graviton**.
- **Strategy:** Quantise each pulse, then glue many pulses together.
- Full non-linear and non-perturbative quantization of impulsive data.
 - Non-linear constraint between shear and conformal factor.
 - Due to flux-balance laws, any alteration of the quantisation of the corner data can also alter the quantisation of the radiative data.
 - Barbero-Immirzi parameter γ affects the spectrum of radiated power.



We consider a single pulse

$$\frac{d}{d\mathcal{U}}\sigma_I = 0, \quad \sigma_I \equiv \sigma_I(\vartheta, \varphi).$$

Conformal factor: Ω

$$\begin{aligned} \Omega^2(\mathcal{U}, \vartheta, \varphi) &= \frac{E_+(\vartheta, \varphi) + E_-(\vartheta, \varphi)}{2} \frac{\cos(\sqrt{2\sigma_I\bar{\sigma}_I}\mathcal{U})}{\cos(\sqrt{2\sigma\bar{\sigma}})} + \\ &+ \frac{E_+(\vartheta, \varphi) - E_-(\vartheta, \varphi)}{2} \frac{\sin(\sqrt{2\sigma_I\bar{\sigma}_I}\mathcal{U})}{\sin(\sqrt{2\sigma_I\bar{\sigma}_I})}, \end{aligned}$$

for initial conditions

$$\Omega^2(\mathcal{U} = \pm 1, \vartheta, \varphi) = E_{\pm}(\vartheta, \varphi).$$

$SL(2, \mathbb{R})$ holonomy: $S = e^{J\Delta} S_I$

$$S_I = HS_-, \quad S(\mathcal{U} = \pm 1) = S_{\pm},$$

$$H = \text{ch}(\sqrt{\sigma_I\bar{\sigma}_I}(\mathcal{U} + 1)) \mathbb{1} + \frac{1}{\sqrt{\sigma_I\bar{\sigma}_I}} (\bar{\sigma}_I X + \sigma_I \bar{X}) \text{sh}(\sqrt{\sigma_I\bar{\sigma}_I}(\mathcal{U} + 1)).$$

Double role of shear: Frequency of oscillation in Ω , boost angle in H .
Recall $A = \Gamma + \gamma K$, Immirzi parameter compactifies extrinsic curvature.

Pull-back of symplectic potential $\Theta_{\mathcal{N}}$ to configurations of a single pulse.
 Finite-dimensional mechanical system on each null ray.

$$\begin{aligned} \{a(\mathbf{z}), \bar{a}(\mathbf{z}')\} &= i \delta^{(2)}(\mathbf{z}|\mathbf{z}'), \\ \{b(\mathbf{z}), \bar{b}(\mathbf{z}')\} &= i \delta^{(2)}(\mathbf{z}|\mathbf{z}'), \\ \{c(\mathbf{z}), \bar{c}(\mathbf{z}')\} &= 2i \delta^{(2)}(\mathbf{z}|\mathbf{z}') L(\mathbf{z}), \\ \{L(\mathbf{z}), c(\mathbf{z}')\} &= -i \delta^{(2)}(\mathbf{z}|\mathbf{z}') c(\mathbf{z}), \\ \{L(\mathbf{z}), \bar{c}(\mathbf{z}')\} &= +i \delta^{(2)}(\mathbf{z}|\mathbf{z}') \bar{c}(\mathbf{z}), \end{aligned}$$

with $\mathbf{z} = (\vartheta, \varphi)$ and

$$\begin{aligned} \{c(\mathbf{z}), U(\mathbf{z}')\} &= XU(\mathbf{z}) \delta^{(2)}(\mathbf{z}|\mathbf{z}'), \\ \{\bar{c}(\mathbf{z}), U(\mathbf{z}')\} &= \bar{X}U(\mathbf{z}) \delta^{(2)}(\mathbf{z}|\mathbf{z}'), \\ \{L(\mathbf{z}), U(\mathbf{z}')\} &= -\frac{1}{2}JU(\mathbf{z}) \delta^{(2)}(\mathbf{z}|\mathbf{z}'). \end{aligned}$$

Highly non-linear definition of canonical variables.

$SL(2, \mathbb{R})$ holonomy

$$U = e^{\gamma \ln(\tan(\sqrt{2\sigma\bar{\sigma}})/\sqrt{2\sigma\bar{\sigma}})J} S_-.$$

Heisenberg charges

$$a = \frac{\sqrt{E_+}}{\sqrt{8\pi\gamma G}} \operatorname{ch}(2\sqrt{\sigma\bar{\sigma}}) e^{-i[\Delta_+ + 2\gamma \ln(\cos(\sqrt{2\sigma\bar{\sigma}}))]},$$
$$b = \frac{\sqrt{E_+}}{\sqrt{8\pi\gamma G}} \operatorname{sh}(2\sqrt{\sigma\bar{\sigma}}) e^{i\left[\Delta_+ + \phi + 2\gamma \ln\left(\frac{\sin(\sqrt{2\sigma\bar{\sigma}})}{\sqrt{2\sigma\bar{\sigma}}}\right)\right]}.$$

Area operators are mere number operators.

$$\begin{aligned}E_- + E_+ &= 16\pi\gamma G (L + a\bar{a}), \\E_- - E_+ &= 16\pi\gamma G (L + b\bar{b}).\end{aligned}$$

Shear operator: quotient of the two oscillators

$$\text{th}(2\sqrt{\sigma\bar{\sigma}}) = \sqrt{\frac{\bar{b}b}{\bar{a}a}}.$$

There are operator ordering ambiguities, but they are mild.

One residual constraint

$$c \bar{a} \bar{b} = -\gamma (L + \bar{a}a) \sqrt{2 \bar{a}a \bar{b}b} \tan \left(\frac{1}{2\sqrt{2}} \ln \left(\frac{\sqrt{\bar{a}a} + \sqrt{\bar{b}b}}{\sqrt{\bar{a}a} - \sqrt{\bar{b}b}} \right) \right) - i \bar{a}a \bar{b}b.$$

Key Observations:

- **Left hand side:** ladder operators.
- **Right hand side:** number operators.
- $SL(2, \mathbb{R})$ Casimir commutes with the constraint.
- This is a simple recurrence relation.

Critical shear and physical states

Physical states can be classified by the value of the $SL(2, \mathbb{R})$ Casimir.

$$L^2 - c\bar{c} = \frac{1}{8\pi G} \left[\frac{1}{4\gamma^2} (E_- - E_+)^2 + \right. \\ \left. - \frac{1}{\gamma^2} \text{sh}^2(2\sqrt{\sigma\bar{\sigma}}) E_+ E_- - \frac{1}{2} (E_+ + E_-)^2 \tan^2(\sqrt{2\sigma\bar{\sigma}}) \right].$$

Discrete series unitary representations: $L^2 > c\bar{c}$, the $U(1)$ generator $|L|$ is bounded from below, recurrence relations terminate.

Continuous series unitary representations: $L^2 < c\bar{c}$, the $U(1)$ generator $|L|$ is unbounded, recurrence relations do not terminate. Physical states are quantum superpositions that contain caustics.

For small shear, the critical value is

$$|\sigma_{crit.}|^2 = \frac{1}{4} \frac{(E_- - E_+)^2}{\gamma^2(E_+ + E_-)^2 + 4E_+E_-} + \mathcal{O}(|\sigma_{crit.}|^3).$$

Use Bondi $1/r$ -expansion to evaluate $|\sigma_{crit.}|$ near future null infinity.

Note: $E_{\pm} = \mathcal{O}(r^2)$, but $E_- - E_+ = 2r(\Delta u)(\vartheta, \varphi) + \mathcal{O}(r^0)$.

■ Bondi time u vs. teleological time \mathcal{U} :

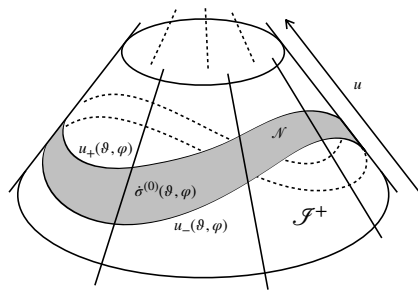
$$\partial_{\mathcal{U}}^a = \frac{(\Delta u)(\vartheta, \varphi)}{2} \partial_u^a + \mathcal{O}(r^{-1}).$$

■ Asymptotic shear:

$$\sigma_{crit.} = \frac{(\Delta u)(\vartheta, \varphi)}{2} \frac{\dot{\sigma}_{crit.}^{(0)}}{r} + \mathcal{O}(r^{-2}).$$

■ Critical news:

$$\dot{\sigma}_{crit.}^{(0)} = \frac{1}{\sqrt{\gamma^2 + 1}}.$$



Critical luminosity

$$\mathcal{L}_{crit.} = \frac{c^5}{4\pi G} \oint_{S_2} d^2\Omega |\dot{\sigma}_{crit.}^{(0)}|^2 = \frac{\mathcal{L}_P}{\gamma^2 + 1}.$$

Outlook and Summary

Summary:

- 1 Non-perturbative quantisation of impulsive null initial data.
- 2 Quantum geometry includes radiative data and corner data.
- 3 Planck power separates discrete and continuous $SL(2, \mathbb{R})$ representations.

Key messages:

- 1 Above the Planck power, physical states contain configurations where shear is unbounded. Contradiction with implicit assumption of smooth \mathcal{I}^+ above the Planck power. Possibly further problems for trans-Planckian states: semi-classical limit, inner product.
- 2 Possibility of rich phenomenology. We expect that Planck luminosity plays for quantum gravity in $D = 4$ same role as Planck mass in $D = 3$. Possible relation with relative locality, non-commutative geometry, deformed symmetries, ...